Written Exam Economics summer 2016

Pricing Financial Assets

13 June 2016

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 2 pages in total

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Let the spot exchange rate for a foreign currency be S, denoting the value of one unit of the foreign currency as measured in the domestic currency.

   Assume that the exchange rate can be be modelled (under the original probability measure $\mathbb{P}$) by the geometric Brownian motion
   $$dS = \mu_S S dt + \sigma_S S dz$$
   where μ_S and $\sigma_S > 0$ are constants, and where dt and dz are the standard shorthand notations for a small time-step and a Brownian increment.

   (a) Describe the qualitative characteristics of this model, and discuss its possible shortcomings as a model of an exchange rate.

   (b) Assume that the domestic and foreign risk free interest rates are constants r and r_f, respectively. What will the drift rate of the exchange rate be under the domestic risk neutral probability measure ($\mathbb{Q}$) and a no-arbitrage assumption? Comment on the influence of the difference of domestic and foreign interest rates on the result.

   (c) Let $Z = 1/S$ be the exchange rate as measured in the foreign currency. Use Ito's lemma to find the process followed by Z where you eventually substitute S by $1/Z$.

2. In the Black-Scholes-Merton for options on a stock paying no dividends before the expiry of the option we get the following formula $c = S_0\Phi(d_1) - K\mathrm{e}^{-rT}\Phi(d_2)$ for the price c of a call at current time, where S_0 is the current stock price, K is the exercise price of the call, Φ is the cumulative standard normal probability function, r is a constant continuously compounded risk free interest rate, $d_1 = \frac{\ln\frac{S_0}{K}+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, and the stock price follows the Geometric Brownian Motion $dS = \mu S dt + \sigma S dz$, where z is a Brownian Motion.

   (a) In a model formulated by Merton the value of equity in a limited liability company with zero coupon debt of a notional of N maturing at T is considered as a call option on the value V of company assets. Assuming that the value of these assets can be described by a geometric Brownian motion derive a formula for pricing the zero coupon debt using the above model.

   (b) Comment on the model and its potential limitations.

   (c) Let $L = N\mathrm{e}^{-rT}/V_0$ and show that the credit spread on the zero coupon bond is
   $$-\ln\left[\Phi(d_2) - \Phi(-d_1)/L\right]/T$$

3. Assume that the stochastic instantaneous risk free rate is modelled by the one-factor
   $$dr = m(r,t)dt + s(r,t)dz$$
   where $s(r,t) > 0$, and dt and dz are the standard shorthand notations for a small time-step and a Brownian increment.

   Consider two zero coupon bonds with different maturities T_i and values $V_i(r,t,T_i)$, $i \in \{1,2\}$, at $t < min[T_1, T_2]$.

   (a) Construct a portfolio of a long position in 1 of the first bond and a short of Δ of the second. Denote the value of this portfolio Π and use Ito's lemma to find the delta that makes Π locally risk free.

   (b) Find the drift rate of Π with this delta and use an arbitrage argument to characterise it.