Written exam for the M. Sc. in Economics. Summer 2013

Economic Growth

Master's Course

June 12, 2013

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total including this page.

The weighting of the problems is: Problem 1: 45%, Problem 2: 40%, and Problem 3: 15%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 We consider a market economy where people first attend school full-time and then work full-time until death. People are born with the same abilities. If a person attends school for S years, he or she obtains human capital $h = h(S)$, $h' > 0$. A person "born" at time v (v arbitrary) chooses S to maximize

$$HW_v = \int_{v+S}^{\infty} \hat{w}_t h(S) e^{-(r+m)(t-v)} dt, \tag{*}$$

where $\hat{w}_t$ is the market-determined real wage per year per unit of human capital at time t, r is a constant real interest rate, and m is a parameter such that the probability of surviving at least until age $\tau > 0$ is $e^{-m\tau}$. It is assumed that owing to technical progress,

$$\hat{w}_t = \hat{w}_0 e^{gt}, \tag{**}$$

where g is a constant satisfying $0 < g < r + m$.

a) Interpret the decision problem, including the parameter m. Is there a sense in which the infinite horizon in (*) can to some extent be defended as an approximation?

b) Let the optimal S for a person be denoted S^*. Given (*) and (**), show that S^* satisfies the first-order condition $h'(S^*)/h(S^*) = r + m - g \equiv \tilde{r}$. *Hint:* Substitute (**) into (*) and move $h(S)$ outside the integral; then perform the integration by applying that for given constants a, b, and c, where $a < 0$, $\int_b^\infty e^{a(t-c)} dt = \left. \frac{e^{a(t-c)}}{a} \right|_b^\infty = \frac{e^{a(b-c)}}{-a}$; finally, maximize w.r.t. S.

c) Provide the economic intuition behind this first-order condition.

Let

$$h(S) = S^\eta, \qquad \eta > 0. \tag{***}$$

d) Solve for S^*. It can be shown that for the second-order condition to ensure that the first-order condition gives an optimum, the elasticity of h' w.r.t. S must be smaller than the elasticity of h w.r.t. S at least at $S = S^*$. Check whether this condition is satisfied in the case (***).

e) With one year as the time unit, let the parameter values be $\eta = 0.6$, $r = 0.06$, $m = 0.008$, and $g = 0.018$. What is the value of the optimal S measured in years? Comment.

f) How will an increase in life expectancy affect m and the optimal S, respectively? What is the intuition?

Suppose there is perfect competition in all markets and that the representative firm chooses capital input, K_t, and labor input (measured in man-years), L_t, in order to maximize profit, given the production function

$$Y_t = F(K_t, A_t h L_t),$$

where Y_t is output, A_t is the technology level, and F is a neoclassical production function with constant returns to scale. There is a constant capital depreciation rate $\delta > 0$. Suppose further that the country considered is a small open economy which is fully integrated in the world market for goods and financial capital. Let the real interest rate in this market be a constant equal to r.

g) Let the equilibrium real wage *per year* at time t for a typical member of the labor force be denoted w_t. Find w_t. *Hint:* Determine $\tilde{k} \equiv K_t/(A_t h L_t)$ from one of the firm's first-order conditions.

h) Express w_t in terms of $\hat{w}_t$. What is the growth rate of w_t according to the information given in the introductory paragraph above? And what is the implied growth rate of A_t?

i) Let $y_t \equiv Y_t/L_t$. Does education affect the level of y_t in this economy? Find the growth rate of y_t. Does education affect the growth rate of y_t? Comment.

Problem 2 Consider a closed economy with profit maximizing firms, indexed $i = 1, 2, \ldots, N$, where N is "large". Markets are competitive. Time is continuous and there is no uncertainty. Aggregate output (GDP) at time t is Y_t per time unit. Output is used for consumption and investment in physical capital, K_t, so that $\dot{K}_t = Y_t - C_t - \delta K_t$, where C_t is aggregate consumption and δ is a constant capital depreciation rate, $\delta \geq 0$. The size of the labor force (= employment = population) is $L_t = L_0 e^{nt}$, where $n \geq 0$ is a constant. There is a perfect market for loans with a real interest rate r_t.

The production function of firm i is

$$Y_{it} = K_{it}^{\alpha}(A_t L_{it})^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{*}$$

where A_t is the economy-wide technology level, $\sum_i K_{it} = K_t$, and $\sum_i L_{it} = L_t$. We assume that every firm is small relative to the economy as a whole and perceives it has no influence on aggregate variables, including A_t.

a) In general equilibrium, determine r_t and the aggregate production function at time t for a given A_t.

Suppose A_t evolves according to

$$A_t = K_t^{\lambda}, \qquad 0 < \lambda \leq 1, \tag{**}$$

where λ is a constant.

b) Briefly interpret (**) in connection with (*).

c) Find the social marginal productivity of capital. Compare with the private marginal productivity of capital.

d) There is a certain feature of the economy which "invites" government intervention in the market mechanism. What is this feature?

We introduce a government which contemplates to (i) pay an investment subsidy $s \in (0, 1)$ to the firms so that their capital costs are reduced to $(1 - s)(r + \delta)$ per unit of capital per time unit; (ii) finance this subsidy by a constant consumption tax; and (iii) supplement the consumption tax with a time-dependent lump-sum tax or transfer whenever necessary to ensure a balanced budget. For simplicity we assume there is no income taxation.

Suppose you, as an economic advisor, are asked by the government to find out what an economically efficient size of s would be.

e) Derive a formula for the size of s you would suggest. *Hint:* Given an investment subsidy s, determine r_t in general equilibrium; apply that a necessary condition for economic efficiency is that the intertemporal rate of transformation faced by the consumer equals the *net* social marginal productivity of capital.

f) What size is your chosen s in case $\alpha = 1/3$ and $\lambda = \frac{1}{2}$? What size is it in case $\alpha = 1/3$ and $\lambda = 1$? What is the intuition behind the sign of the difference in the two cases?

From now on assume that $0 < \lambda < 1$ and $n > 0$.

g) Determine the growth rate of $y \equiv Y/L$ under balanced growth, assuming gross saving is and remains positive. *Hint:* If a production function $Y = F(K, AL)$ is homogeneous of degree one, then

$$\frac{Y}{K} = F(1, \frac{AL}{K});$$

combine this with a certain general balanced growth property.

h) How, if at all, is the growth rate of y related to s and n, respectively? Comment.

Problem 3 *Short questions*

a) Do you view the presence of scale effects on levels in an endogenous growth model as a strength or weakness of the model? Why?

b) There exists a class of growth models that build on the idea that "variety is productive". What is meant by that phrase?

c) In the R&D-based growth model by Charles Jones, research labor is the only rival input in R&D. Yet at the aggregate level the model makes allowance for decreasing returns to research labor in R&D. What is the interpretation?

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