ANSWERS: Exam Labour Economics, Spring 2015

Nikolaj Harmon and Daniel le Maire

Part I - Food, robots and the demand for labor (max 1200 words)

Imagine that in 2016 the following two things happen at once: 1) An exceptionally good harvest causes food prices to drop by one percent worldwide. 2) A new, Chinese robot rental firm enters the world market causing rental prices for fully-automatic, computer-controlled robots to drop by one percent worldwide.

Using data on the mining and restaurant industries, you then observe the following:

- In 2015 employee salaries made up about 40 % restaurants' total costs, while the other 60 % were made up of food (ingredient) costs.
- In 2015 employee salaries made up about 40 % of mining firms' total costs, while the other 60 % were made up of rental costs for fully-automatic robots.
- In 2016 the total number of meals sold at restaurants is the same as in 2015.
- In 2016 the total number of raw materials sold by mining firms is also the same as in 2015.

A

Example answer:

From the question, we see that restaurants produce meals using two inputs: labor and food. Similarly, mining firms produce raw materials using two inputs: labor and rented robots. If the price of one input (food or robots) falls, this will affect how much is demanded of the other input at any given price. From labor demand theory, we know that there are two potential effects: substitution effects and scale effects. Substitution effects stem from the fact that when the relative price of labor vs. food/robots changes, firms will want change the input mix that they use to produce a given amount of output. Scale effects stem from the fact that when input prices change, firms may want to produce more or less output and therefore may require more or less inputs. Since we see from the data that the change in input prices does not lead to a change in output in either of the restaurant or mining industry, however, there are no scale effects here. Any change in factor demands will therefore come from how firms respond to changes in input prices at a given level of production, which is captured by the conditional factor demand functions.

From labor demand theory we know that (in the case of two inputs) cross-price elasticities of conditional factor demands are positive, that is a decrease in the price of one factor causes a decrease in the demand for the other factor. As the price of renting robots and buying food decreases, we can therefore conclude that both the demand for mining and restaurant employees has decreased in 2016.

In examining the relative size of the percentage changes in demand, we can utilize a formula for the cross-price elasticity of conditional demand. If we let $\hat{\eta}$ denote the elasticity of labor demand with respect to the non-labor input, let s denote the labor share of total cost, let σ denote the elasticity of substitution between labor and food/robots and let subscripts r and m denote the restaurant and mining industry, we have the following formulas regarding the percentage change in labor demand:

$$\hat{\eta_r} = (1 - s_r)\sigma_r$$
$$\hat{\eta_m} = (1 - s_m)\sigma_m$$

To make a prediction regarding the relative size of the percentage change in labor demand in the restaurant and mining industry, we need to determine which of the two elasticities above are bigger in magnitude. From the data in the question we know that $s_r = s_m = 0.4$ so the magnitude of the two effects will depend on the elasticities of substitution in the two industries, σ_r and σ_m.

The elasticity of substitution is defined as the change in the ratio of the conditional factor demands in response to a one percent change in the relative prices of the two inputs and is a measure of how easy it is to substitute between the two factors given the production technology of the firm. The case $\sigma = 0$ corresponds to the case where firms can not substitute at all and requires a fixed amount of each factor to produce an additional unit of output. The case $\sigma = \infty$ corresponds to the case where the firm can always replace one unit of one factor by some fixed amount of the other factor without changing the output level.

Focusing on the two industries in question, it seems like the mining industry might have a very high elasticity of substitution between labor and robots. Almost any (manual) task performed by a mining employee should be possible to transfer to a (possibly quite complicated) robot. Conversely, it seems likely that the elasticity of substitution is more limited in the restaurant industry. Having workers be more careful in cutting and preparing the raw ingredients will most likely make it possible to have less wasted raw ingredients so some substitution is clearly possible, however, in the end you will need at least 100 g of ingredients to produce 100 g of a meal. This suggests that $\sigma_m > \sigma_r$, which from the formulas above suggest a larger elasticity in the mining industry than in restaurant industry. This in turn suggests that the demand for mining employees dropped more than the demand for restaurant employees in 2016 (in percentage terms).

B

Example answer:

If restaurants had sold more meals in 2016 than in 2015, then the demand for restaurant employees would have been affected by a scale effect as described under A. This effect is the difference between conditional and unconditional labor demand so to analyze the effect of the drop in food prices on labor demand in the restaurant industry, we can now look at the unconditional labor demand. In particular, if we assume that the increase in number of meals sold is due to the drop in food prices (it could in principle be due to other things), we have a useful formula that decomposes the total effect of food price changes on unconditional labor demand. Letting η_r denote the elasticity of unconditional labor demand with respect to the food price, letting ρ_r denote the elasticity of output with respect to the food price and letting γ_r denote the elasticity of labor demand with respect to output, we have:

$$\eta_r = \hat{\eta}_r + \rho_r \cdot \gamma_r$$

This expression shows that the percentage change in labor demand following a one percent drop in food prices is now equal to the change found in A, $\hat{\eta}_r$, plus an additional term capturing the effect of increased production in the restaurant industry. Since ρ_r is known to always be negative (higher input price means less production) and γ_r is known to always be positive (higher production means more input demand), this additional term is negative and tends to offset the effect found in part A. In this alternative scenario there would either have been a smaller percentage drop in restaurants' labor demand in 2016 or restaurants' labor demand would actually have increased in 2016 as restaurants hire more workers to meet their higher production target.

Part II - Job Search and hiring costs (max 1000 words)

Consider the (Diamond-Mortensen-)Pissarides model. Assume that there exist two types of hiring costs. Whereas h_1 is the flow costs of having a vacancy, h_2 is a one-off fixed cost which is paid immediately when the employment spell begins. This implies that the value of an employed job, Π_e, and the value of a vacant job, Π_v, are given by

$$r\Pi_e = y - w + q\left(\Pi_v - \Pi_e\right) \tag{1}$$

and

$$r\Pi_v = -h_1 + m\left(\theta\right)\left(\Pi_e - h_2 - \Pi_v\right) \tag{2}$$

where y denotes the productivity, w the wage, q the job destruction rate, and $m\left(\theta\right)$ is the rate at which vacancies are filled. This rate depends on the labor market tightness, θ. There is free-entry in vacancy-creation.

