Written Exam, Department of Economics, summer 2019

Economic Growth

(3-hour open/closed book exam)

August 21, 2019

Answers in English only.
This exam consists of 7 pages in total

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- Or if you otherwise violate the rules that apply to the exam

Question 1: Essay questions

1.a

What is a hedonic price index, and how might it help us to capture quality changes of products in the national accounts? Discuss the advantages and disadvantages of hedonic price indices compared to other methods for adjusting for quality changes.

Answer: A hedonic price index is an econometric method to adjust price changes for observed quality changes of components of a product. We can for instance observe how man RAM, how big a hard drive, and how fast a CPU a computer has. By regressing price changes for computers on these observables, we can estimate their values to consumers. With this knowledge, we can separate price changes for computers in a part related to improved components, which reflect a quality increase rather than a price increase, an unexplained part. The unexplained part is assumed to reflect a true price increase.

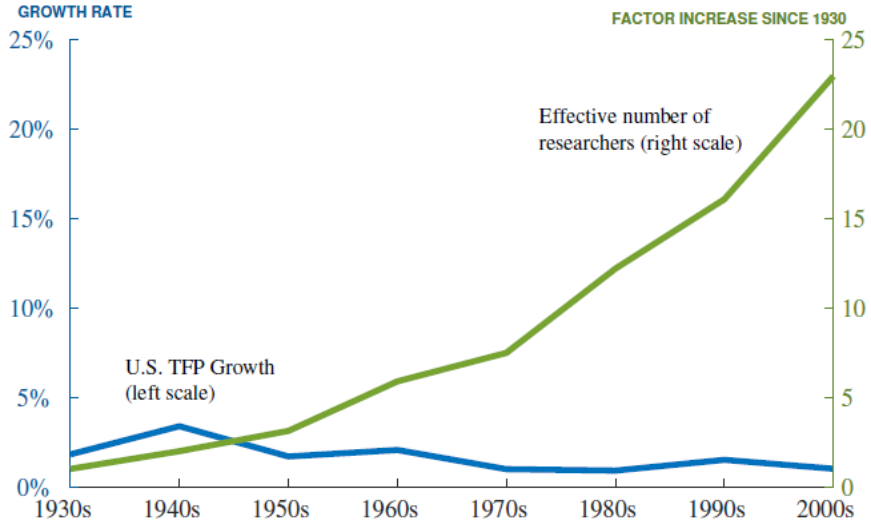
A hedonic price index is clearly superior to not taking quality changes into account, and neither does it suffer the same bias as linking, which implicitly assumes that a higher price of a new computer model is entirely accounted for by increased quality. A drawback of hedonic price indices is that it is impossible to include all things consumers value about a product in a regression. Moreover, only few types of products, such as computers or cars, are suitable for hedonic price indices. Most products do not have components with traits that are quantifiable. For example, how would you quantify the taste of a chocholate bar in a regression?

1.b

Automation may hurt workers' income and employment prospects due to a displacement effect. Through what mechanisms might the displacement effect be mitigated?

Answer: According to the model in Acemoglu and Restrepo (2018a), automation causes both a displacement effect and a productivity effect. The productivity effect comes from the fact that the relative price of the goods previously produced by workers can now be produced at lower cost by machines. Their relative prices therefore decrease, pushing up the purchasing

Figure 1: Aggregate Data on Growth and Research Effort



power of worker's wages. Whether the displacement effect dominates the productivity effect depends on the capital intensity of the economy, according to the model.

Additionally, the displacement effect of automation might be mitigated through other types of technical change. Labour augmenting technical change will benefit workers, and so will capital augmenting technical change insofar it does not cause any worker displacement (i.e., when new and better machines replace old machines). The emergence of new tasks may also benefit workers.

1.c

Figure 1 in Bloom et al (2019), reproduced here, shows TFP growth and the number of (effective) researchers in the US. Explain why the trends in this figure are consistent with, but do not prove that research productivity is declining.

Answer: Suppose that constant output from the R&D sector leads to constant growth in TFP. Under this assumption, research productivity can be defined as TFP growth divided by the (effective) number of researchers. The approximately constant rate of TFP growth and the rising number of researchers is therefore consistent with declining research productivity. However, one could also imagine that economic growth caused the number of goods produced in the economy to expand. The productivity of research that improves the

quality of existing goods could in that case be constant, while demand for researchers rise because of the larger number of goods in the economy.

