# Written Exam at the Department of Economics summer 2017 

# Industrial Organization 

Final (Resit) Exam

August 24, 2017
(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

## This exam paper consists of three pages in total, including this one

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

## Attempt both questions.

Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

## Question 1: Downstream Cournot competition in a vertically related market

In the country of Malumia, apples are produced by a monopoly firm called $U$. The apples are sold to the final consumers by $n$ retailers: $D_{1}, D_{2}, \ldots, D_{n}$. We model the interaction between the $n$ retailers as a Cournot game. In particular, each retailer $i$ chooses its output $q_{i}$, and then the market price is determined by the inverse demand function $p=1-Q$, where $Q \stackrel{\text { def }}{=} \sum_{j=1}^{n} q_{j}$. If a retailer chooses the output $q_{i}$, then it must pay the amount $w q_{i}$ to $U$, where $w$ is the per-unit wholesale price. The profit of retailer $D_{i}$ can thus be written as $\pi_{i}=(1-w-Q) q_{i}$. The upstream firm $U$ is assumed not to have any production costs and its profit can therefore be written as $\pi_{U}=w Q$.

The timing of events is as follows.
(i) The upstream firm $U$ chooses $w$.
(ii) The retailers $D_{1}, D_{2}, \ldots, D_{n}$ observe $w$ and then, simultaneously and independently, choose their own output $q_{i}$.

Each firm's objective is to maximize the own profit.
(a) Solve for the (subgame perfect) equilibrium value of $q_{i}$ (this value will be the same for all retailers). What is the equilibrium value of the price $p$ ? Does $p$ converge to the marginal production cost (which equals zero) as $n$ approaches infinity?

Now there is an important change in the apple market in Malumia: The upstream firm $U$ and one of the retailers, $D_{1}$, merge and become one single firm, called $\widehat{U}$. The new firm is active both at
stage (i) and (ii), where it chooses $w$ and $q_{1}$ respectively. Its profit equals $\pi_{\widehat{U}}=(1-Q) q_{1}+w \sum_{j=2}^{n} q_{j}$; that is, the merged firm can potentially earn profit from two sources: from selling to the final consumers (at the retail price $p$ ) and from selling to the other $n-1$ firms (at the wholesale price $w$ ). The profit of each of the other (non-merged) retailers $D_{i}$ (for $i \in\{2,3, \ldots, n\}$ ) can, as before, be written as $\pi_{i}=(1-w-Q) q_{i}$.

The timing of events is as follows.
(i) The merged firm $\widehat{U}$ chooses $w$.
(ii) The merged firm $\widehat{U}$ and the $n-1$ downstream firms $D_{2}, D_{3}, \ldots, D_{n}$ observe $w$ and then, simultaneously and independently, choose their own output $q_{i}$.

Each firm's objective is to maximize the own profit.
(b) Solve for the (subgame perfect) equilibrium values of $q_{i}$ for all the $n$ firms in the market (there will be one such value for the merged firm $\widehat{U}$ and one common such value for all the other firms). What is the equilibrium value of the price $p$ ?

You are encouraged to answer the question below also if you have failed to solve parts (a) and (b).
(c) Compare the equilibrium prices $p$ in the models that you solved in the (a) and (b) parts. Is the model with or without the merger more competitive, in the sense of giving rise to a lower consumer price? Can you identify any effects that are present in the models and which make a merger between $U$ and $D_{1}$ pro-competitive and anti-competitive, respectively? Explain these effects.

## Question 2: Price competition with and without capacity constraints

In a market there are two firms that produce a homogeneous good and compete in prices. Firm $i$ 's (for $i=1,2$ ) price is denoted by $p_{i}$. Total market demand is given by $Q(p)=1-p$, where $p=\min \left\{p_{1}, p_{2}\right\}$. Firm $i$ 's marginal cost is constant and equal to $c \in[0,1)$. Hence, firm 1's demand is

$$
Q_{1}\left(p_{1}, p_{2}\right)=\left\{\begin{array}{cl}
1-p_{1} & \text { if } p_{1}<p_{2} \\
\frac{1}{2}\left(1-p_{1}\right) & \text { if } p_{1}=p_{2} \\
0 & \text { if } p_{1}>p_{2}
\end{array}\right.
$$

and analogously for firm 2 . The firms' profit functions are therefore

$$
\begin{gathered}
\pi_{1}\left(p_{1}, p_{2}\right)=\left(p_{1}-c\right) Q_{1}\left(p_{1}, p_{2}\right) \quad \text { and } \\
\pi_{2}\left(p_{1}, p_{2}\right)=\left(p_{2}-c\right) Q_{2}\left(p_{1}, p_{2}\right)
\end{gathered}
$$

The game is one-shot (i.e., it is played only once), and the firms choose their prices simultaneously.
(a) Solve for the Nash equilibria of the model described above.

Now modify the model described above as follows. Assume that the firms are capacity constrained: Firm $i$ 's marginal cost is constant and equal to $c$ up to the production level $\bar{q}_{i}$; however, any quantity larger than $\bar{q}_{i}$ is impossible to produce. The capacities do not exceed one-third:

$$
\bar{q}_{1} \in\left(0, \frac{1}{3}\right] \quad \text { and } \quad \bar{q}_{2} \in\left(0, \frac{1}{3}\right] .
$$

Assume efficient rationing and that $c=0$ (exactly as we did in the lecture and as in the textbook).
(b) Prove that both firms charging the price

$$
p^{*}=1-\bar{q}_{1}-\bar{q}_{2}
$$

is a Nash equilibrium.
(c) Explain (i) what kind of model Kreps and Sheinkman (Bell Journal of Economics, 1983) studied and (ii) what result they could show. Also, (iii) discuss the limitations and implications of their analysis and result

## End of Exam