The fixed hiring costs do not alter the Bellman equations for an unemployed worker and employed worker and they are given by, respectively,

$$rV_u = z + \theta m\left(\theta\right)\left(V_e - V_u\right) \tag{3}$$

and

$$rV_e = w + q\left(V_u - V_e\right) \tag{4}$$

where z is the value of leisure and unemployment benefits net of search costs and $\theta m\left(\theta\right)$ is the job arrival rate.

The fixed hiring cost has the implication that the surplus is given by

$$S = V_e - V_u + \Pi_e - \Pi_v - h_2 \tag{5}$$

Finally, the equilibrium unemployment, u, is given by

$$u = \frac{n+q}{n+q+\theta m\left(\theta\right)}$$

where n is the growth rate of the labor force.

A

B

C

D

E

F

G

Example answers (all questions):

A: Using the free-entry condition, we can write the two Bellman equations as

$$\begin{aligned} r\Pi_e &= y - w - q\Pi_e \Leftrightarrow \\ \Pi_e &= \frac{y-w}{r+q} \end{aligned}$$

and

$$\begin{aligned} 0 &= -h_1 + m(\theta)\left[\Pi_e - h_2\right] \Leftrightarrow \\ \Pi_e &= \frac{h_1}{m(\theta)} + h_2 \end{aligned}$$

Equating the two and eliminating Π_e, we arrive at

$$\frac{y-w}{r+q} = \frac{h_1}{m(\theta)} + h_2 \tag{6}$$

The l.h.s. is the expected benefits of having a match and the r.h.s. is the expected total hiring costs.

Increasing h_1 and h_2 increase the r.h.s., so ceteris paribus, $m(\theta)$ needs to increase. Since $m(\theta)$ is the number of matches per vacancy, we have that $m'(\theta) < 0$, so increasing h_1 or h_2 implies that the labor market tightness θ needs to decrease. This seems intuitive: when the costs of creating vacancies are larger, fewer vacancies will be created.

B: The total expected costs are given by $\frac{h_1}{m(\theta)} + h_2$. The first term measures the variable costs of hiring a worker. These costs depend on the how long time the firm needs to wait before filling their vacancy, whereas the h_2 is unrelated to how long time the firm needs to wait before filling the vacancy. As it takes longer to fill vacancies when θ is high, the variable costs will be procyclical, whereas the fixed costs, obviously, are completely acyclical. Compared to the model without fixed hiring costs, the hiring costs therefore fluctuate less over the business cycle relative to its average level.

C: The worker side is unchanged and we have that

$$V_e - V_u = \gamma S \tag{7}$$

and

$$V_e - V_u = \frac{w - rV_u}{r+q} \tag{8}$$

Subtracting $r\Pi_v$ from both sides of the Bellman equation for Π_e gives

$$\begin{aligned}
r\Pi_e - r\Pi_v &= y - w + q\left(\Pi_v - \Pi_e\right) - r\Pi_v \Leftrightarrow \\
r\left(\Pi_e - \Pi_v\right) + q\left(\Pi_e - \Pi_v\right) &= y - w - r\Pi_v - qh_2 \Leftrightarrow \\
\Pi_e - \Pi_v &= \frac{y - w - r\Pi_v}{r+q} \qquad (9)
\end{aligned}$$

Using equations (8), (9) and (5), we can write

$$\begin{aligned}
V_e - V_u + \Pi_e - \Pi_v &= \frac{w - rV_u}{r+q} + \frac{y - w - r\Pi_v}{r+q} \Leftrightarrow \\
V_e - V_u + \Pi_e - \Pi_v &= \frac{y - rV_u}{r+q} \Leftrightarrow \\
S &= \frac{y - (r+q)\,h_2 - rV_u}{r+q} \qquad (10)
\end{aligned}$$

Using equation (7), (8) and (10), we can write

$$\begin{aligned}
V_e - V_u &= \gamma S \Leftrightarrow \\
\frac{w - rV_u}{r+q} &= \gamma \frac{y - (r+q)\,h_2 - rV_u}{r+q} \Leftrightarrow \\
w &= rV_u + \gamma\left(y - (r+q)\,h_2 - rV_u\right)
\end{aligned}$$

Using the Bellman equation for an unemployed worker and equation (10), we can write

$$\begin{aligned}
rV_u &= z + \theta m(\theta)\left(V_e - V_u\right) \Leftrightarrow \\
rV_u &= z + \theta m(\theta)\,\gamma S \Leftrightarrow \\
rV_u &= z + \theta m(\theta)\,\gamma\left[\frac{y - (r+q)\,h_2 - rV_u}{r+q}\right] \Leftrightarrow \\
(r+q)\,rV_u &= (r+q)\,z + \gamma\theta m(\theta)\left[y - (r+q)\,h_2 - rV_u\right] \Leftrightarrow \\
rV_u &= \frac{(r+q)\,z + \gamma\theta m(\theta)\left[y - (r+q)\,h_2\right]}{r+q+\gamma\theta m(\theta)}
\end{aligned}$$

