Written Exam at the Department of Economics summer 2019

Pricing Financial Assets

Final Exam

14 August, 2019

(3-hour closed book exam)

Answers only in English.

This exam question consists of 2 pages in total

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- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
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You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
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- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Let the price of a traded financial instrument, S, be modelled (under the probability measure $\mathbb{P}$) by the geometric Brownian motion

$$dS = \mu S dt + \sigma S dz$$

where μ and $\sigma > 0$ are constants, and z is a Brownian motion.

(a) Describe the qualitative characteristics of this model, and discuss its possible shortcomings as a model of a stock price.

(b) Assume a constant risk free rate of r, and that the instrument pays a continuous dividend stream of δ proportional to the price S. What will the drift rate of the price be under the standard risk neutral probability measure ($\mathbb{Q}$) and a no-arbitrage assumption?

(c) Consider a derivative on S with value V equal to S^n. Use Ito's lemma to find the process followed by V. Is the volatility of V higher, if n is higher?

2. The HJM-model describes the simultaneous evolution of the term structure of interest rates. Let the evolution of instantaneous forward rates contracted at t for time T be described by the Ito-process

$$df(t,T) = m(t,T,\Omega)dt + s(t,T,\Omega)dz$$

where Ω is a set of state variables.

(a) Under certain conditions we have the following no-arbitrage condition for the drift term:

$$m(t,T,\Omega) = s(t,T,\Omega)\int_t^T s(t,\tau,\Omega)d\tau$$

Comment on this result, and in particular explain under which probability measure it is derived.

(b) The short rate in this model is in general non-Markov. Explain what this means, and why it is a complication for implementation.

(c) As a special case let $s(t,T,\Omega)$ be a constant s. Derive the process followed by forward rates. Comment on the distribution of the forward rates.

3. (a) Consider a derivative with value $V(S,t)$ as some function of the current stock price S and time t (and further implicit parameters). Define the Delta, Gamma and Theta of the derivative. What can they be used for?

(b) Assume that the stock pays no dividends before time T, and that there is a constant risk free interest rate of r. Let $c(S,K,T,r)$ and $p(S,K,T,r)$ be the price at time $t=0$ of a European call and a European put, respectively, on the stock with the same strike K and expiry T. Derive the call-put-parity.

(c) Use the call-put-parity to find a relationship between the Deltas of the call and put. Repeat this for Gamma and Theta, respectively.

(d) Suppose a portfolio of the stock and/or derivatives of that stock is Delta-neutral, and that there are no arbitrage possibilities. Let the value of the portfolio be $\Pi(S,t)$. Use the Black-Scholes-Merton PDE to characterise the relation between the Theta and Gamma of the portfolio.