Suggested Solutions to:
Final (Resit) Exam, Spring 2014
Industrial Organization
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Question 1: Collusion with fluctuating demand

To the external examiner: The students had not seen this model before. But the model is a (relatively small) extension of the Rotemberg-Saloner model that we studied in the lecture and which is discussed in the textbook.[1] *The intuition in the (b) question was discussed in the course. The intuition in the (c) question was not discussed.*

Part (a)

- We must investigate under what conditions a typical firm does not want to deviate from the trigger strategy described in the question, given that the other firm follows the trigger strategy.

- To that end, first note that, if following the equilibrium strategy when the state is s, a firm's overall payoff equals

$$\boxed{\tfrac{1}{n}\pi_s^m + \delta V,} \tag{1}$$

where

$$V \overset{\text{def}}{=} \frac{(1-\lambda)\frac{\pi_L^m}{n} + \lambda\frac{\pi_H^m}{n}}{1-\delta} = \frac{(1-\lambda)\pi_L^m + \lambda\pi_H^m}{n\,(1-\delta)}.$$

In words, the firm will in the current period get the fraction $1/n$ of the monopoly profits given state s. In the following periods the state is not yet known, so what enters as the second term of (1) is the fraction $1/n$ of the the stream of expected monopoly profits, discounted to the present period.

- If making the best possible deviation (which is to just undercut the rival's price), the firm can get (almost)

$$\boxed{\pi_s^m + 0,}$$

because from next period onwards the firm gets a zero profit according to the trigger strategy.

- That is, there is no incentive to deviate if

$$\frac{1}{n}\pi_s^m + \delta V \geq \pi_s^m \Leftrightarrow \boxed{\delta V \geq \tfrac{n-1}{n}\pi_s^m.}$$

This condition must hold both for $s = L$ and $s = H$. Because $\pi_H^m > \pi_L^m$, the high-state condition is the most stringent. Therefore the condition holds for both states if and only if it holds for the high state:

$$\underbrace{\delta\frac{(1-\lambda)\pi_L^m + \lambda\pi_H^m}{n\,(1-\delta)}}_{=\delta V} \geq \frac{n-1}{n}\pi_H^m$$

or, equivalently,

$$\delta \geq \frac{(n-1)\pi_H^m}{(n-1+\lambda)\pi_H^m + (1-\lambda)\pi_L^m} \overset{\text{def}}{=} \delta_0. \tag{2}$$

The last inequality is the one that we were asked to derive. The reasoning above (which investigates the incentives to deviate *on* the equilibrium path) shows that this condition is necessary for the trigger strategy to be part of an SPNE. To be able to conclude that the condition also is sufficient, we must consider the incentives to deviate off the equilibrium path — in particular, we must show that it is optimal for a firm to follow the trigger strategy when being in a punishment phase (given that the above condition is satisfied). However, that is indeed, almost trivially, optimal, since

[1] We get that version if we set $n = 2$ and $\lambda = \frac{1}{2}$.

the trigger strategy specifies that the firms should revert to the one shot Nash equilibrium (p=MC) in case of a deviation, so the firms are by construction of the trigger strategy making best replies in that situation.

Part (b)

One can, as in a standard repeated game, sustain a collusive equilibrium if the firms care sufficiently much about future profits (high enough discount factor δ). However, in this model, the requirement on the discount factor when having a high demand state is more stringent — the firms must be more patient than in the known-demand model for co-operation to be possible. The reason for this is that in the uncertainty model, in a high demand state, demand will be *unusually* high. The demand realization is by assumption independent over time, so the expected profits tomorrow and onwards are the same regardless of today's demand state. This means that when the demand is known to be high today, then the incentive to deviate from the equilibrium is higher than in the standard model, as the "one-period temptation" is unusually high whereas the "long-term reward of not deviating" is the same. The conclusion is that there is a tendency for collusion to break down in a high demand state (hence price war during booms and counter-cyclical prices).

Part (c)

Suppose, as stated in the question, that there is a strong positive correlation between the state in one period and the following period. That means that if the state is high today, then it is very likely that the state is high also tomorrow (which in turn means that it is likely that the state is high also the day after tomorrow, and so on). Therefore, the future expected profit if staying in the cartel, given a *high* state today, is larger than the future expected profit if staying in the cartel, given a *low* state today. This effect should, all else equal, make it less tempting to deviate in a high state and more tempting to deviate in a low state. However, the effect discussed above under (b) — which creates a relatively strong incentive to deviate in a high state because of the high deviation profit — would still be present. Still, if the correlation over time is strong enough, then it seems plausible that the correlation effect would dominate the other effect. Hence, the result under (b) would be reversed and the incentive to deviate would be stronger in a low state than in a high state.