Finally, substitute rV_u in the wage equation to obtain

$$
\begin{aligned}
w &= rV_u + \gamma\left(y - (r+q)h_2 - rV_u\right) \\
w &= \frac{(r+q)z + \gamma\theta m(\theta)\left[y - (r+q)h_2\right]}{r+q+\gamma\theta m(\theta)} + \gamma\left(\left[y - (r+q)h_2\right] - \frac{(r+q)z + \gamma\theta m(\theta)\left[y-(r+q)h_2\right]}{r+q+\gamma\theta m(\theta)}\right) \\
w &= \frac{(r+q)z + \gamma\theta m(\theta)\left[y - (r+q)h_2\right]}{r+q+\gamma\theta m(\theta)} + \gamma\left(\frac{\begin{array}{c}(r+q+\gamma\theta m(\theta))\left[y-(r+q)h_2\right] \\ -(r+q)z - \gamma\theta m(\theta)\left[y-(r+q)h_2\right]\end{array}}{r+q+\gamma\theta m(\theta)}\right) \\
w &= \frac{(r+q)z + \gamma\theta m(\theta)\left[y - (r+q)h_2\right]}{r+q+\gamma\theta m(\theta)} + \gamma\left(\frac{(r+q)\left[y-(r+q)h_2\right] - (r+q)z}{r+q+\gamma\theta m(\theta)}\right) \\
w &= z\frac{(r+q)(1-\gamma)}{r+q+\gamma\theta m(\theta)} + \left[y-(r+q)h_2\right]\frac{\gamma\left[r+q+\theta m(\theta)\right]}{r+q+\gamma\theta m(\theta)} \\
w &= z\frac{(r+q)(1-\gamma)}{r+q+\gamma\theta m(\theta)} + \left[y-(r+q)h_2\right]\frac{\gamma\left[r+q+\theta m(\theta)\right]}{r+q+\gamma\theta m(\theta)} + z\frac{\gamma\theta m(\theta) - \gamma\theta m(\theta)}{r+q\gamma\theta m(\theta)} \\
w &= z\frac{r+q+\gamma\theta m(\theta)}{r+q+\gamma\theta m(\theta)} + \left[y-(r+q)h_2 - z\right]\frac{\gamma\left[r+q+\theta m(\theta)\right]}{r+q+\gamma\theta m(\theta)} \\
w &= z + \left[y-(r+q)h_2 - z\right]\frac{\gamma\left[r+q+\theta m(\theta)\right]}{r+q+\gamma\theta m(\theta)} \\
w &= z + \left(y - (r+q)h_2 - z\right)\Gamma(\theta) \qquad (11)
\end{aligned}
$$

where $\Gamma(\theta) \equiv \frac{\gamma[r+q+\theta m(\theta)]}{r+q+\gamma\theta m(\theta)}$.

D: Whereas the variable hiring costs are paid before the match is made, the fixed costs are incurred only when the match is created. The implication is that the variable costs are sunk costs, so they do not directly affect the bargaining between the matched agents. This is not the case for the fixed costs, which directly affect the bargaining between the worker and firm.

E: The three equilibrium equations are:

$$u = \frac{n+q}{n+q+\theta m(\theta)}$$

$$\frac{y-w}{r+q} = \frac{h_1}{m(\theta)} + h_2$$

and

$$w = z + \left(y - (r+q)h_2 - z\right)\Gamma(\theta)$$

where $\Gamma(\theta) \equiv \frac{\gamma[r+q+\theta m(\theta)]}{r+q+\gamma\theta m(\theta)}$

The flow hiring costs, h_1, only shift the labor demand curve to the left. This lowers the labor market tightness and the wage. The intuition is that the expected higher costs of having a vacancy make vacancy creation less attractive. This decreases the labor market tightness, which worsen the worker's bargaining position and lowers the wage. The lower labor market tightness leads to a lower job finding rate for the unemployed, so the unemployment rate increases.

Higher fixed costs, h_2, will shift the labor demand curve to the left similarly to increasing h_1 as this will make it less attractive for firms to create vacancies. In contrast to h_1, increasing h_2 also has a direct negative effect on wages, which tend to make it more attractive for firms to create vacancies. Hence, the effect of a higher fixed hiring cost is to decrease the wage, whereas there are opposite effects for the effect on the labor market tightness. However, we can combine equations (6) and (11) to eliminate w in order to examine the effect on θ

$$\begin{aligned}
\frac{y-[z+(y-(r+q)\,h_2-z)\,\Gamma(\theta)]}{r+q} &= \frac{h_1}{m(\theta)}+h_2 \Leftrightarrow \\
\frac{(y-(r+q)\,h_2-z)\,(1-\Gamma(\theta))}{r+q} &= \frac{h_1}{m(\theta)} \Leftrightarrow \\
\frac{(y-(r+q)\,h_2-z)\left(\frac{r+q+\gamma\theta m(\theta)}{r+q+\gamma\theta m(\theta)}-\frac{\gamma[r+q+\theta m(\theta)]}{r+q+\gamma\theta m(\theta)}\right)}{r+q} &= \frac{h_1}{m(\theta)} \Leftrightarrow \\
\frac{(1-\gamma)\,(y-(r+q)\,h_2-z)}{r+q+\gamma\theta m(\theta)} &= \frac{h_1}{m(\theta)} \Leftrightarrow
\end{aligned}$$

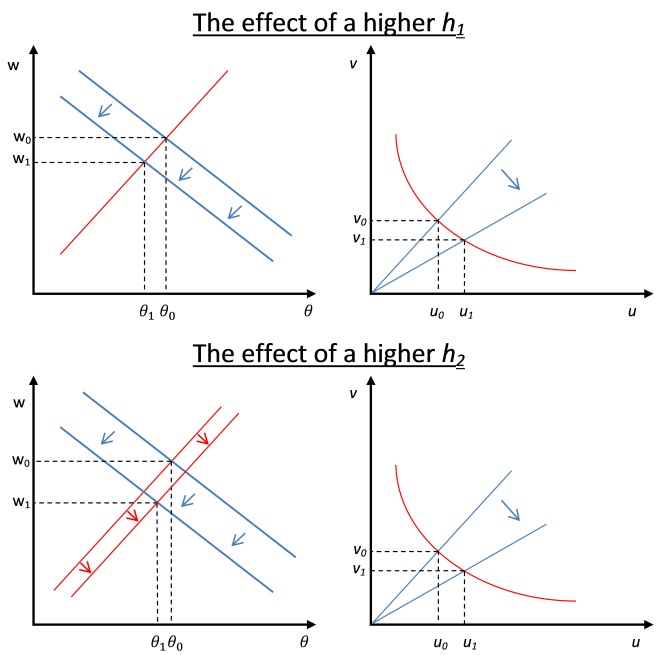
We see that a higher h_2 decreases the l.h.s. This can be counteracted by decreasing $\theta m(\theta)$ or by increasing $m(\theta)$. Both these cases imply that θ must decrease since $m'(\theta) < 0$. Therefore, higher fixed costs increase the unemployment as we move to the right a long the Beveridge curve.

