

Written Exam for the M.Sc. in Economics summer 2014

Pricing Financial Assets

Final Exam/ Elective Course/ Master's Course

23 June 2014

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. (a) Define a forward contract on a stock.
- (b) Consider a stock with current price S_0 that does not pay dividends. Assume a constant continuously compounded interest rate of r . Using an arbitrage argument find the forward price F_0 at time 0 on a forward contract on the stock that matures at time T .
- (c) Consider the value of a forward contract with forward price K at time $t < T$ and assume that the stock price follows the geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dz$$

Using Ito's lemma find the process followed by the value of the forward contract. What will the drift be under the risk neutral measure?

- (d) Assume now that the stock pays a constant continuous dividend rate of q . What will the forward price be?
2. (a) Let the probability of a borrower not defaulting at or before time t be given by $V(t)$. What is the probability that the borrower will default between time t and $t + \Delta t$ conditional on not being in default at time t ? Use this to define the continuously compounded default hazard rate.
 - (b) How, and under which assumptions, may we estimate the hazard rate from the interest rate spread on bonds issued by the borrower? Under what probability measure would we say this estimate is derived? Compare this to a hazard rate that is derived from default frequencies and recovery ratios published by a rating agency.
 - (c) In one model by Merton the value of a claim on a company with limited liability is modelled using a variation of the Black-Scholes-Merton option model. What is the option features embedded in such a claim? What parameters that are not directly observable must be determined to price the claim in this model? Comment on the model.
3. (a) Give a definition of an interest rate swaption (also called a swap option)
 - (b) Give an argument that a receiver swaption can be seen as a call option on a properly defined fixed rate bond. What can you say about the coupon and principal of such a bond?
 - (c) Let PS denote the value of a swaption to pay a fixed rate s_K and receive LIBOR between times T_1 and T_2 , let RS denote the value of a swaption to receive a fixed rate of s_K and pay LIBOR between times T_1 and T_2 , and let RFS denote the value of a forward starting swap that receives a fixed rate of s_K and pays LIBOR between times T_1 and T_2 . Assume that there are no arbitrage opportunities and i) show that $PS + RFS = RS$ and ii) deduce that $PS = RS$ when s_K equals the current forward swap rate.