Written Exam at the Department of Economics summer 2021

Industrial Organization

Final (Resit) Exam

August 13, 2021

(three-hour closed-book exam)

Answers only in English.

This exam question consists of five pages in total

Falling ill during the exam
If you fall ill during an examination at Peter Bangsvej, you must:

- submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

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You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Attempt both questions.
Make sure that you explain all the steps of your analysis and that you define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Collusion with fluctuating, asymmetric, and persistent cost

Consider the following variation of the Rotemberg-Saloner model. In a market there are two identical, profit-maximizing firms, indexed by $i \in \{1, 2\}$. The firms produce and sell a homogeneous good, and they compete in prices. There are infinitely many discrete time periods t (so $t = 1, 2, 3, \ldots$), and at each t the firms simultaneously choose their respective price, p_i^t. The firms' common discount factor is denoted by $\delta \in \left[\frac{2}{3}, 1\right)$.

As the good is homogeneous, demand is a function of the lowest price, $p^t = \min\left\{p_1^t, p_2^t\right\}$. Moreover, demand is inelastic and normalized to one:

$$Q\left(p^t\right) = \begin{cases} 1 & \text{if } p^t \leq 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Each firm has a constant marginal cost $c_i^t \in [0, 1)$. In particular, firm 1's marginal cost is known to always equal zero: $c_1^t = 0$ for all t. However, the value of firm 2's marginal cost, in a given period t, depends on the period t state: in a low state, $c_2^t = 0$; and in a high state, $c_2^t = c \in (0, 1)$. It is assumed that

$$\Pr\left[c_2^{t+1} = c \mid c_2^t = c\right] = \Pr\left[c_2^{t+1} = c \mid c_2^t = 0\right] = \frac{1}{2}.$$

In words, with probability one-half, firm 2's cost parameter is the same in the next period as in the current period; and with the same probability, the cost parameter switches to its other possible value. If the firms charge the same price in a low-state period (i.e., when also their costs are the same), then they share the demand equally; moreover, if the firms charge the same price in a high-state period (i.e., when firm 1 has a lower cost than firm 2), then all demand goes to firm 1.[1] The firms can observe the rival firm's choice of price once it has been made. Moreover, the firms can observe the current period's cost realization before choosing their price. However, the realizations of firm 2's cost in future periods are not known to the firms.

The best possible collusive outcome for the two firms would be if they, in all periods, charged the prices $p_1^t = p_2^t = 1$ and then shared their joint profits between them in some way that made both parties content. Given the prices $p_1^t = p_2^t = 1$, the firms' joint profits would equal one, both in a low state and in a high state. Suppose firm 1 is allocated the shares $\alpha_L \in [0, 1]$ and $\alpha_H \in [0, 1]$ of these joint profits in a low and in a high state, respectively. Then firm 1's and firm 2's collusive per-period profits in a low-state period can be written as $\pi_1^c = \alpha_L$ and $\pi_2^c = 1 - \alpha_L$, respectively; and in a high-state period, these profits can be written as $\pi_1^c = \alpha_H$ and $\pi_2^c = 1 - \alpha_H$, respectively.

Consider now the following *grim trigger strategy*: Each firm i starts out charging the price $p_i^c = 1$ (regardless of the state). However, if there has been any deviation from that behavior by either one of the firms in any previous period, then both firms play $p^n = 0$ in a low state and $p^n = c$ in a high state. These prices yield the per-period one-shot Nash equilibrium profits

$$\pi_1^n = \pi_2^n = 0 \tag{2}$$

in a low-state period, and

$$\pi_1^n = c, \qquad \pi_2^n = 0 \tag{3}$$

in a high-state period.

[1] This assumption about how to break ties can be justified by the fact that, if we think of the consumers as players of the game, the behavior is part of an equilibrium (since the consumers are indifferent from which firm to buy when the prices are the same).

In order to investigate under what conditions the grim trigger strategy specified above is part of a subgame-perfect Nash equilibrium, we can use the methodology that we employed in the course when studying collusion with unobservable actions (the Green-Porter model). Thus, let V_i^L (V_i^H, respectively) denote firm i's expected present-discounted future stream of profits at the point in time when it is choosing the price and when being in a low state (high state, respectively). V_1^L and V_1^H must satisfy the following two equations:

$$V_1^L = \alpha_L + \frac{\delta}{2}\left(V_1^L + V_1^H\right), \qquad V_1^H = \alpha_H + \frac{\delta}{2}\left(V_1^L + V_1^H\right). \tag{4}$$

Solving these equations for V_1^L and V_1^H yields

$$V_1^L = \frac{(2-\delta)\alpha_L + \delta\alpha_H}{2(1-\delta)}, \qquad V_1^H = \frac{\delta\alpha_L + (2-\delta)\alpha_H}{2(1-\delta)}. \tag{5}$$

The corresponding expressions for firm 2 can be derived in a similar manner, leading to:

$$V_2^L = \frac{(2-\delta)(1-\alpha_L) + \delta(1-\alpha_H)}{2(1-\delta)}, \qquad V_2^H = \frac{\delta(1-\alpha_L) + (2-\delta)(1-\alpha_H)}{2(1-\delta)}. \tag{6}$$

For our analysis, we also need expressions for the value of each firm's present-discounted stream of profits if being in the trigger strategy's punishment phase (i.e., if the firms play the one-shot Nash equilibrium in all periods). Denote these expressions by $V_{i,L}^n$ and $V_{i,H}^n$, for firm i. We clearly have $V_{2,L}^n = V_{2,H}^n = 0$, as firm 2's per-period profits in the one-shot Nash equilibrium are zero. One can also show that

$$V_{1,L}^n = \frac{\delta c}{2(1-\delta)}, \qquad V_{1,H}^n = \frac{(2-\delta)c}{2(1-\delta)}. \tag{7}$$

(a) Derive the expression for $V_{1,L}^n$ stated in (7).