F: Note: The exam text included an unfortunate sign mistake: the correct expression is

$$\eta_y^\theta = \frac{-1}{\eta_\theta^{m(\theta)}} \frac{y-w\eta_y^w}{y-w-(r+q)\,h_2}$$

Therefore, exam grading will initially be done excluding this question. However, to compensate for students' time use on this question, additional credit will subsequently be given to students who have derived the elasticity with or without the sign mistake, or just noted the counter-intuitive sign of the elasticity. Below is given the correct derivation.

Figure 1: Effect of increases in hiring costs



Total differentiate equation (6) [1]

$$
\begin{aligned}
\frac{dy}{r+q} - \frac{dw}{r+q} &= \frac{-h_1 m'(\theta)}{m(\theta)^2} d\theta \Leftrightarrow \\
\frac{-h_1 m'(\theta)}{m(\theta)^2} \frac{d\theta}{dy} &= \frac{1}{r+q} - \frac{\frac{dw}{dy}}{r+q} \Leftrightarrow \\
\frac{d\theta}{dy} &= \frac{-1}{r+q} \left[1 - \frac{dw}{dy}\right] \frac{m(\theta)^2}{h_1 m'(\theta)} \Leftrightarrow \\
\frac{d\theta}{dy} \frac{y}{\theta} &= \frac{-1}{r+q} \frac{m(\theta)}{h_1} \left[y - \frac{dw}{dy} y\right] \frac{m(\theta)}{m'(\theta)\theta} \Leftrightarrow \\
\eta_y^\theta &= \frac{-1}{r+q} \frac{m(\theta)}{h_1} \left[y - w\eta_y^w\right] \frac{1}{\eta_\theta^{m(\theta)}}
\end{aligned}
$$

[1]It is easier to derive the final result if taking the logs before total differentiating.

Finally, we can use equation (6) to write

$$\begin{aligned}
\frac{y-w}{r+q} &= \frac{h_1}{m(\theta)} + h_2 \Leftrightarrow \\
y-w &= (r+q)\frac{h_1}{m(\theta)} + (r+q)\,h_2 \Leftrightarrow \\
(r+q)\frac{h_1}{m(\theta)} &= y-w-(r+q)\,h_2 \Leftrightarrow \\
\frac{1}{r+q}\frac{m(\theta)}{h_1} &= \frac{1}{y-w-(r+q)\,h_2}
\end{aligned}$$

which we can insert in the elasticity above

$$\eta_y^\theta = \frac{-1}{\eta_\theta^{m(\theta)}} \frac{y - w\eta_y^w}{y-w-(r+q)\,h_2}$$

G: Note: Due to the sign mistake in II.F, exam grading will initially be done excluding this question. However, to compensate for students' time use on this question, additional credit will subsequently be given for answers noting the counter-intuitive sign, or discussing the question in the spirit of the Shimer critique. An answer based on the correct expression is given below.

The elasticity is increasing in the fixed costs, h_2. Shimer (2005) argues that the Diamond-Mortensen-Pissarides model cannot match the business cycle fluctuations in key variables such as the labor market tightness and the unemployment rate. Extending the model with fixed hiring costs implies that the model will do a better job in matching the business cycle fluctuations.

Most of the proposed solutions for solving the Shimer puzzle involve some form of wage rigidity. However, in this case the improvement does not come from a more rigid wage. Instead, the presence of a fixed cost breaks the proportionality between hiring costs and the expected duration of a vacant job. When productivity increases, creating vacancies become more attractive for the firm. However, this has the effect of increasing the expected duration of a vacancy, which, in turn, moderates the effect of productivity on the labor market tightness. If part of the hiring costs are fixed, the total hiring costs will fluctuate less over the business cycle relative to their level. Therefore, they will also to a lesser extent moderate the effect of the productivity on the labor market tightness.

This mechanism is also directly visible from the labor demand curve

$$\text{Total costs} = \frac{h_1}{m(\theta)} + h_2 = \frac{y-w}{r+q}$$

If the r.h.s. increases by, say, 1 percent due to a higher productivity, $m(\theta)$ would just need to decrease by approximately 1 percent in absence of fixed firing costs when holding w fixed. However, if we suppose that the fixed costs constitute about 50 percent of the total costs, $m(\theta)$ would need to decrease by approximately 2 percent. Since $m'(\theta) < 0$, this implies that θ will need to increase more.

Part III - Beer and compensating differentials (max 1500 words)

In the imaginary country of Harmonix, beer brewing is a very important export industry.

Among the breweries in Harmonix, by far the most important step in beer production is the fermentation of the beer, which is done by leaving the beer inside big storage rooms for 4 weeks. During this time, brewery workers go around inside the storage rooms and take care of the beer (checking that it develops correctly, adding additional ingredients at various points, etc.).

Historically, the breweries in Harmonix have brewed three different types of beers: Ales, lagers and saisons. The main difference between these is the temperature that they need to be stored at during fermentation. For ales, the storage rooms need to be kept at a pleasant 21 $^{\circ}C$. All inhabitants in Harmonix always prefer the ambient temperature to be exactly 21 $^{\circ}C$ so all living rooms are kept at 21 $^{\circ}C$ and this is also known as room temperature.

For lagers, the storage rooms need to be kept significantly cooler than room temperate, around 12 $^{\circ}C$. This unpleasantly cold environment makes the yeast work in a way that results in a very neutral or clean taste.

Finally, for saisons, the storage rooms need to be kept much warmer than room temperature, at around 30 $^{\circ}C$. This unpleasantly warm environment gives the yeast the possibility of adding more complex, interesting flavors to the beer.

