Economic Growth, August 2016.

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# A suggested solution to the problem set at the re-exam in Economic Growth, August 29, 2016

(3-hours closed book exam)[1]

As formulated in the course description, a score of 12 is given if the student's performance demonstrates precise understanding of the concepts and methods needed for analyzing the factors that matter for economic growth.

## 1. Solution to Problem 1 (40 %)

We consider

$$Y = \tilde{F}(K, L_1, L_2, t),$$

where $K$ is capital input, $L_1$ is input of unskilled labor, and $L_2$ is input of skilled labor. Suppose technological change is such that the production function can be rewritten

$$\tilde{F}(K, L_1, L_2, t) = F(K, H(L_1, L_2, t)),$$

where $H(L_1, L_2, t)$ represents input of a "human capital" aggregate. $F$ is CRS-neoclassical w.r.t. $K$ and $H$, and $H$ is CRS-neoclassical w.r.t. $(L_1, L_2)$ and has $\partial H/\partial t > 0$. Markets are *competitive*. Finally, the real wages of unskilled and skilled labor are denoted $w_1$ and $w_2$, respectively.

a) The skill-premium can be written

$$\frac{w_2}{w_1} = \frac{\partial Y/\partial L_2}{\partial Y/\partial L_1} = \frac{F_H \partial H/\partial L_2}{F_H \partial H/\partial L_1} = \frac{H_2(L_1, L_2, t)}{H_1(L_1, L_2, t)} = \frac{H_2(1, L_2/L_1, t)}{H_1(1, L_2/L_1, t)}, \tag{1.1}$$

where we have used Euler's theorem (saying that if $H$ is homogeneous of degree one in its first two arguments, then the partial derivatives of $H$ are homogeneous of degree zero w.r.t. these arguments).

[1] The solution below contains *more* details and more precision than can be expected at a three hours exam. The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

b) Hicks' definitions are now: If for all $L_2/L_1 > 0$,

$$\frac{d\left(\frac{H_2(1,L_2/L_1,t)}{H_1(1,L_2/L_1,t)}\right)}{dt}\Bigg|_{\frac{L_2}{L_1}\text{ constant}} \gtreqless 0, \text{ then technical change is}$$

$$\begin{cases} \text{skill-biased in the sense of Hicks,} \\ \text{skill-neutral in the sense of Hicks.} \\ \text{blue collar-biased in the sense of Hicks,} \end{cases} \tag{1.2}$$

respectively.

c) We know that the actual skill-premium in the US as well as the supply of skilled labor relative to unskilled labor have been rising since 1950. In terms of the mentioned Hicksian concepts, an economic evolution with these two properties reveals that technical change has been *skill-biased.* Had we observed a rising skill-premium accompanied by *no* change in the relative supply of skilled-labor, the evolution would – by direct application of the definition – reveal skill-biased technical change. On top of that comes that the relative supply of skilled labor *has been rising.* If there were no skill-biased technical change, this would have created a declining skill-premium because of the diminishing marginal productivity of skilled labor in the absence of technical change. And if there had been blue collar-biased technical change, this declining tendency would have been even stronger, cf. (1.2).

The fact that in spite of these potential effects of the rising relative supply of skilled labor, the skill-premium has actually been *increasing*, reveals a *strong* skill-bias in technical change.

d) Yes, skill-biasedness could be part of the *cause* of the observed increase in the relative supply of skilled labor. The increased skill-premium strengthens the incentive to go to college. And if the skill-biasedness is strong enough, we will simultaneously observe a rising skill-premium and a rising relative supply of skilled labor.

The point of departure for the next questions is the following example of a production function exhibiting *capital-skill complementarity:*

$$Y = \tilde{F}(K, L_1, L_2, t) = F(K, A_{1t}L_1, A_{2t}L_2) = (K + A_{1t}L_1)^{\alpha}(A_{2t}L_2)^{1-\alpha}, \ 0 < \alpha < 1,$$

e) $\tilde{F}$ exhibits *capital-skill complementarity* if $\partial^2\tilde{F}/(\partial K\partial L_2) > 0$. Here, we have

$$\begin{aligned} \partial\tilde{F}/\partial K &= \alpha(K + A_{1t}L_1)^{\alpha-1}(A_{2t}L_2)^{1-\alpha}, \qquad \text{and so} \\ \partial^2\tilde{F}/(\partial K\partial L_2) &= \alpha(K + A_{1t}L_1)^{\alpha-1}(1-\alpha)(A_{2t}L_2)^{-\alpha}A_{2t} > 0. \end{aligned}$$

