Written Exam for the M.Sc. in Economics summer 2014

Pricing Financial Assets

Final Exam/ Elective Course/ Master's Course

7 August 2014

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Consider an economy over two time periods, $\mathcal{T} = \{0, 1, 2\}$, described by a recombining binomial tree. Two securities are traded: A non-dividend-paying stock with a price S_t that in each period can move up by a factor u (an up-move) or down with a factor d (a down-move), and risk free asset with a price B_t that will accumulate by a factor of e^r in each period. Assume $u > \mathsf{e}^r > d > 0$, $S_0 > 0$, and $B_0 = 1$.
   (a) Consider a European put option written on the stock with expiry at $t = 2$ and strike X. Use an arbitrage argument to find the value of the put at $t = 0$ expressed using a risk-neutral probability q of an up-move. Explain why the real-world probability p of an up-move does not enter directly in the expression.
   (b) Find the state prices (or Arrow-prices) at $t = 0$ for payments at $t = 1$
   (c) Now consider an American put in this model. Find and comment on a (non-trivial) example for the value of the American put to exceed the value of the European put.

2. (a) In the one-factor Gaussian model of defaults the default of an obligor i is indicated by

$$x_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

   where F and Z_i, $i = 1, \ldots, n$, have independent standard normal distributions. Give an interpretation of the model and its parameters. What is the (copula) correlation between x_i and x_j for some $j \neq i$?
   (b) In this model the indicator x_i is compared to the probability $Q_i(T)$ that the obligor has defaulted before or at time T. Suppose that the obligor defaults at time t. Which value of the indicator x_i does this correspond to?
   (c) Describe in general terms how a Monte Carlo simulation can be used to model correlated defaults given the individual default probabilities and (copula) correlations.

3. (a) In the Cox-Ingersoll-Ross (CIR) Model the (instantaneous) short term interest rate r is described by the process:

$$dr = a(b - r)dt + \sigma\sqrt{r}dz$$

   where a,b and σ are constants ($\sigma^2 < 2ab$), and dz a Wiener process. What does this mean for the behaviour of the short term interest rate?
   (b) In this model the solution for the price of a zero-coupon bond can be written

$$P(t, T) = A(t, T)\mathsf{e}^{-B(t,T)r(t)}$$

   Derive the duration of the bond.
   (c) In some models of the short term interest rate r (e.g. the Hull-White model) the drift rate is made a function of calendar time. What is the purpose of the extra flexibility compared to the CIR (or Vasicek) type of models?