Written Exam for the B.Sc. or M.Sc. in Economics summer 2015

Pricing Financial Assets

Master's Course

June 23rd, 2015

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Let the price of a traded financial instrument, S, be modelled (under the probability measure $\mathbb{P}$) by the geometric Brownian motion
$$dS = \mu S dt + \sigma S dz$$
where μ and $\sigma > 0$ are constants, and where dt and dz are the standard short hand notations for a small time-step and a Brownian increment.
   (a) Describe the qualitative characteristics of this model, and discuss it's possible shortcomings as a model of a stock price.
   (b) Assume that the instrument pays a continuous dividend stream of q proportional to the price S. What will the drift rate of the price be under the standard risk neutral probability measure ($\mathbb{Q}$) and a no-arbitrage assumption?
   (c) Consider the transformation G of S given by the natural logarithm (ln), i.e. $G(x) = \ln(x)$. Use Ito's lemma to find the process followed by $G(S)$.
   (d) Suppose that for $t = 0$ the price of the instrument is S_0. What is the expectation (under $\mathbb{P}$) of the natural log of the price at $t = T \geq 0$?

2. (a) Consider a derivative with price $V(S, t)$ as some function of the current stock price S and time t (and further implicit parameters). Define and interpret the Delta, Gamma and Theta of the derivative.
   (b) Assume that the stock pays no dividends before time T, and that there is a constant risk free interest rate of r. Let $c(S, K, T, r)$ and $p(S, K, T, r)$ be the price at time $t = 0$ of a European call and a European put, respectively, on the stock with the same strike K and expiry T. Derive the call-put-parity.
   (c) Use the call-put-parity to find a relationship between the Deltas of the call and put. Repeat this for Gamma and Theta, respectively.
   (d) Suppose a portfolio of the stock and/or derivatives of that stock is Delta-neutral, and that there are no arbitrage possibilities. Let the value of the portfolio be $\Pi(S, t)$. What can we say about the relation between the Theta and Gamma of the portfolio?

3. Consider an interest rate floor with a life of T, a principal of N, and a floor rate of R_K. Consider reset dates $0 = t_0 < t_1 < t_2, \ldots < t_n$, and let R_k be the Libor rate for the period from t_k to t_{k+1} known at t_k.
   (a) Describe the payments of the floor, and define a floorlet.
   (b) Show that the floor can be considered as a portfolio of European call options on zero-coupon bonds (for notational ease you may assume $N = 1$) *(Hint: Start by analyzing a single floorlet)*.
   (c) A standard market practice is to price a floorlet (at $t = 0$) with the Black formula:
$$Floorlet^{Black}(0, k, N, R_K, \sigma_k) = NP(0, t_{k+1})\tau_k(R_K\Phi(-d_2) - F_k\Phi(-d_1))$$
   with
$$d_1 = \frac{ln(F_k/R_K) + 0.5\sigma_k^2 t_k}{\sigma_k\sqrt{t_k}}$$
$$d_2 = d_1 - \sigma_k\sqrt{t_k}$$
   where $P(t, T)$ is the price at t of a zero coupon bond maturing at T, F_k is the forward rate at 0 for the time interval (t_k, t_{k+1}) with length τ_k, and σ_k the volatility of this rate. The function Φ is the standard Normal distribution function. What assumptions can justify this formula?