One can show that the trigger strategy specified above is part of a subgame-perfect Nash equilibrium if, and only if, the following four conditions hold:

$$\alpha_L \geq \psi_1^L(\alpha_H), \qquad \text{where } \psi_1^L(\alpha_H) \stackrel{\text{def}}{=} \frac{2(1-\delta) + \delta c - \delta\alpha_H}{2-\delta}, \tag{8}$$

$$\alpha_L \geq \psi_1^H(\alpha_H), \qquad \text{where } \psi_1^H(\alpha_H) \stackrel{\text{def}}{=} \frac{2(1-\delta) + \delta c - (2-\delta)\alpha_H}{\delta}, \tag{9}$$

$$\alpha_L \leq \psi_2^L(\alpha_H), \qquad \text{where } \psi_2^L(\alpha_H) \stackrel{\text{def}}{=} \frac{\delta(2-\alpha_H)}{2-\delta}, \tag{10}$$

$$\alpha_L \leq \psi_2^H(\alpha_H), \qquad \text{where } \psi_2^H(\alpha_H) \stackrel{\text{def}}{=} \frac{2\delta + 2(1-\delta)c - (2-\delta)\alpha_H}{\delta}. \tag{11}$$

(b) Derive the conditions stated in (10) and (11), i.e., the conditions $\alpha_L \leq \psi_2^L(\alpha_H)$ and $\alpha_L \leq \psi_2^H(\alpha_H)$. As the notation indicates, these conditions refer to the incentives of firm 2. To "derive the conditions" here means to show rigorously that firm 2 does not have an incentive to deviate unilaterally from the trigger strategy if, and only if, (10) and (11) hold.

Let $A \subset [0,1]^2$ denote the set of (α_L, α_H) such that the four conditions (8)–(11) are satisfied. Moreover, assume that

$$c < \frac{2(1-\delta)}{2-\delta}. \tag{12}$$

(c) What values of α_L and α_H maximize firm 1's expected stream of profits, $\frac{1}{2}\left(V_1^L + V_1^H\right)$, subject to the constraint $(\alpha_L, \alpha_H) \in A$?

Question 2: Vertically related firms and RPM

In a market there are two vertically related monopoly firms. The upstream firm (firm U) produces its good using a constant-returns-to-scale technology with marginal cost equal to zero. The firm chooses a linear wholesale price, denoted by w. The downstream firm (firm D) is a retailer and sells the good that the upstream firm produces to the final consumers. The demand of the final consumers is either "high", meaning that $Q(p) = 1 - p$ (where p denotes price), or "low", meaning that there is no demand at all. The probability that demand is high equals $\frac{e}{a+e}$, where $a \in \left(0, \frac{1}{4}\right)$ is an exogenous parameter and $e \geq 0$ is an effort level chosen by firm D. The cost of exerting effort level e is equal to e. Firm D does not have any costs on top of the effort cost and the cost of buying the good from firm U at the (per-unit) wholesale price w. The firms try to maximize their expected profits.

The sequence of events is as follows. (i) Firm U chooses w. (ii) Knowing w, firm D chooses p and e. (iii) Demand is realized. If demand is low, there is no trade and D pays nothing to U (but incurs the effort cost e). If demand is high, D pays $(1-p)w$ to U (and incurs the cost e).

Thus, the objective functions of U and D can be written as

$$\pi^U = (1-p)w\frac{e}{a+e}$$

and

$$\pi^D = (1-p)(p-w)\frac{e}{a+e} - e,$$

respectively.

Answer the following questions:

(a) Solve for the subgame-perfect equilibrium values of p and e.

- You do not need to show that the second-order conditions are satisfied (and you will not get any credit if you nevertheless do that).

Suppose the firms integrate and become one single firm. One can show that then the subgame-perfect equilibrium values of the consumer price and the effort level are given by $p^I = 1/2$ and $e^I = \sqrt{a}\left(1 - 2\sqrt{a}\right)/2$, respectively.

(b) Are the values of p and e that you derived in part (a) larger or smaller than, or equal to, the corresponding values under integration stated above (i.e., p^I and e^I)? Explain the economic logic for why we have those relationships. Answer verbally only. Moreover, would you expect expected consumer surplus to be largest under integration or under non-integration? Spell out your reasons and the logic. Answer verbally only. *You are encouraged to attempt these questions even if you have not been able to solve part (a).*

Suppose now that, as under (a), the firms are not integrated. Moreover, the retail price p is now chosen *not* by firm D at stage 2, but by firm U at stage 1 (we can interpret this as resale price maintenance, RPM). Everything else in the model is unchanged. Denote the equilibrium values of the price and the effort level in this model by p^R and e^R, respectively.

(c) Derive an expression for p^R and relate this to p^I. Moreover, solve sufficiently much of the model in order to learn which one of the statements (i)-(iv) below is true.

(i) $e^R < e^I$ for all $a \in \left(0, \frac{1}{4}\right)$.

(ii) $e^R > e^I$ for all $a \in \left(0, \frac{1}{4}\right)$.

(iii) $e^R = e^I$ for all $a \in \left(0, \frac{1}{4}\right)$.

(iv) Whether e^R is smaller or larger than e^I depends on the value of a.

You do not need to show that the second-order conditions are satisfied (and you will not get any credit if you nevertheless do that).

End of Exam