Students could in this question also comment on potential measurement problems (e.g., maybe firms just classify more and more activities as research to get tax exemptions), but, as argued by Bloom et al (2019) such problems are likely to be of a much smaller magnitude than the decline in research productivity they find.

1.d

Statement: "*Faster growth inevitably leads to higher welfare*". Do you agree or disagree? Please explain why/why not.

Answer: No not inevitably. Here are some reasons that have been highlighted in the course: (a) It is possible that the engine of growth – capital accumulation say– is associated with congestion effects. In this setting the market growth rate can be too high from a social planner perspetive, since private agents do not internalize this negative externality from capital accumulation. (B&S ch. 4) (b) A similar situation can arise in an Agion-Howitt model where innovation exerts a negative externality on the return to future innovators due to creative destruction. (Aghion and Howitt). (c) Faster growth may be associated with faster technological change which brings the risk of disaster (which may be very low though). If agents are risk averse it can be optimal to bring growth to an end (Jones).(d) Faster growth may be associated with rising inequality. As rising inequality lowers social welfare it may be relevant to slow down growth. (e) Faster growth may, due to creative destruction, leave people unemployed which can lower welfare absent government policy. (Aghion et al.).

1.e

Please, explain how espionage during the cold war can help us understand the convergence process.

Answer: The study by Glitz and Meyerson examines the influence of information flows from

spies in the west (German federation) to the former GDR on industry level total factor productivity. The reports from the spies in the West were assigned with keywords that the authors use to assign the information to sectors. Their key independent variable is thus the number of reports pertaining to a specific sector for a particular time period, which is then regressed on sector specific tfp-gaps between the west and the east. Endogeniety is attempted dealt with by way of IV regressions. The key insigt is that industrial espionage considerably reduced the TFP gap between the west and the east, highlighting the role of idea flows between nations for convergence.

1.f

It is conventional wisdom that older firms tend to be less innovative; especially if they hold market power. What could explain a lack of innovative activity on the part of an incumbent? (Please, explain.)

Answer: A candidate explanation is the so-called *Arrow replacement effect*. Suppose the value of innovating is V^{t+1}, which captures expected discounted profits from a new idea to an entrant. An incumbent, however, will obtain $V^{t+1} - V^t$ in as much that a new innovation renders the existing product obsolete and without value. Hence, the fact that an incumbent "replaces" itself, may likely cause it to be less innovative since it rationally expects entrants to have a greater incentive to exert time and resources. This are different, however, if the incumbent has lower costs when innovating. Nevertheless, the Arrow replacement effect can be part of an explation for the causal empirical observation highlighted in the question.

Question 2: The skill premium in the canonical model

Suppose that the labor force consists of skilled and unskilled workers. Each worker inelastically supplies one unit of labor. Aggregate output, produced by a representative firm acting as a price-taker, is given by:

$$Y = \left[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

L and H are the number of unskilled and skilled workers, respectively, and A_L and A_H are the associated productivity levels.

2.a

Show that skill premium, defined as the wage of skilled workers relative to the wage of unskilled workers, is given by:

$$\omega = \frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}$$

Figure 2 below shows the evolution of the relative wage rate of college graduates in the US. How might this picture be explained, according to the expression you have just derived?

Answer: The first order conditions for the representative firm's profit maximization problems yield the wages for the two groups of workers:

$$w_L = \left[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} A_L^{\frac{\sigma-1}{\sigma}} L^{\frac{1}{\sigma}},$$

and

$$w_H = \left[(A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} A_H^{\frac{\sigma-1}{\sigma}} H^{-\frac{1}{\sigma}},$$

from which the skill premium is easily derived. The skill premium is driven by two opposing forces: a higher share of skilled workers in the economy decreases the skill premium due to a supply effect, whereas skill biased technical change, defined as an increase in $\frac{A_H}{A_L}$, increases the skill premium under the realistic assumption that σ is larger than one (empirical estimates of σ are typically in the 1.5-3 range). According to this framework, the rising skill premium shown in Figure 2 must be due to skill biased technical change. The education level in the US population has increased over the period, which would have reduced the skill premium in the absence of skill biased techical change.

2.b

How would you use the expression for the skill premium to test the model empirically? What will the regression look like, and what variables would you use? What are the strengths and weaknesses of this approach to analyzing the evolution of the skill premium?

