Written Exam Economics Winter 2019-20

Auctions: Theory and Practice

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This exam question consists of 7 pages in total (including this front page)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is, for example, if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Introduction

Throughout the assignment, please show your work. Simply stating the correct answer without sufficiently explaining your calculations/reasoning is not enough to get full credit. Correspondingly, an incorrect or incomplete answer that uses some of the correct argumentation may be given partial credit.

If you believe that there may be a typo in one of the questions, or if something is stated unclearly, please let us know as quickly as possible by sending an email to both Neil and Holger. Any material responses to such queries will be published on Absalon.

Good luck!

Problem 1 (True or false)

Please state whether each of the following statements is true or false and show the arguments and/or calculations which justify your conclusion.

1a. 5 bidders with private values uniformly distributed between 0 and 1 partake in a 1st price auction. Assuming that everyone is playing the symmetric equilibrium bidding strategy, the optimal bid for a bidder who makes a draw of 0.75 is 0.7.

1b. The expected revenue of a 1st price auction with 2 bidders uniformly distributed between 2 and 6 and a reserve price of 3 is 3.5.

1c. In a multi-unit setting, a discriminatory auction is less susceptible to collusion than a Vickrey auction. (a qualitative explanation is sufficient)

1d. An information rule can be used to avoid or mitigate so-called "push bidding". (a qualitative explanation is sufficient)

1e. In a multi-unit uniform auction, it is a dominant strategy for bidders with single-unit demand to bid their true value vector. (a qualitative explanation is sufficient)

Problem 2

Renowned graffiti artist Banksta has created a new painting. It depicts two monkeys in suits eating caviar. The painting serves as a subtle criticism of excessive wealth in the West.

Banksta wants to allocate the painting via auction to ensure efficiency (i.e. to make sure that the person who values the painting the most wins it). But he wants to avoid a high selling price as this would go against the very ideology of his painting.

Banksta thus devises an auction format whereby the highest bidder will win but pay a price only equivalent to the lowest bid submitted at the auction. Banksta believes that this format will guarantee efficiency whilst also achieving a low selling price.

We can think of this as an N-th price auction, i.e. the highest bidder wins but pays a price equivalent to the N-th highest bid (where N bidders partake in the auction).

Consider N risk-neutral Banksta fans with valuations x independently and uniformly distributed between 0 and 5 million pounds.

2a. Find the symmetric equilibrium bidding strategy for the N bidders in a sealed-bid N-th price auction. Please also explain the intuition of your result.

Banksta expects that the more people he can get to turn up at the auction, the lower the selling price will be, since it is more likely that low-value bidders will turn up and thus submit the lowest bid.

2b. Is Banksta correct that more bidders will lead to lower revenue? In other words, what is expected revenue in this auction, and how does it depend on the number of bidders? Please also explain the intuition of your result.

Banksta concedes that he does not know exactly how many bidders will participate at the auction, and he realises that bidders will probably also be unsure regarding the true number of bidders when submitting their bid. He does not think that this should matter too much for achieving his main goal of efficiency.

2c. Do you think that the N-th price auction is likely to achieve an efficient outcome? Given Banksta's main goal of efficiency, would you recommend any changes to the auction design? (a qualitative explanation is sufficient)

Problem 3

Horse-house is a famous auction house, which sells racehorses. Horse-house has used the single-unit Dutch auction format for generations. Horse-house's main objective is revenue maximisation.

A risk-averse accountant has informed Horse-house's management that it should try to steady its revenue streams in coming years. To this aim, the accountant has suggested that Horse-house should consider switching to English auctions instead of Dutch auctions as this may lead to less variable revenue.

Horse-house's management asks you, the stable boy/girl (who has studied auction theory), what you think of the accountant's advice.

3a. Is the accountant correct that English auctions will generate more stable (i.e. less variable) revenue than Dutch auctions? Why/why not? (a qualitative explanation is sufficient)

The accountant also advises management that there is a common value element to the valuations of racehorses (since the value of a horse depends in part on its forecasted ability to win races). The accountant advises that English auctions may lead to higher revenues in such a situation.

Again, Horse-house's management asks you, the stable boy/girl, for your thoughts.