When breweries in Harmonix export their beer, the prices they can charge vary depending on the type. Lagers are very popular internationally so sell for 1 dollar per bottle. Ales are also reasonably popular so sell for 0.80 dollars per bottle. Saisons, however, are not so well-known or popular so only sell for 0.50 dollars per bottle.

A

Example answer:

To begin with please note that this question is open-ended and requires students to translate the wording in the question into various assumptions on their own. There is therefore more than one good way to answer this question. In terms of grading, a good answer should be clear, precise, well-structured and well argued. It should also convey that the student understands how to apply the concept of compensating differentials to the situation at hand in a relevant way. It does not in itself detract from the quality of an answer whether students make strong or questionable assumptions unless these go directly against the spirit of the question (one example of this would be to assume that workers do not care about the temperature in which they work) and/or they preclude the students from demonstrating an understanding of the theory of compensating differentials (one example of this would be to assume that the different types of beers require different types of workers so that no worker ever faces a choice between jobs producing different kinds of beers). Any assumption *must* be properly stated, explained and discussed, however. For example it is important to be explicit about what is assumed about the production technology (constant output per worker, decreasing returns to scale, etc.).

Below are given examples of good answers for each part:

Because brewery workers making different kinds of beer potentially has to work in environments that are warmer or colder than their preferred 21 $^{\circ}C$, there is scope for some workers to be compensated for the unpleasant working environment via a higher salary. This is called compensating differential.

Since firms export their beers, it is reasonable to assume that they can sell as many of them as they like at the export price. If additionally we assume that each additional worker is able to produce x beer bottles per month and that the non-labor costs of a bottle of beer are k per unit then the additional per month profits a brewery makes by hiring one worker to brew ales are $(0.8-k)\cdot x$. A brewery will never pay a worker producing ales more than these but - assuming a competitive environment - will be willing to outbid other breweries paying less than this to attract workers as long as they have room for additional workers. If we assume that breweries (in the long run) will always have room for additional workers (they can build a bigger building), competition among breweries will therefore imply that the wage paid to any workers hired to brew ales will therefore be:

$$w_a \equiv (0.8-k)\cdot x$$

Similar arguments show that the wages paid to workers producing lagers and saisons will be:

$$w_l \equiv (1-k)\cdot x$$

$$w_s \equiv (0.5-k)\cdot x$$

Here, it has been assumed that the non-labor costs are the same despite the required room temperature levels being different. At wages w_a, w_l and w_s firms will be willing to hire any workers who are interested in working for them.

Next we turn to analyze workers' decisions about which jobs to choose. We can do this either graphically or mathematically. Below are shown examples of both methods but only one of them is needed for a complete answer.

Graphical:

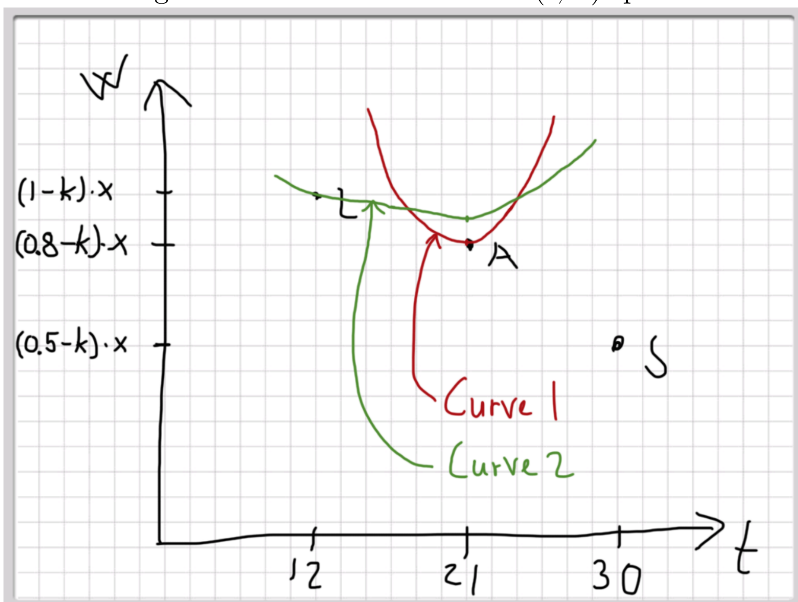
Brewery workers can choose between jobs producing different types of beers. Given the description in the question and the discussion above, it is reasonable to assume that the only difference between these jobs is the wage paid, w, and the temperature in which the worker has to work, t. Figure 2 depicts the different jobs in t, w-space together with indifference curves for two different workers (Point L corresponds to lager producing jobs, A to ale producing jobs and S to saison producing jobs. As usual, we will assume that workers like wages, whereas for the temperature, the question states that people in Harmonix all prefer 21 $°C$ so the best temperature to work in is $t = 21$; any higher or lower temperature than this will make people worse off. As result, workers' indifference curves will be u-shaped in t, w-space and have their lowest point at $t = 21$; workers working at the preferred $t = 21$ will can be paid the lowest wage to achieve a given level of utility. The exact shape of the indifference curve depends on how tolerant workers are towards higher or lower temperatures.

Indifference curve 1 corresponds to a worker who cares a lot about temperature, while indifference curve 2 corresponds to a worker that is not as sensitive to temperature. Moving away from $t = 21$ along this indifference curve only results in small wage increases.

The first thing we conclude is that no worker will ever want to work on producing saison beers. Because workers prefer temperatures closer to $t = 21$, workers will always be better off moving from a saison-producing job to a different job that pays the same wage but involves a slightly lower temperature. Since ale-producing jobs involve a temperature closer to $t = 21$ than saison-producing jobs *and* pays a higher wage, no worker will choose a saison-producing job, regardless of the exact shape of their indifference curve.