Answer: First, take the log of the skill premium to transform it into a linear relationship:

$$\ln \omega = \frac{\sigma - 1}{\sigma} \ln \left(\frac{A_H}{A_L} \right) - \frac{1}{\sigma} \ln \left(\frac{H}{L} \right)$$

We observe the skill premium and the supplies of workers, but the relative productivity level $\frac{A_H}{A_L}$ is unobserved. We therefore need to impose some more structure on the model. This can be done in various ways, but the simplest approach is to assume that the skill bias in technology increases at a constant rate γ_1. We can write:

$$\ln \left(\frac{A_{Ht}}{A_{Lt}} \right) = \gamma_0 + \gamma_1 t$$

where t denotes time. Plugging this expression into the log skill premium (with added time subscripts) yields:

$$\ln \omega_t = \frac{\sigma - 1}{\sigma} \gamma_0 + \frac{\sigma - 1}{\sigma} \gamma_1 t - \frac{1}{\sigma} \ln \left(\frac{H_t}{L_t} \right)$$

The corresponding OLS regression is:

$$\ln \omega_t = \beta_0 + \beta_1 t - \beta_2 \ln \left(\frac{H_t}{L_t} \right) + \varepsilon_t.$$

The strength of this approach is that it is simple, and yet it has a very high explanatory power. The main weakness is that we have to assume a functional form for skill biased technical change, so we do not learn much about skill bias except that the parameter β_1 will capture its average growth rate. There are neither theoretical or empirical evidence that skill bias grows at a constant rate over long time horizons, so while the assumption appears to be a reasonable approximation over the period covered in Figure 2, we cannot extrapolate to the

future (or to the past, where empirical evidence shows that the skill premium were fell between 1900 and 1940).

2.c

We also observe increased wage dispersion within educational groups, meaning that wage differences among, e.g., college graduates, are increasing. Acemoglu and Autor (2011) argue that the canonical model is consistent with this pattern if true skill levels are unobserved, but correlated with education levels. To be specific, they assume that a fraction ϕ_c of college graduates are skilled, and a smaller fraction ϕ_n of those without college are skilled. Firms observe whether a worker is skilled or not, and pay them accordingly, but in the statistics we only the worker's education level. Use these assumptions to write down the observed college wage premium as a function of ϕ_c, ϕ_n and the true, unobserved skill premium. Show that skill biased technical change increases the observed college wage premium.

Answer: Let $\widehat{\omega}$ denote the observed wage premium of college graduates, and ω the true unobserved skill premium. We can then write

$$\widehat{\omega} = \frac{\phi_c w_H + (1-\phi_c)\, w_L}{\phi_n w_H + (1-\phi_n)\, w_L} = \frac{\phi_c \omega + (1-\phi_c)}{\phi_n \omega + (1-\phi_n)}$$

A higher ω increases the numerator by more than the denominator because $\phi_c > \phi_n$, so an increasing unobserved skill premium also increases the observed college wage premium.

2.d

Suppose now that ϕ_c and ϕ_n are endogeneous variables. How might these have changed over the past decades? Can such changes can help us to understand the trend observed in Figure 2?

Answer: The share of college graduates in the population has increased over the past decades, so one should expect that the cognitive ability of the average college graduate has declined over time. In other words, we should expect ϕ_c to have fallen. However, it is also highly likely within the group that formerly would have ended up with less than a college degree, it is the most skilled individuals who now go to college. This suggests that ϕ_n has fallen,