3b. Given the common value element (with affiliated signals) of racehorse valuations, would you expect English auctions to generate more, less or the same revenue as Dutch auctions? Why? (a qualitative explanation is sufficient)

Horse-house's management has been in the business for many years and has noticed that bidders at racehorse auctions tend to be quite superstitious. As soon as they have a good feeling about a horse, bidders generally, become quite risk-averse in their quest to win it.

Horse-house asks you, the stable boy/girl, whether and how you think this observation impacts the decision of whether to transition to English auctions.

3c. Given the potential risk aversion of bidders (and ignoring the common value element), would you expect English auctions to generate more, less or the same revenue as Dutch auctions? Why? (a qualitative explanation is sufficient)

Regardless of your advice (and that of the accountant), Horse-house ultimately decides to stick with its tried-and-tested Dutch auction format (since the Dutch format is such an integral part of the Horse-house brand).

Horse-house is now arranging the sale of two young colts (i.e. young male horses): Thunder and Lightning. Thunder and Lightning are twins and are deemed to have exactly equal potential as racehorses. Horse-house is going to sell the two colts in two sequential Dutch auctions. In each auction, the price will start at 1 million pounds and then slowly descend until a bidder submits a bid, at which point the horse will be sold at that price.

Three bidders will take part at the auctions: Tommy, Charlie and Alfie. Tommy, Charlie and Alfie each have single-unit demand, i.e. each of them wants to buy only one of the two horses. Each bidder holds a valuation for one of the colts, which is uniformly distributed between 0 and 1 million pounds. Each of the bidders is indifferent between the two horses. For the purposes of the rest of this problem, you can ignore the potential common value element and also assume bidder risk neutrality.

3d. Find the symmetric equilibrium bidding strategy in each of the two sequential auctions. Please also explain the intuition of your result.

Horse-house proceeds with the first auction.

To everyone's shock, Thunder is sold to Tommy in the first auction at a price of just 1 pound! Horse-house suspects that the three bidders may have formed a bidding ring, i.e. they may have agreed not to compete with one another at the live auction.

Horse-house wonders how it can counter such a bidding ring and considers implementing a reserve price for the auction of Lightning. Horse-house wants to set the revenue-maximising reserve price in the second auction, under the assumption that there is a bidding ring, and assuming that Tommy was the highest value bidder.

Horse-house's management asks you, the stable boy/girl, which reserve price it should set.

3e. What is the revenue-maximising reserve price in the second auction? Hint: you may need to use Wolfram Alpha (or similar) to identify the roots of a polynomial.

Problem 4

Danish shipping company mAhoy is struggling financially. Shipping volumes have been declining due to the ongoing trade war between China and the US, and the company is faced with an oversupply of ships. To reduce supply and free up financial liquidity, the company has decided to divest one of its largest container carriers, Queen Margrethe II. At the same time, the company is planning an IPO in the near future to generate more capital and thereby weather the storm.

You are a student employed in the department tasked with selling the ship. Due to the upcoming IPO, mAhoy needs a good estimate of the expected selling price and the CFO has asked your department to come up with a number. For now, mAhoy's goal is to get as high a selling price as possible.

The team at your department has done some research about the prospective bidders. You expect N risk-neutral symmetric bidders to participate in the auction. Your department assumes that the bidders' values for the ship are interdependent and equal to:

$$v_i = \frac{1}{2}x_i + \frac{1}{2} * \frac{1}{N-1}\sum_{j \neq i} x_j$$

Where each bidder's signal, x_i, is uniformly and independently distributed between 0 and 100 million US dollars.

The head of your department is leaning towards selling the ship using a 2nd price auction. He has heard that with this format it is a dominant strategy for all of the bidders to bid truthfully – i.e. to place a bid of x_i. You are quite sure that he is incorrect, since you know of the winner's curse when bidders' values are interdependent.

To convince your boss, you have decided to do some calculations.

4a. Derive the bidders' expected payoff from winning the auction depending on the number of bidders in the auction, N, under the assumption that each bidder bids its signal x_i. How many bidders are needed for the expected payoff to become negative? Please also explain the intuition of your result.

You have managed to convince your boss and he is quite happy with your work. Thus, he has asked you to derive the correct symmetric equilibrium bidding strategy, i.e. what bidders should actually bid (given that they do not simply bid x_i).

4b. Derive the bidders' symmetric equilibrium strategy and the degree of the shading depending on N. Please also explain the intuition of your result.

Your department expects five bidders to participate in the auction.

