Financial Frictions - exam solutions (Dec 17, 2015)

General remarks

Please note that the maximum possible grade of the exam is 180. The scaling grade was used to match minutes and points and thus guide students in the use of time.

Mathematical errors reduce the grade for the item in which they were made, not for subsequent items that carry the mistake (unless the student arrives at an infeasible solution and is unaware of the inconsistency, e.g. stating that deposit withdrawals are negative).

1. False (or Uncertain). The fact that we are told that there are binding borrowing constraints implies both the presence of a friction (in the model it is adverse selection), and that at least some entrepreneurs need to borrow funds to undertake their investments. Bernanke and Gertler (1990) show that in this case a negative shock to entrepreneurs' wealth implies a worsening of the asymmetric information problem. This leads to an unambiguous decline in output. But the effect on investment is ambiguous since fewer entrepreneurs screen the quality of their projects, but those that do so will then be more likely to invest.

2. False. While it is true that "ambiguity" in the lender of last resort facility introduces some market discipline, and this improves welfare, this comes at the cost of a wealth transfer from small and medium-sized banks to large banks since there is no doubt that the latter will always be bailed out due to the increase in systemic risk should they become bankrupt (thus, big banks are "too big to fail"). And this wealth transfer might decrease aggregate welfare.

3. True. In the Geanakoplos (2009) model entrepreneurs are heterogeneous and this leads to only a subgroup of them demanding a risky asset. The model considers heterogeneity in their subjective probability for the realization of the state in which the asset pays off. Thus there will be a cutoff probability that distinguishes the marginal entrepreneur that is indifferent between buying or not the asset, such that the asset will be demanded by those entrepreneurs with a subjective probability higher than this cutoff (the optimists). Allowing entrepreneurs to borrow more will increase borrowing by those that are more optimistic. This will increase asset prices, even if in the new equilibrium the set of optimist shrinks (i.e. there is an increase in the cutoff probability).

4. a) Denoting $R(p)$ the contractual repayment to foreign lenders when the project is successful, the individual rationality constraint is

$$p(Y - R(p)) \geq (1 + r^*)W.$$

b) For the marginal investor the above inequality holds as an equality, we denote this investor by its probability of success p^*.

Competition among foreign lenders implies that they get in expectation the opportunity cost of their funds:

$$pR(p) = (1 + r^*)B.$$

The repayment when successful has to be higher when the probability of success is lower to compensate for the higher risk.

Replacing the zero profit condition for lenders in the incentive rationality constraint for the marginal investor gives

$$p^*Y = (1 + r^*)(W + B) = 1 + r^*.$$

Thus, when there are no information asymmetries entrepreneurs would only invest when their projects have positive NPV (trivially this is seen from adding the zero profit condition for lenders to the incentive rationality constraint of entrepreneurs).

Investment, v, and output, q, are given by

$$\begin{aligned} v &= \int_{p^*}^{1} f(p)dp, \\ q &= Y \int_{p^*}^{1} pf(p) + (1 + r^*) \int_{0}^{p^*} f(p)dp. \end{aligned}$$

Note: if students only have the first term for output give full credit.

c) Yes, the equilibrium is optimal. Since agents are risk neutral we only need to verify that aggregate output is maximized and this is guaranteed when only projects with positive NPV are undertaken.

d) With information asymmetries there will be a unique debt contract with repayment $\hat{R}$ if the project is successful. Given this contract the marginal entrepreneur (one indifferent between undertaking her project or saving at risk free rate) is denoted by $\hat{p}$. Both

$\hat{R}$ and $\hat{p}$ are determined by the following equations

$$\begin{aligned} \hat{p}(Y-\hat{R}) &= (1+r^*)W, \\ \hat{R}\frac{\int_{\hat{p}}^{1} pf(p)}{1-F(\hat{p})} &= (1+r^*)B. \end{aligned} \tag{1}$$

By adding both equation we get

$$\hat{p}Y + \hat{R}\frac{\int_{\hat{p}}^{1}(p-\hat{p})f(p)}{1-F(\hat{p})} = 1+r^*.$$

Since the integral is positive this implies $\hat{p}Y < 1+r^*$. Thus $\hat{p} < p^*$ and there is overinvestment. Output will be lower now since some projects with negative NPV are undertaken.

