Written Exam Economics summer 2016

Pricing Financial Assets

11 August 2016

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Assume that the price of a non-dividend paying stock S can be modelled (under the original probability measure $\mathbb{P}$) by the geometric Brownian motion

$$dS = \mu S dt + \sigma S dz$$

where μ and $\sigma > 0$ are constants, and where dt and dz are the standard short hand notations for a small time-step and a Brownian increment.

   (a) Describe the qualitative characteristics of this model, and discuss its possible shortcomings as a model of a stock.

   (b) Consider the transformation G of S given by the natural logarithm (ln), i.e. $G(x) = \ln(x)$. Use Ito's lemma to find the process followed by $G(S)$.

   (c) Consider a forward contract on the stock maturing at time T. Compare the pricing of a call option on the stock with a pricing of a call on the forward contract on the stock both maturing at T and having the same strike K.

2. Consider European call and put options on the same futures contract. Both options have strike price K and expiry at time T. Assume that there is a constant risk free interest rate of r. Let the futures price at time t be F_t (You may ignore margining and treat the futures as forward contracts).

   (a) At time 0 derive the put-call parity for the futures options.

   (b) Use the put-call parity to find the relationship between

      1. The delta of the European futures call and the delta of the European futures put (use the delta with respect to the futures price rather than the spot price)
      2. The gamma of the European futures call and the gamma of the European futures put (use the gamma with respect to the futures price rather than the spot price)
      3. The vega of the European futures call and the vega of the European futures put
      4. The theta of the European futures call and the theta of the European futures put

      and comment on the results.

3. The HJM-model describes the simultaneous evolution of the full term structure of interest rates. Let the evolution of instantaneous forward rates contracted at t for time T be described by the Ito-process

$$df(t,T) = m(t,T,\Omega)dt + s(t,T,\Omega)dz$$

where Ω is a set of state variables, and dt and dz are the standard shorthand notations for a small time step and a Brownian increment.

   (a) Under certain conditions we have the following no-arbitrage condition for the drift term:

$$m(t,T,\Omega) = s(t,T,\Omega) \int_t^T s(t,\tau,\Omega) d\tau$$

   Comment on this result, and in particular explain under which probability measure it is derived.

   (b) As a special case let $s(t,T,\Omega)$ be a constant denoted s. Derive the process followed by forward rates. Comment on the distribution of the forward rates.

   (c) Certain models of the term structure can be calibrated to be consistent with an initial given term structure. How is this achieved in simple one-factor models as e.g. the Ho-Lee model? How is this achieved in the HJM-model?