4c. Derive expected revenue from selling the ship with five expected bidders.

Your boss has heard that an open auction format such as the English auction is preferred when bidders' values are interdependent for two reasons in particular. Firstly, an open format will lead to increased expected revenue due to price discovery. Secondly, an open format reduces strategic complexity for the bidders. Impressed by your work so far, he has asked for your opinion.

4d. Are the two statements correct given the assumptions about the prospective bidders? Does this conclusion hinge on a specific assumption made by your department? (A qualitative explanation is sufficient)

The CFO has called you and your boss in for a meeting to present your results. She is impressed with your work, but wants to move forward with a 1st price sealed-bid auction since she is worried that only one or two bidders will participate in the auction due to the dire market circumstances facing all shipping companies.

At the meeting, the CFO has also presented you with a new complexity. The board of mAhoy is worried that some of the bidders will not take proper care of the ship. Queen Margrethe II is one of mAhoy's finest ships and she was the last ship to be produced at mAhoy's shipyard, which was put out of operation in the wake of the global financial crisis. Thus, the board believes that it is important for mAhoy's reputation that the ship is kept in fine shape by her new owner. The CFO wants you to think of a way to incorporate this goal into your design of the 1st price auction.

To incorporate this concern into the design, your department has devised a multi-criteria auction (you have experience using this approach from procurement auctions).

The design will work as follows: All prospective bidders will be assigned a score between 0 and 10 depending on how well mAhoy thinks the company will maintain the ship. 10 is the highest score. The price bids will also be translated into a linear score between 0 and 10, where 10 will be awarded to bids of 100m and 0 will be awarded to bids of 0. The two scores will then be combined to produce a total score, S_i^t:

$$S_i^t = \frac{1}{2}S_i^b(b_i) + \frac{1}{2}S_i^m$$

Where $S_i^b(b_i)$ denotes the price bid score for bidder i as a function of the price bid and S_i^m denotes the maintenance score for bidder i. The bidder with the highest total score will be awarded the ship and will pay its own price bid, b_i. The maintenance scores will be published prior to the auction.

4e. How would the multi-criteria auction outlined above affect price bidding behaviour for hypothetical bidders with relatively low maintenance scores or relatively high maintenance scores respectively? (a qualitative explanation is sufficient)

Problem 5

The municipality of Copenhagen has received many complaints from its citizens over a series of new companies that offer short-term rentals of electric scooters (e-scooters). The majority of the complaints relate to an issue of overcrowded sidewalks throughout the city. To mitigate this issue, the city council has decided to introduce a licensing system to restrict the number of e-scooters in the city to 1,000 units. The licenses will expire after five years, after which the municipality will assign new licenses.

There are currently five companies offering the rental service in Copenhagen. Since the e-scooter rental service is a new and developing industry, the municipality has limited knowledge of how profitable the companies are today and how profitable the companies will be depending on the number of licenses they are allocated. At this stage, the city council is also unsure of its main considerations/goals in relation to the auction other than that it wants a well-functioning low-priced rental service.

The municipality has decided to allocate the licenses to the rental companies using an auction and has asked you to submit a proposal for auction expert assistance. In the proposal, you are asked to write:

- A short of description of the main characteristics of the licenses and prospective bidders as well as what that means for auction design
- Your opinion on what you think the main goal(s) for the municipality should be when designing the auction

5a. Write the proposal as outlined in the two bullets above in a maximum 250 words. (A qualitative explanation is sufficient.)

After reading your proposal, the municipality has decided to hire you. The municipality has done some more research on the market for e-scooter rentals and can share two findings with you. Firstly, the municipality expects that prospective bidders will be highly uncertain regarding the business case of the rental service since the market is still immature. Secondly, the municipality expects that each company will require a certain scale of e-scooters to offer a profitable service.

5b. Based on the municipality's two findings, which auction format would you recommend? Why? (A qualitative explanation is sufficient.)

One of the electric scooter rental companies, Scoolectric, is infamous for employing an aggressive market strategy of swarming each city with scooters trying to push competitors out of the local market. The council is worried that Scoolectric will outbid all competitors in the auction to secure a monopoly in Copenhagen and subsequently push up rental prices.

5c. Could you address this concern using auction design? Could this affect the municipality's expected revenue? Could there be any other issues associated with this approach? (A qualitative explanation is sufficient)