Written Exam at the Department of Economics summer 2021

Pricing Financial Assets

Final Exam

June 24th, 2021

(3-hour closed book exam)

Answers only in English.

This exam question consists of two pages in total

Falling ill during the exam

If you fall ill during an examination at Peter Bangsvej, you must:

- submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Consider a stock with price $\mathbf{S}_t$ at time t and a zero coupon bond maturing at time $T > t$ with price $\mathbf{P}(t, T)$ at time t.

   (a) Assume that the stock pays no dividend. What is the forward price $F(t, T)$ at t for the stock with delivery at T?

   (b) Now assume that the stock pays a continuous dividend rate of δ. What is the forward price in this case?

   (c) Let $c(t, K, T)$ and $p(t, K, T)$ denote the prices respectively of a call and a put option on the stock, both with exercise price K and maturity T. What is the put-call-parity for the case with the dividend paying stock?

   (d) If the prices of the call, put and stock as defined above are known at t, derive the implied dividend rate.

2. Consider a situation with credit risk, and let the probability of a borrower not defaulting at or before time t be given by $V(t)$, $V(0) = 1$.

   (a) What is the probability that the borrower will default between time t and $t + \Delta t$ conditional on not being in default at time t? Use this to define the continuously compounded default hazard rate λ.

   (b) How, and under which assumptions, may we estimate the hazard rate from the interest rate spread s on bonds issued by the borrower? Under what probability measure would we say this estimate is derived? Compare this to a hazard rate that is derived from default frequencies and recovery ratios published by a rating agency.

   (c) In one model by Merton the value of a claim on a company with limited liability is modelled using a variation of the Black-Scholes-Merton option model. What is the option features embedded in such a claim? What parameters that are not directly observable must be determined to price the claim in this model? Comment on the model.

3. (a) Give a definition of a payer and a receiver interest rate swaption (also called a swap option)

   (b) Give an argument that a receiver swaption can be seen as a call option on a properly defined fixed rate bond with no credit risk. What can you say about the coupon and principal of such a bond?

   (c) Let PS denote the value of a swaption to pay a fixed rate s_K and receive LIBOR between times T_1 and T_2, let RS denote the value of a swaption to receive a fixed rate of s_K and pay LIBOR between times T_1 and T_2, and let RFS denote the value of a forward starting swap that receives a fixed rate of s_K and pays LIBOR between times T_1 and T_2.

   Assume that there are no arbitrage opportunities and i) show that $PS + RFS = RS$ and ii) deduce that $PS = RS$ when s_K equals the current forward swap rate.