So, yes, $\tilde{F}$ exhibits capital-skill complementarity.

f) Under perfect competition the skill premium is

$$\begin{aligned} \frac{w_2}{w_1} &= \frac{\partial Y/\partial L_2}{\partial Y/\partial L_2} = \frac{(K + A_{1t}L_1)^\alpha (1-\alpha)(A_{2t}L_2)^{-\alpha} A_{2t}}{\alpha (K + A_{1t}L_1)^{\alpha-1} A_{1t}(A_{2t}L_2)^{1-\alpha}} \\ &= \frac{1-\alpha}{\alpha}\left(\frac{K + A_{1t}L_1}{A_{2t}L_2}\right)\frac{A_{2t}}{A_{1t}}. \end{aligned} \tag{1.3}$$

The right-hand side depends only on $\alpha$ and two generally variable "factors", namely the ratios $(K + A_{1t}L_1)/(A_{2t}L_2)$ and $A_{2t}/A_{1t}$. This is the required property.

g) Profit maximization gives, under perfect competition,

$$\partial Y/\partial K = \partial \tilde{F}/\partial K = \alpha(K + A_{1t}L_1)^{\alpha-1}(A_{2t}L_2)^{1-\alpha} = \alpha\left(\frac{K + A_{1t}L_1}{A_{2t}L_2}\right)^{\alpha-1} = r_t + \delta.$$

Hence, along the path $P$, where $r_t = r$, a constant, the ratio $\frac{K + A_{1t}L_1}{A_{2t}L_2}$ will also be constant.

h) Here we consider a situation where the skill premium is rising along the path $P$. This phenomenon requires that technical change brings about a *rising* $A_{2t}/A_{1t}$. So the explanation of the phenomenon is that technical change favors skilled labor by raising $A_{2t}$ faster than $A_{1t}$.

## 2. Solution to Problem 2 (45 %)

Closed economy, profit maximizing firms, perfect competition. Perfect loan market. At the aggregate level,

$$\dot{K}_t = Y_t - C_t - \delta K_t, \qquad \delta \geq 0, \qquad K_0 > 0 \text{ given.} \tag{2.1}$$

Firm $i$:

$$Y_{it} = K_{it}^\alpha (A_t L_{it})^{1-\alpha}, \qquad 0 < \alpha < 1, \qquad i = 1, 2, \ldots, N, \qquad N \text{ "large"}.$$

$\sum_i Y_{it} = Y_t$, $\sum_i K_{it} = K_t$, $\sum_i L_{it} = L_t$ = aggregate employment = population. Each firm "small" relative to the economy as a whole.

a) We suppress the time index when not needed for clarity. Consider firm $i$. Its maximization of profits, $\Pi_i = K_i^\alpha (AL_i)^{1-\alpha} - (r+\delta)K_i - wL_i$, leads to the first-order conditions

$$\begin{aligned} \partial \Pi_i/\partial K_i &= \alpha K_i^{\alpha-1}(AL_i)^{1-\alpha} - (r+\delta) = 0, \\ \partial \Pi_i/\partial L_i &= (1-\alpha)K_i^\alpha A^{1-\alpha} L_i^{-\alpha} - w = 0. \end{aligned} \tag{2.2}$$

We can write (2.2) as

$$\alpha A^{1-\alpha} k_i^{\alpha-1} = r + \delta, \tag{2.3}$$

where $k_i \equiv K_i/L_i$. From this follows that the chosen $k_i$ will be the same for all firms, say $\bar{k}$. In equilibrium, $\sum_i K_i = K$ and $\sum_i L_i = L$, where $K$ and $L$ are the available amounts of capital and labor, respectively (both pre-determined). Since $\sum_i K_i = \sum_i k_i L_i = \sum_i \bar{k} L_i = \bar{k} L$, the chosen capital intensity, $k_i$, satisfies

$$k_i = \bar{k} = \frac{K}{L} \equiv k, \qquad i = 1, 2, ..., N. \tag{2.4}$$

Since $k$ is predetermined, we can use (2.3) to *determine* the equilibrium interest rate:

$$r_t = \alpha A^{1-\alpha} k^{\alpha-1} - \delta. \tag{2.5}$$

b) The implied aggregate production function is

$$\begin{aligned} Y &= \sum_i Y_i \equiv \sum_i y_i L_i = \sum_i k_i^{\alpha} A^{1-\alpha} L_i = k^{\alpha} A^{1-\alpha} \sum_i L_i = k^{\alpha} A^{1-\alpha} L \\ &= k^{\alpha} L^{\alpha} A^{1-\alpha} L^{1-\alpha} = K^{\alpha} (AL)^{1-\alpha} = A^{1-\alpha} K^{\alpha} L^{1-\alpha} \equiv T K^{\alpha} L^{1-\alpha}. \end{aligned} \tag{2.6}$$

c) $\text{TFP}_t = T_t = A_t^{1-\alpha}$.

d) We get

$$g_Y = \alpha g_K + (1-\alpha) g_L + g_T,$$

where $g_T$ is the residual.

e) The TFP growth rate can be expressed as

$$g_{\text{TFP}} \equiv \text{ Solow residual } = g_Y - \alpha g_K - (1-\alpha) g_L.$$

In the present case, since we already from c) have a formula for the TFP *level*, we can also write

$$g_{\text{TFP}} = g_T = (1-\alpha) g_A,$$

when using c).

The gross income share of capital is

$$\frac{(r+\delta)K}{Y} = \frac{\frac{\partial Y}{\partial K} K}{Y} = \frac{\alpha \frac{Y}{K} K}{Y} = \alpha. \tag{2.7}$$

The labor income share is

$$\frac{wL}{Y} = \frac{\frac{\partial Y}{\partial L} L}{Y} = \frac{(1-\alpha)\frac{Y}{L} L}{Y} = 1 - \alpha. \tag{2.8}$$

We now assume that $A_t$ evolves according to

$$A_t = e^{\varepsilon t} K_t^{\lambda}, \qquad \varepsilon > 0, \quad 0 < \lambda < 1, \tag{*}$$

where $\varepsilon$ and $\lambda$ are given constants.

f) The assumption (*) says that the technology level has an exogenous component, $e^{\varepsilon t}$, growing at the exogenous rate $\varepsilon$, and an endogenous component, $K_t^{\lambda}$. The latter can be interpreted as reflecting "learning by investing". The idea is that investment – the production of capital goods – as an unintended *by-product* results in *experience* or what we may call *learning.* This adds to the knowledge about how to produce the capital goods in a cost-efficient way and how to design them so that in combination with labor they are more productive and satisfy better the needs of the users. The idea stems from Arrow (1962) who hypothesized that the primary basis for learning is *gross* investment. Yet, the term $K_t^{\lambda}$ in (*), where $\lambda$ is called the "learning parameter", indicates that the basis for learning in the present model is *net* investment, so that the accumulated learning – a proxy for the technology level – is here an increasing function of cumulative *net* investment, $\int_{-\infty}^{t} I_s^n ds = K_t$. This latter hypothesis is more popular for the only reason that it leads to simpler dynamics.

The learning is assumed to benefit essentially all firms in the economy due to *knowledge spillovers* across firms. Empirics indicate that such spillovers are reasonably fast relative to the time horizon relevant for growth theory.

Combined with (*), (2.6) implies

$$Y_t = (e^{\varepsilon t} K_t^{\lambda})^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} = e^{(1-\alpha)\varepsilon t} K_t^{\alpha + (1-\alpha)\lambda} L_t^{1-\alpha} \tag{2.9}$$

so that

$$g_Y = (\alpha + (1-\alpha)\lambda)\, g_K + (1-\alpha) g_L + \text{ residual}, \tag{2.10}$$

where the residual is $(1-\alpha)\varepsilon$, the growth rate of the exogenous term, $e^{(1-\alpha)\varepsilon t}$, in (2.9).

g) To compare standard growth accounting with this, let the weights attached to $g_K$ and $g_L$ be denoted $\eta_K$ and $\eta_L$, respectively. Then, in standard growth accounting we have $\eta_K = (r+\delta)K/Y$ and $\eta_L = wL/Y$, respectively. Hence, by (2.7), the "contribution" to output growth from growth in capital is set equal to $\alpha g_K$. This is less than the "true contribution" to output growth from growth in capital which, by (2.10), is

$$(\alpha + (1-\alpha)\lambda)\, g_K. \tag{2.11}$$

In this sense, standard growth accounting underestimates the "contribution" to output growth from growth in capital. This is because the market price $r$ does not reflect the positive externality from capital investment.