e) When equilibrium requires the equality of demand and supply for credit in domestic markets this implies that the interest rate will be determined by

$$D(r) = B\int_{\hat{p}(r)}^{1} f(p) = S(r),$$

where now $\hat{p}$ depends on r, since r measures the opportunity cost of funds. Since we are told that in the first best $r = r^*$ and we have a problem of overinvestment, then in equilibrium it has to be the case that $D(r^*) > S(r^*)$. Thus in equilibrium $r > r^*$. Intuitively, the demand for funds is a decreasing function of the interest rate (as seen from condition for marginal entrepreneur keeping $\hat{R}$ constant), and this implies that there is less overinvestment than in d). Students that reason this without math should get full credit. A formal verification requires differentiating equations (1) to get, after some manipulation to get rid of the term $\frac{d\hat{R}(r)}{dr}$,

$$\frac{d\hat{p}(r)}{dr}\left[(Y-\hat{R}(r))E[p|p\geq\hat{p}(r)] + \hat{R}\frac{dE[p|p\geq\hat{p}(r)]}{d\hat{p}(r)}\right] = 1$$

Since $Y > \hat{R}(r)$, and $\frac{dE[p|p\geq\hat{p}(r)]}{d\hat{p}(r)} \geq 0$, we confirm that $\frac{d\hat{p}(r)}{dr} > 0$ and thus since $r > r^*$ there is less overinvestment than in d).

f) Yes. Any regulation that increases the opportunity cost of borrowing will reduce the overinvestment problem (and potentially eliminate it and lead to first best investment and output). An example would be a tax on borrowing and lending that increases r both for borrowers and for lenders (see (1)).

5. a) Denoting by I the fraction of the endowment invested in the long run technology, the optimal allocation is the one that maximizes ex ante expected utility subject to the budget constraints that allocate $c_1 = \frac{1-I}{\pi}$ to impatient consumers, and $c_2 = \frac{IR}{1-\pi}$ to patient consumers (alternatively one can use the intertemporal budget constraint $\pi c_1 + (1-\pi)\frac{c_2}{R} = 1$ and maximize only with respect to c_1 and c_2).

$$\begin{aligned} \max_{c_1, c_2, I} \quad & \pi\sqrt{c_1} + (1-\pi)\rho\sqrt{c_2} \\ s.t. \quad & c_1 = \frac{1-I}{\pi} \\ & c_2 = \frac{IR}{1-\pi} \end{aligned} \tag{2}$$

FOC gives (denoting optimal allocation by c_i^*)

$$\frac{1}{\sqrt{c_1^*}} = \rho R \frac{1}{\sqrt{c_2^*}}$$

Since $\rho\sqrt{R}$ might be either larger or smaller than 1 ($R > 1$, and $\rho R > 1$ do not imply anything on $\rho\sqrt{R}$), then c_1^* could be larger or smaller than $c_1^M = 1$, and c_2^* could be larger or smaller than $c_2^M = R$.

b) Since $\rho R > 1$ implies that $c_1^* < c_2^*$, the optimal allocation can be implemented by a financial intermediary because a deposit contract offering either c_1^* or c_2^*, depending on when funds are withdrawn, is incentive compatible (patient depositors do not gain by pretending to be impatient).

A run equilibrium exists if when a depositor expecting that everybody else would run finds it in her best interest to run herself. This requires that a bank that has to liquidate its long run investment be unable to pay c_1^* to a mass one of depositors. Thus, bankruptcy requires that

$$\pi c_1^* + (1 - \pi c_1^*)L < c_1^*, \tag{3}$$

where the left hand side is the value of assets at liquidation in period 1, and the right hand side is the value of deposits withdrawn in period 1. From a) we know that there are parameters such that $c_1^* < 1$. In that case for L sufficiently large the bank will not be bankrupt if every depositor withdraws in period 1 (i.e. (3) is not satisfied), and thus a run cannot be an equilibrium since knowing a bank would be solvent in the event of a run makes patient depositors to be strictly better off by waiting to withdraw in period 2.

