Written Exam at the Department of Economics summer 2018

Pricing Financial Assets

Final Exam

June 9, 2018

(3-hour closed book exam)

Answers only in English.

This exam question consists of 2 pages in total

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Be careful not to cheat at exams!

- You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Consider a commodity with current price S_0. Assume that it can be stored, that there are commodities at storage (so that it may be sold), and that the net storage costs of the commodity can be described as a constant proportional continuously paid cost of δ.
   (a) Describe the workings of a forward contract on the commodity.
   (b) Assume a constant continuously compounded risk-free interest rate of r. Using an arbitrage argument find the forward price F_0 at time 0 on the forward contract that matures at time T *Hint: You may think at an analogy with a stock paying a continuously paid dividend rate.*
   (c) Consider the value of a forward contract with forward price K at time $t < T$ and assume that the commodity price follows the geometric Brownian motion

   $$dS_t = \mu S_t dt + \sigma S_t dz$$

   Using Ito's lemma find the process followed by the value of the forward contract. What will the drift be under the risk neutral measure?

2. Suppose certain derivatives have values that depend on a single state variable given by the process

   $$\frac{d\theta}{\theta} = mdt + sdz$$

   where dz is a Wiener process.
   (a) Consider two such derivatives, assume a continuously compounded risk-free interest rate of r, and use an arbitrage argument to derive and define the market price of ($\theta-$)risk, λ (You may assume that the prices of the derivatives follow geometric Brownian motions).
   (b) If θ is itself a traded asset, what can we say about the relation between m, s and the market price of risk?

3. Suppose a company i is in default before or at time t with probability $Q_i(t)$.
   (a) We will often assume that $Q_i(t)$ is weakly increasing in t. Why?
   (b) In a model of defaults a random variable x_i is used to indicate a default before or at time T. It is assumed that x_i has a standard normal distribution, and that a default occurs at or before time T if

   $$\Phi(x_i) \le Q_i(T)$$

   where Φ denotes the standard normal cumulative distribution function. Interpret this. Why is such a model of computational value?
   (c) In the Vasicek one-factor model of credit risk the default indicators x_i for different companies $i = 1, \dots, n$ are given by

   $$x_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

   where the a_is are constants, and F,$(Z_i)_{i=1,\dots,n}$ are independent standard normal random variables. Describe and interpret this model. When can it be a reasonable description of default?