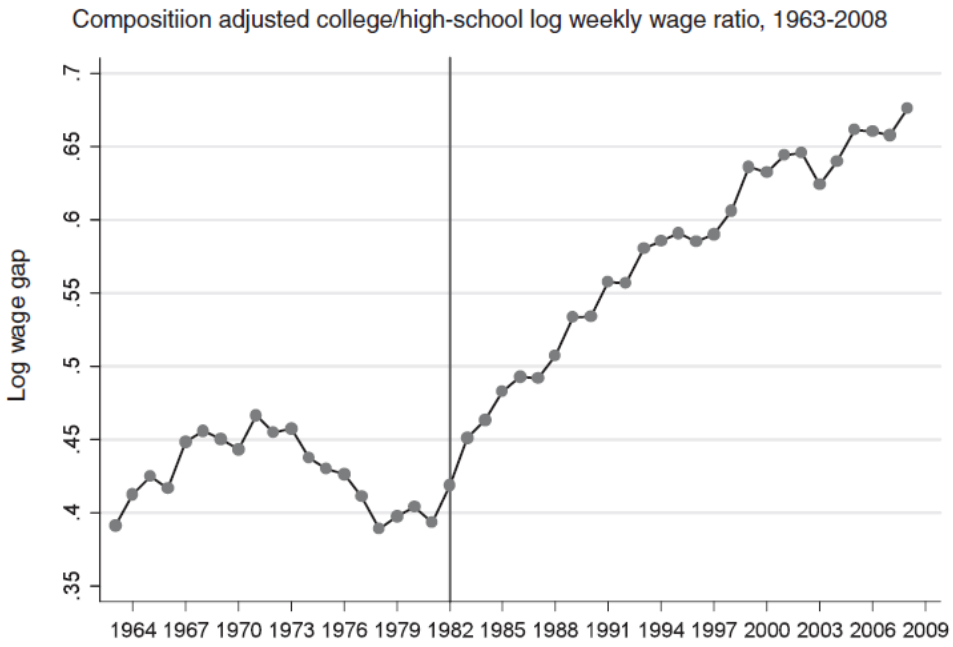


Figure 2: The evolution of the skill premium in the US

too. The impact on the observed college wage premium depends on which effect dominate. If this selection process is contributing positively to the observed skill premium, we should expect the fall in ϕ_n to have been greater than the fall in ϕ_c, a possibility that appears plausible given the discussion above.

Question 3: The Romer model

Consider a Romer-model of growth through research and development (R&D). Time is continous and supressed in the notation that follows. The economy is closed. Final goods are produced using the technology:

$$Y = L_Y^{1-\alpha} \sum_{i=1}^{A} x_i^{\alpha},$$

where L_Y is labor input in the final goods sector, x_i is an specialized intermediate input and A is the number of such inputs, as well as an index signifiying the level of technology in the economy. The final goods sector is competitive, and the price of output is normalized to one. The real wage is denoted w and the price of the ith specialized input is p_i.

3.a

Solve the profit maximization problem of the final goods firm and derive the demand for labor and intermediate goods, $i = 1., ..., A$.

Answer:

$$\pi = L_Y^{1-\alpha} \sum_{i=1}^{A} x_i^{\alpha} - wL_Y - \sum_{i}^{A} p_i x_i$$

FOC

$$w = (1-\alpha) L_Y^{-\alpha} \sum_{i=1}^{A} x_i^{\alpha}$$

$$\alpha L_Y^{1-\alpha} x_i^{\alpha-1} = p_i, \ i = 1, ..., A.$$

3.b

x_i is produced in an intermediate good sector, which comprises A firms. Each firm is a monopolist, and thus selects the price p_i and quantity, x_i. A unit of intermediate good is produced using one unit of physical capital, so $x_i = K_i$. The cost of a unit of capital is the real rate of interest r. Solve the problem of the intermediate goods producer, and show that her profits are

$$\pi_i = (1-\alpha) \alpha L_Y^{1-\alpha} \bar{x}^{\alpha}, \text{ for all } i = 1, ...A.$$

where $\bar{x}$ is the optimal quantity supplied by the monopolist, which is the same for all $i = 1, ..., A$.

Answer:

$$\pi_i = p_i x_i - r x_i$$

s.t.

$$\alpha L_Y^{1-\alpha} x_i^{\alpha-1} = p_i$$

Accordingly, the first order condition is

$$\alpha^2 L_Y^{1-\alpha} x_i^{\alpha-1} = r$$

from which we conclude $x_i = \bar{x}$ for all i. Price (substituting into demand)

$$p_i = \frac{1}{\alpha} r$$

that is, a mark-up over marginal cost, and $p_i = \bar{p}$ for all i. Profits

$$\pi_i = (p - r)\, x_i = (1 - \alpha)\, \bar{p}\bar{x} = (1 - \alpha)\, \alpha L_Y^{1-\alpha} \bar{x}^{\alpha}.$$

3.c

Ideas are produced using the technology $\dot{A} = \delta L_A A$. There is free entry into the research sector, for which reason perfect competition rules. Accordingly, the R&D firm owner takes the value of an idea, p_A, as given along with the value price of labor, w. Show that profit maximization implies

$$p_A \delta A = w.$$

Answer:

$$\max_{L_A} p_A \dot{A} - w L_A$$

st

$$\dot{A} = \delta L_A A$$

straight forward differentiation yields the stated FOC.