A formal characterization of the parameter constellations for which a run is possible requires solving from FOC and budget constraint to find $c_1^* = \frac{1}{\pi + (1-\pi)\rho^2 R}$. Replacing this in (3) gives

$$\pi + (1-\pi)\rho^2 RL < 1.$$

A run is inefficient because it implies the liquidation of the long run investment which is, by definition, inefficient. (Efficient runs occur when there is a deterioriation of the bank's fundamentals, as might happen if the return of the long run technology is random. In that case it might be efficient to run on a bank and force its closure in period 1.)

c) Now $\rho R < 1$. The program for the first best, (2), as its FOC, is unchanged. But now $\rho\sqrt{R} < 1$ and thus unambiguously $c_1^* > c_1^M = 1$, and $c_2^* < c_2^M = R$.

d) Because $\rho R < 1$, $c_1^* > c_2^*$. Thus all patient depositors have an incentive to lie and declare to be impatient, withdraw c_1^* and by saving it using the short run technology consume $c_1^* > c_2^*$ in period 2. The constrained optimal allocation is the one that solves (2) with the added constraint that the deposit contract be incentive compatible, i.e. $c_2 \geq c_1$. Given that the constraint will be binding, $c_1 = c_2 = c$ and the level of consumption will be determined by the intertemporal budget constraint

$$c(\pi + (1-\pi)\frac{1}{R}) = 1 \ \rightarrow \ c = \frac{R}{R\pi + (1-\pi)}.$$

The condition for a bank run is still given by (3). Using the above expression this gives

$$\pi + (1-\pi)\frac{L}{R} < 1.$$

Since $L < 1 < R$ this is always satisfied and a run equilibrium is always possible independent of parameters (as long as $\rho R < 1$).

e) If a central bank can commit to a pre-announced suspension of convertibility this would prevent the inefficient bank run equilibrium. The reason for this is that a patient consumer that expects everybody else to run on the bank can rest assured that the bank will have resources left to honor period 2 deposits in full (since the central bank forces the bank to stop paying deposits once a mass of πc resources have been withdrawn the bank does not have to liquidate any of its long run investments).

f) Now students are asked to characterize the ex post optimal suspension of convertibility policy for a central bank that observes that a run takes place in period 1 (this follows Ennis and Keister (2009)). In this situation the central bank knows that only a fraction of impatient consumers have been able to withdraw and thus has an incentive to keep the withdrawal window at the bank open for longer than what was ex ante optimal (πc). Under the assumption that depositors still receive an amount of c for withdrawals in amount $\pi^S \geq \pi$, the ex post program of the central bank is

$$\max_{\pi^S} \quad \pi^S\sqrt{c} + \rho(1-\pi)(1-\pi^S)\sqrt{c_2^S(\pi^S)}$$
$$s.t \quad c_2^S(\pi^S) = \frac{(1-\pi c)R - (\pi^S - \pi)c\frac{R}{L}}{(1-\pi^S)(1-\pi)}$$

where the objective recognizes that only a fraction $1-\pi$ of depositors not served in period 1 are trully patient (and thus would withdraw their funds in period 2). The expression for $c_2^S(\pi^S)$ comes from observing that to keep the bank window open beyond πc requires some liquidation of the long run technology. Call ϕ the fraction of $I = (1-\pi c)$ that is liquidated. This must satisfy

$$\phi(1-\pi c)L = (\pi^S - \pi)c,$$

since extra withdrawals in period 1 pay c. The remainder fraction $1-\phi$ of $I = (1-\pi c)$ that is not liquidated is used to pay remaining patient depositors in period 2. Thus

$$(1-\phi)(1-\pi c)R = (1-\pi^S)(1-\pi)c_2^S(\pi^S).$$

Combining these two equations to eliminate ϕ we get the expression for $c_2^S(\pi^S)$ in the central bank's program. Note that $c_2^S(\pi) = \frac{c}{1-\pi} > c$, and that $\frac{dc_2^S(\pi^S)}{d\pi^S} < 0$.

The ex post suspension of convertibility would prevent bank runs if parameters are such that $c_2^S(\pi^S) \geq c$. In this case, a patient depositor that expects everybody else to run on the bank will be strictly better off by waiting to withdraw in period 2. The ex post policy will prevent the bank from being bankrupt and will leave enough resources to pay even more than under the original contract. Thus, no patient consumer has an incentive to run and the run equilibrium is eliminated.

Conversely, if parameters are such that $c_2^S(\pi^S) < c$, all patient depositor have an incentive to run if they expect everybody else is doing so. Thus, in this case, the ex post policy is unable to eliminate the inefficient bank run equilibrium.