# Written Exam at the Department of Economics summer 2017 

# Industrial Organization 

Final Exam

June 1, 2017
(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

## This exam paper consists of three pages in total, including this one

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

## Question 1: A vertical relationship and downstream duopoly

In the country of Pantalonia, jeans are produced by a monopoly firm called $U$. The jeans are sold to the final consumers by two retailers, $D_{1}$ and $D_{2}$. We model the interaction between the two retailers as a quantity-competition game with (potentially) differentiated products. In particular, the inverse demand function of retailer $D_{1}$ and $D_{2}$, respectively, is given by

$$
p_{1}=1-q_{1}-d q_{2}
$$

and

$$
p_{2}=1-d q_{1}-q_{2},
$$

where $p_{i}$ and $q_{i}$ (for $i=1,2$ ) are $D_{i}$ 's price and output, respectively, and $d \in[0,1]$ is a parameter.

If retailer $D_{i}$ chooses the output $q_{i}$, then this firm must pay the amount $w q_{i}$ to $U$, where $w$ is the per-unit wholesale price. The profit of $D_{i}$ can thus be written as $\pi_{i}=\left(1-w-q_{i}-d q_{-i}\right) q_{i} .{ }^{1}$ The upstream firm $U$ is assumed not to have any production costs and its profit can therefore be written as $\pi_{U}=\left(q_{1}+q_{2}\right) w$.

The timing of events is as follows.
(i) The upstream firm $U$ chooses $w$.
(ii) The retailers $D_{1}$ and $D_{2}$ observe $w$ and then, simultaneously and independently, choose their own output $q_{i}$.
Each firm's objective is to maximize the own profit.
(a) Assume that $d=0$. Solve for the (subgame perfect) equilibrium values of $q_{1}$ and $q_{2}$. What are the equilibrium values of the prices $p_{1}$ and $p_{2}$ ?

[^0]Now there is an important change in the jeans market in Pantalonia: The upstream firm $U$ and one of retailers, $D_{1}$, merge and become one single firm, called $\widehat{U}$. The new firm is active both at stage (i) and (ii), where it chooses $w$ and $q_{1}$ respectively. Its profit equals $\pi_{\widehat{U}}=\left(1-q_{1}-d q_{2}\right) q_{1}+w q_{2}$; that is, the merged firm can potentially earn profit from two sources: from selling to the final consumers (at the retail price $p_{1}$ ) and from selling to retailer $D_{2}$ (at the wholesale price $w$ ). The profit of $D_{2}$ can, as before, be written as $\pi_{2}=\left(1-w-q_{2}-d q_{1}\right) q_{2}$.

The timing of events is as follows.
(i) The merged firm $\widehat{U}$ chooses $w$.
(ii) Retailer $D_{2}$ observes $w$. Then $\widehat{U}$ and $D_{2}$, simultaneously and independently, choose their own outputs $q_{1}$ and $q_{2}$, respectively.

Each firm's objective is to maximize the own profit.
(b) Allow for any $d \in[0,1]$. Solve for the (subgame perfect) equilibrium values of $q_{1}$ and $q_{2}$.

You are encouraged to answer the question below also if you have failed to solve parts (a) and (b).
(c) For each case $d=0$ and $d=1$, discuss verbally whether we should expect total surplus in the market to be largest under integration (i.e., the market structure in the (b) part) or under non-integration (i.e., the market structure in the (a) part). Your discussion may draw on your results under (a) and (b), but you should not show any mathematics here. For each case $d=0$ and $d=1$, can you identify any effects that are present in the models and which make a merger between $U$ and $D_{1}$ good or bad, respectively, for total surplus? Explain these effects.

## Question 2: Subsidizing

## Monopoly Firm's Sales

Consider a market in which there is a single (monopoly) firm. Let $q$ denote the firm's output and $p$ denote its price. Market demand is given by $p=a-q$, where $a$ is a positive constant. The firm's marginal cost is constant and equals $c$ (with $0 \leq c<a)$. There are no fixed costs. The firm's sales are subsidized by the government: If a firm sells $q$ units, it receives the amount $s q$ from the government, where $s$ is the per-unit subsidy. The firm wants to maximize its profits, $\pi$, where

$$
\pi=\text { revenues from sales }- \text { costs }+s q .
$$

The firm's choice variable is its output.
(a) Solve for the optimal output, given some $s \geq 0$.

Now suppose that the subsidy $s$ is chosen by the government, and that the government wants to maximize $W$, where

$$
W=\text { consumer surplus }+\pi-s q
$$

The timing of events is as follows. First the government chooses $s$, subject to $s \geq 0$; then, after having observed $s$, the firm chooses $q$.
(b) Solve for the subgame perfect Nash equilibrium of this game.

Next assume that $a=11$ and $c=1$. Also assume that the government's objective is given not by $W$ but by $V$, where

$$
V=\text { consumer surplus }+z \pi-s q
$$

and $z$ is a constant satisfying $1 / 2<z<1$. All other parts of the game are unchanged.
(c) Show that the market price, given the optimal $s$, is decreasing in $z$; that is, the more the government cares about the firm's profit, the lower is the market price. Can you explain this (perhaps counterintuitive) result?

## End of Exam


[^0]:    ${ }^{1}$ The notation $-i$ means "not $i$ ". So if $i=1$, then $-i=2$; and if $i=2$, then $-i=1$.