We now assume $L_t = L_0 e^{nt}$, where $n > 0$, constant.

h) In view of (2.1), under balanced growth with positive saving, $g_Y = g_K$. By (2.10) and $g_L = n$ we then have $g_Y = (\alpha + (1-\alpha)\lambda)\, g_Y + (1-\alpha)n + (1-\alpha)\varepsilon$, from which follows

$$g_Y = \frac{n+\varepsilon}{1-\lambda}.$$

With $y \equiv Y/L$, this implies

$$g_y = g_Y - n = \frac{\lambda n + \varepsilon}{1-\lambda} \; (= g_k = g_A) > 0. \tag{2.12}$$

i) On the basis of the model there are, according to (2.12), two *ultimate sources* of per capita growth (along a BGP), learning by investing, represented by the term $\lambda n$, and an exogenous source, represented by the parameter $\varepsilon > 0$.

The first source, the learning *mechanism*, is more powerful, the higher is the population growth rate, $n$. This role of population growth derives from the fact that at the economy-wide level there are increasing returns to scale w.r.t. capital *and* labor. For the increasing returns to be exploited, growth in the labor force is needed. The more fundamental background is that technical knowledge is partly endogenous in the model and is at the same time a *non-rival good* – its use by one firm does not (in itself) limits the amount of knowledge available to other firms. In a large economic system *more* people and firms benefit from a given increase in knowledge than in a small economic system. At the same time the per capita cost (here per capita net investment) of creating the increase in knowledge is less in the large system than in the small system.

In contrast, the role of the *exogenous* component of the technology is not expanded by the population growth rate $n$.

The learning mechanism, however, expands the role of *both* sources of per capita growth. This is manifested by the appearance of the learning parameter in the "multiplier" $1/(1-\lambda) > 1$ in (2.12).

According to the growth accounting in d),

$$g_y = g_Y - n = \alpha(g_K - n) + g_T = \alpha g_k + g_T, \tag{2.13}$$

where $k \equiv K/L$ and $g_T = (1-\alpha)g_A = (1-\alpha)(\lambda n + \varepsilon)/(1-\lambda) = (1-\alpha)g_y$ under balanced growth.

*Comparison:* The natural interpretation of (2.12) is that (along a BGP) all per capita growth "comes from" growth in the labor-augmenting technology level $A$, that is, from "technical progress". In contrast, (2.13) says that only a fraction of per capita growth is accounted for by "technical progress", the remainder ($\alpha g_k$) being accounted for by increases in the capital-labor ratio, $k$. This way of characterizing the growth process is, however, superficial for two reasons. First, even in a mere accounting perspective, the "direct contribution" to $g_Y$ from growth in $K$ (or "direct contribution" to $g_y$ from growth in $k$) is underestimated, as noted under g). Second, the *growth in $k$ is itself endogenous* and would be absent in the long run if there were no learning, no population growth, and no exogenous technology growth. To see this, in (2.12) let $\lambda = n = \varepsilon = 0$.

On the other hand, not only is the learning mechanism a key factor behind capital accumulation, but the latter is also a key factor in the learning process. It is thus appropriate to say that the learning mechanism is an ultimate source of per capita growth, and capital accumulation is (in this model) a necessary vehicle in the learning process.

## 3. Solution to Problem 3 (15 %)

a) An important strength of the Schumpeterian model is that it is captures that innovations often imply "creative destruction" − the process through which existing businesses and technologies are competed out of the market by new technologies. To capture this is a strength because it corresponds to what elementary observations tell us.

In the horizontal innovations model, at least in the form we meet it in our textbook, innovations create new input types and do so without making any of the old input types obsolete. Instead there will just be a longer list of available and used input "varieties". This is meant to reflect more specialization and division of labor. The model is constructed such that the aggregate production function in the final goods sector will in equilibrium use *all* the existing input types. None have become obsolete.

b) Here we consider that fact that the bulk of empirical evidence suggests that market economies do too little R&D investment compared to the optimal level as defined from the perspective of a social planner respecting the preferences of an assumed representative infinitely-lived household.