Next we consider the choice between ale-producing and lager-producing jobs. Here anything is possible depending on the exact shape of the indifference curves. In figure 2, workers with indifference curve 1 will prefer ale-producing jobs, whereas workers with indifference curve 2 will prefer lager jobs. If we assume that workers in Harmonix differ sufficiently in how much they care about temperature, the second prediction we can make is

Figure 2: Indifference curves in (t, w)-space



that some workers will work in ale-producing jobs and some will work in lager producing jobs. The exact share in each will depend on the distribution of preferences among the workers. As for the relative wages it follows from above that the lager producing workers earn $\frac{w_l}{w_a} = \frac{1-k}{0.8-k}$ times more than ale producing workers. This higher wage reflects a compensating differential, compensating lager producing workers for the fact that they have to work in a cold environment.

Mathematical:

Brewery workers can choose between jobs producing different types of beers. Given the description in the question and the discussion above, it is reasonable to assume that the only difference between these jobs is the wage paid, w, and the temperature in which the worker has to work, t.

The question states that people in Harmonix all prefer 21 $°C$ so the best temperature to work in is $t = 21$. If we therefore assume that worker i's monetary value of the disutility from working at $t = 12$ is constant equal to $\gamma_{il} > 0$, that his monetary value of the disutility from working at $t = 30$ is constant equal to $\gamma_{is} > 0$, we can write the utility of worker i as:

$$u_i(w,t) = \begin{cases} w - \gamma_{il} & \text{if} \quad t = 12 \\ w & \text{if} \quad t = 21 \\ w - \gamma_{is} & \text{if} \quad t = 30 \end{cases}$$

Now we can start by considering the choice between an ale producing job with and a saison producing job. Worker i will be willing to accept a saison producing job if and only if:

$$\begin{aligned} & u_i(w_s, 30) \geq u_i(w_a, 21) \iff \\ & w_s - \gamma_{is} \geq w_a \iff \\ & w_s - w_a \geq \gamma_{is} \end{aligned}$$

The last equation shows that the worker is willing to choose a saison producing job over an ale producing job if and only if the wage gain from doing so is greater than the disutility from working in a 30 $°C$ environment. From the discussion of wages above, however, saison jobs will always pay less than ale jobs so $w_s - w_a < 0$ and this can never be true.

The first prediction we can make is therefore that there will be no brewery workers employed in the production of saisons. Since saisons require that brewery workers work at a higher temperature than they prefer, saison-producing breweries would have to pay workers more than ale-producing breweries in order to attract workers. Since saisons sell for less, however, no brewery is willing to do this.

Next we turn to the choice between an ale producing job and a lager producing job. A similar calculation as above here shows that worker i will be willing to accept a lager producing job if and only if:

$$w_l - w_a \geq \gamma_{il}$$

Plugging in for the wages we can further write:

$$0.2 \cdot x \geq \gamma_{il}$$

Again the interpretation of this is that the worker is willing to choose a lager producing job over an ale producing job if and only if the wage gain from doing so is greater than the disutility from working in a 12 $^{\circ}C$ environment. Since breweries are actually willing to pay more to workers producing lagers, however, this condition can be met depending on the size of γ_{il}. If we assume that workers differ in their temperature sensitivity so differ in how much disutility they get from working at a temperature below room temperature then γ_{il} differs across workers. In this case all workers with $\gamma_{il} < 0.2 \cdot x$ will work in lager producing jobs, while workers with $\gamma_{il} > 0.2 \cdot x$ will choose ale producing jobs. Workers with $\gamma_{il} = 0.2 \cdot x$ will be indifferent.

In sum, the second prediction we can make is that (under the assumptions above), there will be some workers working in lager production and some working ale production. The exact share in each will depend on the distribution of preferences in among the workers (the distribution of γ_{il}). As for the relative wages it follows from above that the lager producing workers earn $\frac{w_l}{w_a} = \frac{1-k}{0.8-k}$ times more than ale producing workers. This higher wage reflects a compensating differential, compensating lager producing workers for the fact that they have to work in a cold environment.

B

Example answer:

Yes. The answer above assumed that workers differed in their preferences regarding temperature. If workers instead all have the same preferences regarding temperature, we will generally get different predictions.

The analysis of what wages would be paid to workers in different kinds of job is unchanged from above. We can again either analyze worker choices graphically or mathematically. Only one of them is needed for a complete answer.

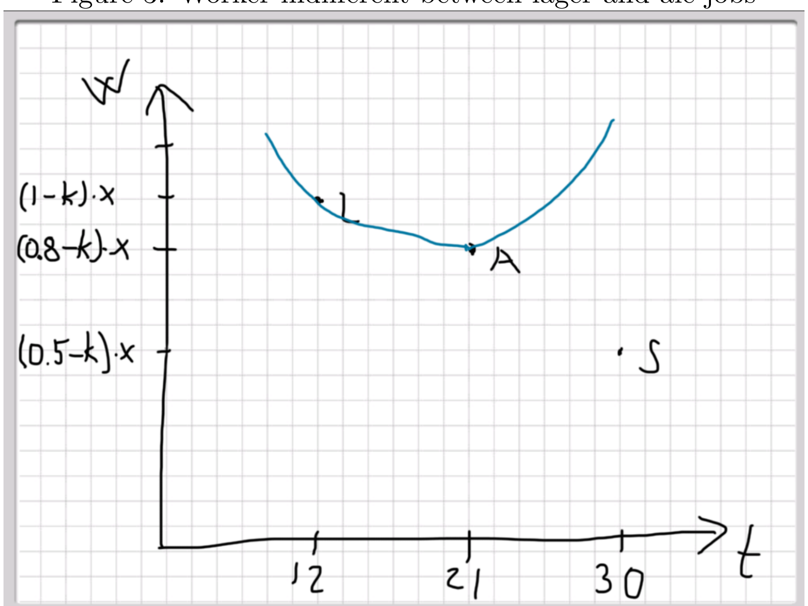
Graphical

The conclusion that workers will never choose a saison job did not rely on any assumption about whether workers had different preferences so this goes through unchanged.

