Written Exam at the Department of Economics summer 2019

Pricing Financial Assets

Final Exam

12 June, 2019

(3-hour closed book exam)

Answers only in English.

This exam question consists of 2 pages in total

Falling ill during the exam

If you fall ill during an examination at Peter Bangs Vej, you must:

- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

The exam consists of 3 problems that will enter the evaluation with equal weights.

1. In Vasicek's model, the risk-neutral process for the short rate r is

$$dr = a(b - r)\,dt + \sigma dz$$

where a, b, σ are constants. Suppose $a > 0$.

   (a) Describe the qualitative characteristics of this model of the short rate.

   (b) Consider a zero-coupon bond that pays \$1 at time T. Vasicek shows that its price at time t, denoted $P(t,T)$, can be expressed

$$P(t,T) = A(t,T)\,e^{-B(t,T)r(t)}$$

   using functions A and B that he finds (which we don't need in detail right here). Show that

$$\frac{\partial P(t,T)}{\partial r(t)} = -B(t,T)\,P(t,T).$$

   (c) Argue that the process for $P(t,T)$ in the traditional risk-neutral world satisfies

$$dP(t,T) = r(t)\,P(t,T)\,dt - \sigma B(t,T)\,P(t,T)\,dz(t).$$

2. Suppose f and g are the prices of two non-dividend paying securities with volatilities σ_f and σ_g, depending on a single source of risk captured by the Wiener process z. Suppose that the market price of risk is σ_g. Let r denote the risk-free rate.

   (a) Argue that f and g satisfy

$$df = (r + \sigma_g\sigma_f)\,f dt + \sigma_f f dz \text{ and } dg = \left(r + \sigma_g^2\right) g dt + \sigma_g g dz.$$

   (b) Apply Itô's lemma to find $d\ln f$ and $d\ln g$. Recall that $\ln\left(\frac{f}{g}\right) = \ln f - \ln g$. Show that

$$d\left(\ln\frac{f}{g}\right) = -\frac{(\sigma_f - \sigma_g)^2}{2}dt + (\sigma_f - \sigma_g)\,dz.$$

   (c) Using the equation for $\ln\left(\frac{f}{g}\right)$ from (b), apply Itô's lemma to determine the equation for f/g. Show that f/g has zero drift, and comment on the interpretation of this result.

3. Consider a futures contract on a non-dividend paying stock with a time to maturity of T. Let the current $(t = 0)$ price of the stock be S_0, and assume that the risk-free rate of interest is a constant r.

   (a) The futures price F_0 at $t = 0$ satisfies $F_0 = S_0 e^{rT}$. What is the delta of the futures contract?

   (b) Compare the price of a European call option on the stock with the price of a European call option on the futures contract when the two options have the same strikes and maturities, and they mature at the same time as the futures contract.

   (c) Explain the following in words (no mathematical derivation is expected): in a risk-neutral world, the futures price behaves in the same way as the price of a stock that pays dividend yield r.