Written Exam for the B.Sc. or M.Sc. in Economics summer 2015

Pricing Financial Assets

Master's Course

August 6th, 2015

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. In the Black-Scholes-Merton Model the price, p, at time $t = 0$ of a European put option with strike K on a stock with price S_0 at time $t = 0$ and expiry at $T > 0$ is given by:

$$
\begin{aligned}
p =& K\mathrm{e}^{-rT}\Phi(-d_2) - S_0\Phi(-d_1) \\
&\text{where} \\
d_1 =& \frac{\ln S_0/K + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\
d_2 =& d_1 - \sigma\sqrt{T}
\end{aligned}
$$

   and Φ is the standard Normal distribution function.

   (a) What are the assumptions behind this model?

   (b) Using the call-put-parity derive a formula for the pricing of a call option on the same stock and with same strike and expiry as the put.

   (c) How can the put price formula be modified to cover the case of a dividend paying stock?

2. (a) Define a Credit Default Swap (CDS) and describe the payment structure.

   (b) Let the probability of a borrower not defaulting at or before time t be given by $V(t)$. What is the probability that the borrower will default between time t and $t + \Delta t$ conditional on not being in default at time t? Use this to define the continuously compounded default hazard rate $\lambda(t)$.

   (c) How, and under which assumptions, may we estimate the hazard rate from a CDS spread s on the relevant borrower? Under what probability measure would we say this estimate is derived? Compare this to a hazard rate that is derived from default frequencies and recovery rates published by a rating agency.

3. (a) In the Ho-Lee Model in continuous time the (instantaneous) short term interest rate r is described by the process:

$$dr = \theta(t)dt + \sigma dz$$

   where $\theta(t)$ is a function of t, σ is constant, and dz the increment of a Brownian motion. What does this mean for the behaviour of the short term interest rate?

   (b) Here as in other models of the short term interest rate r the drift rate is made a function of calendar time. What is the purpose of the extra flexibility compared to the CIR (or Vasicek) type of models?

   (c) In this model the solution for the price of a zero-coupon bond can be written

$$P(t,T) = A(t,T)\mathrm{e}^{-r(t)(T-t)}$$

   Use Ito's lemma to derive the volatility of P and comment on the result.

   (d) Derive the duration of the bond.