Let us in the following way name the two models to be compared:

Model H = the horizontal innovations model.

Model S = the Schumpeterian model.

In addition, let us in the following way name the two sub-questions to be answered:

Q1: Are both models consistent with this evidence?

Q2: Are there in both models theoretically possible combinations of parameter values that may give rise to the opposite conclusion?

Both questions are about whether the market distortions in each of the models *may* end up in "too little" alternatively "too much" R&D activity?

So we first have to expose the market distortions in each model.

**Model H** assumes that the aggregate increase per time unit in technical knowledge, proxied by the number of existing input varieties, is determined as

$$\dot{A} \equiv \frac{dA(t)}{dt} = \bar{\eta} L_A \equiv \eta A^{\varphi} L_A^{1-\xi}, \qquad \eta > 0, \varphi \leq 1, 0 \leq \xi < 1, \qquad A(0) > 0 \text{ given.} \qquad (*)$$

Here $L_A$ is aggregate input of research labor. Each individual R&D lab is "small" and therefore perceives, correctly, its contribution to the aggregate entities, $\dot{A}$ and $L_A$, to be negligible. The innovation production function from the perspective of lab $j$ is

$$\dot{A}_j = \bar{\eta} L_{Aj}, \qquad \bar{\eta} > 0,$$

where $\bar{\eta}$ is the perceived productivity of the lab's own input of research labor, $L_{Aj}$. From the point of view of the economy as a whole, $\bar{\eta} = \eta A^{\varphi} L_A^{-\xi}$, where $\varphi$ indicates the strength and the sign of the *intertemporal knowledge spillover* from the current stock of knowledge. If $\varphi > 0$, the spillover amounts to a "standing on the shoulders" effect (a positive externality on R&D). And if $\varphi < 0$, the spillover amounts to a "fishing out" effect (a negative externality on R&D).

A positive value of the exponent, $\xi$, on $L_A$ reflects an additional externality coming from duplication of effort. This externality is often called the *standing on the toes* effect and is an unambiguously negative externality, which everything else equal points in the direction of too much R&D.

Two additional market distortions are related to the *monopoly position* of each innovator supplying a patented specialized capital-good service (or specialized "intermediate good" as Jones and Vollrath call them).[2]

a) At the *supply side* we have the *surplus appropriability problem*: The innovator's monopoly profit captures only a fraction of the innovator's service flow to the users (the

[2] It is OK to merge them and speak of just one distortion.

final goods producers). This implies too little incentive to *make* inventions and innovations.

b) At the *demand side* we have the *monopoly markup* which implies a *wedge* between the price of the services of specialized capital goods and the marginal costs of providing them. Because of this wedge, these services are not demanded up to the point where a social planner would demand them − which they should. This implies too little incentive to *use* inventions and innovations.

Both a) and b) thus, everything else equal, point in the direction of *too little* R&D.

In **Model S** essentially the same four distortions are present (arising from the analogue of (*) and the monopoly position of the innovator) plus a fifth, which involves a negative externality. This is known as the *business stealing effect* (about which Jones and Vollrath are quite silent but which is implied by creative destruction and was mentioned in lectures). This effect refers to the fact that the innovator does not internalize the loss to the previous monopolist caused by the new innovation. Everything else equal, this effect unambiguously points in the direction of *too much* R&D.

Our answer to Q1 is: Yes. Since both models contain positive externalities (and similar features) on R&D, both models are consistent with the "too little R&D" view.

Our answer to Q2 is: Yes. Since both models contain negative externalities on R&D, both models are consistent with the "too much R&D" view. We may add that because Model S contains an additional *negative* externality, Model S can be perceived as more likely to end up in "too much R&D" than Model H.

c) The static distortion related to market power which the two models have in common is the one named b) above,. The problem is that the monopoly markup creates a wedge between the price of the services of specialized capital goods and the marginal costs of providing them (once the technical design behind them has been invented).

The distortion due to monopoly pricing can be remedied by a subsidy, $\sigma$, to buyers of the services of specialized capital goods. By adjusting the size of the subsidy so as to compensate the the markup, the effective price from the point of view of the buyer can be made equal to the desired level, the marginal cost of supplying the service in question.

To satisfy this efficiency condition, $\sigma$ must be such that

$$(1-\sigma)(r+\delta)/\alpha = r+\delta.$$

The solution is $s = 1-\alpha$.

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