As for the analysis of workers' choice between lager and ale jobs, if workers all have the same preferences one of three things can happen: 1)

All workers are very sensitive to temperature and have preferences like indifference curve 1 in Figure 2. In this case all workers prefer ale jobs so all workers are producing ales. The intuition is that all workers have preferences regarding temperature that make it too costly for breweries to hire workers to produce anything but ales. 2) All workers are relatively insensitive to temperate and have preferences like indifference curve 2. In this case all workers prefer lager jobs so all workers will be employed producing lagers. The intuition is here that while all workers prefer working at room temperature, the higher export price of lager beer causes firms to prefer to hire only lager producing workers and compensate them with a higher wage. 3) Finally there is an in-between case where all workers are indifferent between the two jobs, as illustrated in Figure 3. In this case any number of workers may be employed in either of the two types of jobs, while the relative wages are as discussed under A.

Figure 3: Worker indifferent between lager and ale jobs



Mathematical

The conclusion that workers will never choose a saison job did not rely on any assumption about whether workers had different preferences so this goes through unchanged.

As for the analysis of workers' choice between lager and ale jobs, if workers all have the same preferences one of three things can happen: 1) All workers are very sensitive to temperature and have $\gamma_{il} = \gamma_l > 0.2 \cdot x$. In this case all workers prefer ale jobs so all workers are producing ales. The intuition is that all workers have preferences regarding temperature that make it too costly for breweries to hire workers to produce anything but ales. 2) All workers are relatively insensitive to temperate and have $\gamma_{il} = \gamma_l < 0.2 \cdot x$. In this case all workers prefer lager jobs so all workers will be employed producing lagers. The intuition is here that while all workers prefer working at room temperature, the higher export price of lager beer causes firms to prefer to hire only lager producing workers and compensate them with a higher wage. 3) Finally there is an in-between case where all workers have $\gamma_{il} = \gamma_l = 0.2 \cdot x$. In this case workers are indifferent between the two jobs and any number of workers may be employed in either of the two types of jobs, while the relative wages are as discussed under A.

C

Example answer:

Yes and no. In the derivations above, we simply assumed that each worker could produce x beers per month, where x could be anything. In this sense the results do not depend on the exact number of bottles that a worker can produce per month. It is an important assumption though that workers produce the same number of bottles of beer regardless of which type of beer they are producing. If the number of bottles a worker can produce differs across the three types of beer (for example because one type require more work adding ingredients during fermentation) then this changes the relative wages that firms will be willing to pay to workers in the different types of jobs. This may in turn also impact the prediction regarding how many workers work in the different types of jobs.

D

Example answer:

In terms of grading, please note than in this question it is possible to arrive at a different, sharper prediction than given below if one invokes the additional assumption that workers gets the same disutility from a given temperature increase above and below room temperature. Pursuing this approach is fine, however, a good answer should be clear that this is an additional assumption not implied by the description in the question.

With the higher price of saison beers, the wages paid to brewery workers producing saisons would increase to:

$$w_s \equiv (1.1 - k) \cdot x$$

Since this implies $w_s > w_a$ we can no longer rule out that workers choose saison producing jobs. The analysis of what workers would actually choose can again be done mathematically or graphically:

Graphical

With the higher wage paid to saison workers, it is now possible that workers would choose any of the three jobs depending on the their preferences and the exact shape of their indifference curves. Figures 4 to 6 shows examples of preferences that would make workers choose each of the three types of jobs with the changed saison wage. If we assume that workers differ sufficiently in their preferences we would therefore predict that the workers who are most sensitive to temperature still choose ale jobs, while workers who are less sensitive choose either saison or lager jobs depending on exactly how insensitive they are to (higher) temperatures (i.e. depending on whether the preferences look like the ones in Figures 4, 5 or 6). Under this assumption there will be some workers working in both ale, lager and saison producing jobs. Saison workers would earn $\frac{w_s}{w_a} = \frac{1-k}{0.8-k}$ times as much as ale workers, while as before lager workers would earn $\frac{w_l}{w_a} = \frac{1-k}{0.8-k}$ as much as lager workers. These wage differences again reflect compensating differentials due to the different temperatures in the working environment.

If we instead assume that preferences for temperature does not differ much across workers or does not differ at all, however, we could instead end up in situations where there is one or more type of job that no worker is willing to choose. Depending on our assumption regarding preferences, anything can thus happen in terms of how many workers are employed in each of the three types of jobs.

Mathematical

With the higher wage paid to saison workers, it is now possible that workers would choose any of the three jobs depending on the their preferences.