3.d

Along a balanced growth path – which is what we are considering from now on – the value of a new idea, p_A, equals the discounted value of profits from production of the type of intermediate good that is produced using the idea. The monopolist has a patent of infinite duration. Accordingly, along a balanced growth path $p_A = \pi / r$. Furthermore, labor markets clear so that $L_A + L_Y = L$ where L is total (and time constant) labor supply. Show that the growth rate of

ideas fulfills

$$\frac{\dot{A}}{A} \equiv \gamma = \delta L - \frac{r}{\alpha}. \tag{1}$$

Answer: We start with

$$p_A \delta A = w$$

using that $p_A = \pi/r$ and the expression for profits we obtain

$$\frac{\pi}{r}\delta A = \frac{(1-\alpha)\,\alpha L_Y^{1-\alpha}\bar{x}^{\alpha}}{r}\delta A = w$$

Next using the first order condition from profit max in the final goods sector yields

$$\frac{(1-\alpha)\,\alpha L_Y^{1-\alpha}\bar{x}^{\alpha}}{r}\delta A = (1-\alpha)\,L_Y^{-\alpha}\sum_{i=1}^{A} x_i^{\alpha} = (1-\alpha)\,L_Y^{-\alpha} A\bar{x}^{\alpha}$$

where the last equality follows in that x is constant as established in question 2. Isolating L_Y

$$\begin{aligned}
\frac{(1-\alpha)\,\alpha L_Y^{1-\alpha}\bar{x}^{\alpha}}{r}\delta A &= (1-\alpha)\,L_Y^{-\alpha} A\bar{x}^{\alpha} \\
\frac{(1-\alpha)\,\alpha L_Y^{1-\alpha}}{r}\delta &= (1-\alpha)\,L_Y^{-\alpha} \\
\frac{(1-\alpha)\,\alpha L_Y}{r}\delta &= (1-\alpha)
\end{aligned}$$

$$L_Y = \frac{r}{\delta\alpha}$$

Finally, since

$$\frac{\dot{A}}{A} = \delta L_A = \delta\,(L - L_Y) = \delta L - \delta L_Y = \delta L - \frac{r}{\alpha}$$

3.e

Households maximize utility from consumption over an infinite horizon. Per period utility is logaritmic, $u(c) = \ln(c)$. As a result, they follow the standard consumption Euler equation

$$\frac{\dot{C}}{C} = r - \rho = \gamma.$$

where the last equality follows as we are considering a balanced growth path where all endogenous variables grow at the same rate. Derive the long-run growth rate of the economy, as well as the long-run real rate of interest.

Answer:

$$\begin{aligned} r - \rho &= \gamma \Leftrightarrow r = \gamma + \rho \\ \gamma &= \delta L - \frac{r}{\alpha} \end{aligned}$$

inserting the expression for r into the growth equation:

$$\gamma = \delta L - \frac{\gamma + \rho}{\alpha}$$

$$\gamma = \frac{\alpha\delta L - \rho}{1 + \alpha}$$

Substituting the growth rate into the consumption euler yields

$$r = \frac{\alpha\delta L - \rho}{1 + \alpha} + \rho = \frac{\alpha\delta L - \rho + (1 + \alpha)\rho}{1 + \alpha} = \frac{\alpha}{1 + \alpha}(\delta L + \rho).$$

3.f

Over the last 30 years the real rate of interest has been declining empirically. According to the model: what could generate a reduction in the real rate of interest? What would be the implications for the growth rate of the economy?

Answer: A reduction in the elasticity of output with respect to intermediate goods, α, (i.e.,

in principle the share of capital in national accounts) a reduction in the labor force L, in R&D productivity, δ, and a lower discount rate ρ (more patience) will lead to a reduction in the long-run real rate of interest. As for the implications for growth: it depends on the cause of the decline in the real rate. If the decline is caused by factors that do not affect R&D directly (i.e., involves ρ or α) a reduction in the real rate of interest should speed up growth. However, if the real rate of return declines due to declining R&D productivity, δ, or a lower labor force, it would be associated with slower growth according to the Romer model. Hence, the model holds different implications that e.g. AK-type one sector models, where growth and the real rate of interest *nessesarily* moves in the same direction.