Figure 4: Worker preferring ale job with higher saison wage

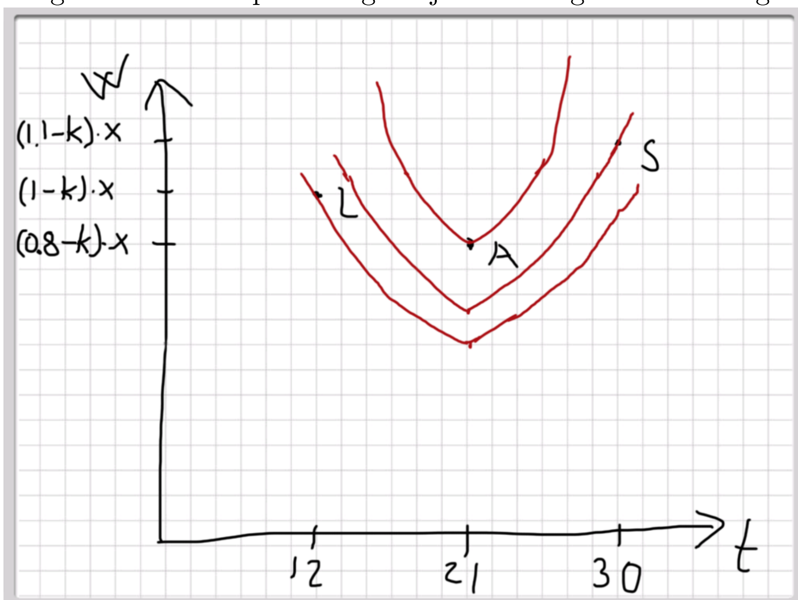


Figure 5: Worker preferring lager job with higher saison wage

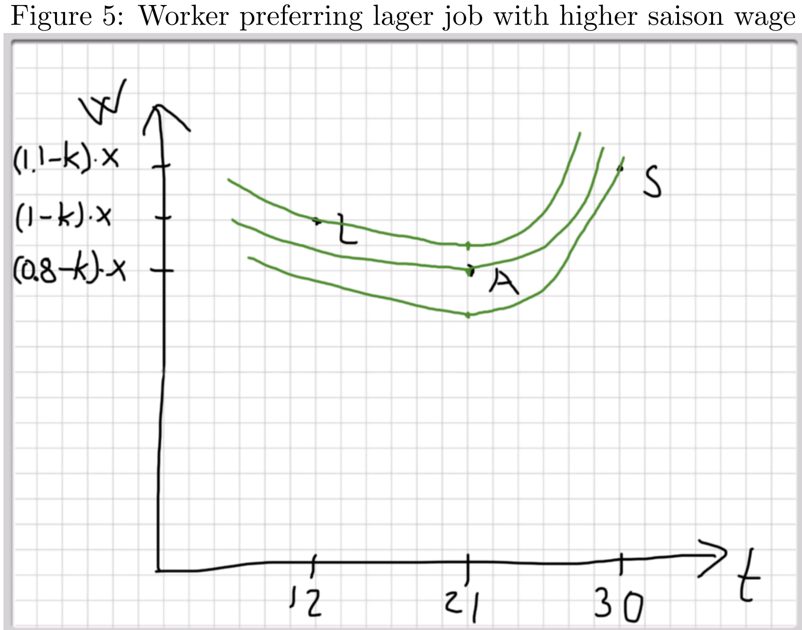
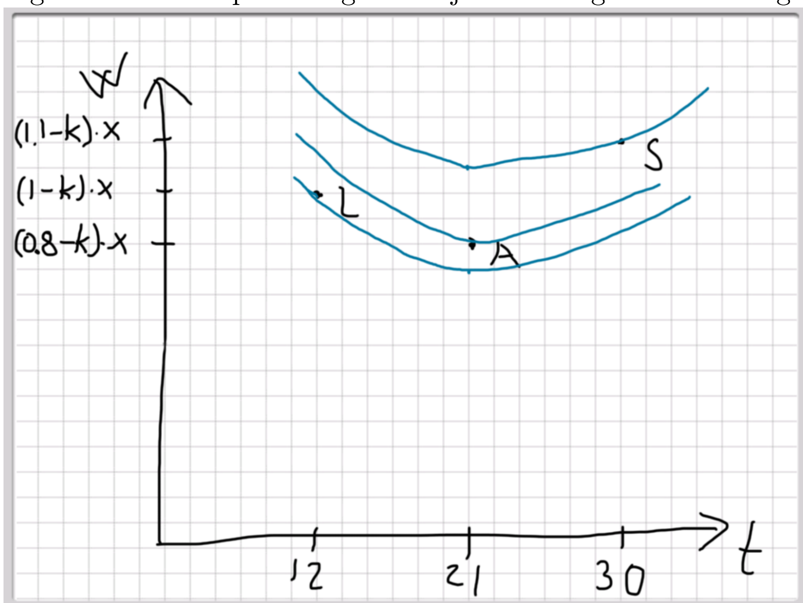


Figure 6: Worker preferring saison job with higher saison wage



Worker i would be willing to accept an ale producing job if and only if:

$$\begin{array}{rcrc}
u_i(w_a, 21) \geq u_i(w_s, 30) & \wedge & u_i(w_a, 21) \geq u_i(w_l, 12) & \iff \\
w_a \geq w_s - \gamma_{is} & \wedge & w_a \geq w_l - \gamma_{il} & \iff \\
w_s - w_a \leq \gamma_{is} & \wedge & w_l - w_a \leq \gamma_{il} & \iff \\
0.3 \cdot x \leq \gamma_{is} & \wedge & 0.2 \cdot x \leq \gamma_{il} &
\end{array}$$

Similar calculations show that worker i would be willing to accept a saison producing job if and only if:

$$0.3 \cdot x \geq \gamma_{is} \qquad \wedge \qquad 0.1 \cdot x \geq \gamma_{is} - \gamma_{il}$$

Similar calculations also show that worker i would be willing to accept a lager producing job if and only if:

$$0.2 \cdot x \geq \gamma_{il} \qquad \wedge \qquad 0.1 \cdot x \leq \gamma_{is} - \gamma_{il}$$

Inspecting the three conditions above, we see that depending on worker i's exact disutility of working at $t = 12$ and $t = 30$, γ_{is} and γ_{il}, it is possible for him to prefer either of the three jobs and/or be indifferent between two or more of them. For example $\gamma_{is}, \gamma_{il} = 0.3 \cdot x$ would imply a strict preference for ale producing jobs, $\gamma_{is}, \gamma_{il} = 0.1 \cdot x$ would imply a strict preference for saison producing jobs and $\gamma_{is} = 0.3 \cdot x, \gamma_{il} = 0.1 \cdot x$ would imply a strict preference for lager producing jobs.

Assuming that workers differ sufficiently in their preferences regarding temperature, there will therefore be some workers working in both ale, lager and saison producing jobs. Saison workers would earn $\frac{w_s}{w_a} = \frac{1-k}{0.8-k}$ times as much as ale workers, while as before lager workers would earn $\frac{w_l}{w_a} = \frac{1-k}{0.8-k}$ as much as lager workers. These wage differences again reflect compensating differentials due to the different temperatures in the working environment.

If we instead assume that preferences for temperature does not differ much across workers or does not differ at all, however, we could instead end up in situations where there is one or more type of job that no worker is willing to choose. Depending on our assumption regarding preferences, anything can thus happen in terms of how many workers are employed in each of the three types of jobs.