Question 2: Discrimination against minorities and strategic incentives

To the external examiner: This question is identical to a question in a problem set that the students discussed in an exercise class. It has also been an exam question in a previous exam (a few years ago), and old exams, with solutions, are available to the students.

Part (a)

We can solve for the subgame-perfect Nash equilibria by using backward induction, i.e., by solving the game from the end. At stage 3 the customers are making their consumption decisions and that behavior is already summarized in the question. At stage 2 we are in one of four subgames, depending on the firms' choices at stage 1. In terms of the notation introduced in the question, the four subgames are $(x_1, x_2) \in \{(n,n), (d,d), (d,n), (n,d)\}$. For each subgame we must calculate the equilibrium prices and the equilibrium profits. However, the amount of calculations that we must do will not be that large, as we only need to investigate one of the subgames (d,n) and (n,d) (due to symmetry of the model) and (as will be explained below) the subgames (n,n) and (d,d) are also very similar.

The stage 2 subgame where neither discriminates: $(x_1, x_2) = (n,n)$

Firm 1's profits are

$$\pi^1(p_1,p_2) = p_1 D_1(p_1,p_2) = p_1\overline{\theta} = p_1\left[\frac{p_2 - p_1 + 1}{2}\right].$$

The FOC is:

$$\frac{\partial \pi^1(p_1,p_2)}{\partial p_1} = \left[\frac{p_2 - p_1 + 1}{2}\right] - p_1\left[\frac{1}{2}\right] = 0.$$

By symmetry of the game, we obtain the Nash equilibrium of the subgame (n,n) by setting $p_1 = p_2 = p_{n|n}$ in this FOC. Doing that yields

$$\left[\frac{p_{n|n} - p_{n|n} + 1}{2}\right] - p_{n|n}\left[\frac{1}{2}\right] = 0 \Rightarrow p_{n|n} = 1.$$

Next, we get the firms' profits at the subgame (n,n) by setting $p_1 = p_2 = p_{n|n} = 1$ in the objective function:

$$\pi^1\left(p_{n|n},p_{n|n}\right)\overset{\text{def}}{=}\pi_{n|n}=p_{n|n}\left[\frac{p_{n|n}-p_{n|n}+1}{2}\right]=\frac{1}{2}.$$

The stage 2 subgame where both discriminate: $(x_1,x_2)=(d,d)$

The profit functions at this subgame are

$$\pi^1(p_1,p_2)=p_1D_1(p_1,p_2)=p_1(1-\gamma)\overline{\theta},$$
$$\pi^2(p_1,p_2)=p_2D_2(p_1,p_2)=p_2(1-\gamma)\left(1-\overline{\theta}\right).$$

Since these are exactly as in the (n,n) subgame but with each profit function being multiplied by $(1-\gamma)$, the equilibrium prices are not affected: Both firms charge the price $p_{d|d}$, where

$$p_{d|d}=p_{n|n}=1.$$

We get the firms' profits at the subgame (d,d) by setting $p_1=p_2=p_{d|d}=1$ in the objective function:

$$\pi^1\left(p_{d|d},p_{d|d}\right)\overset{\text{def}}{=}\pi_{d|d}=(1-\gamma)\pi_{n|n}=\frac{1-\gamma}{2}.$$

The stage 2 subgame where firm 1 only discriminates: $(x_1,x_2)=(d,n)$

Firm 1's profits are

$$\pi_1(p_1,p_2)=p_1D_1(p_1,p_2)=p_1\overline{\theta}=(1-\gamma)p_1\left[\frac{p_2-p_1+1}{2}\right].$$

Firm 2's profits are

$$\begin{aligned}\pi_2(p_1,p_2)&=p_2D_2(p_1,p_2)=p_2\left[1-(1-\gamma)\overline{\theta}\right]\\&=p_2\left[1-\frac{(1-\gamma)(p_2-p_1+1)}{2}\right]\end{aligned}$$

Firm 1's FOC:

$$\begin{aligned}\frac{\partial\pi^1(p_1,p_2)}{\partial p_1}&=(1-\gamma)\left[\frac{p_2-p_1+1}{2}\right]\\&-(1-\gamma)p_1\left[\frac{1}{2}\right]=0\Leftrightarrow 2p_1-p_2=1\end{aligned}\quad(3)$$

Firm 2's FOC:

$$\begin{aligned}\frac{\partial\pi^2(p_1,p_2)}{\partial p_2}&=\left[1-\frac{(1-\gamma)(p_2-p_1+1)}{2}\right]\\&-p_2\left[\frac{1-\gamma}{2}\right]=0\Leftrightarrow 2p_2-p_1=\frac{1+\gamma}{1-\gamma}\end{aligned}\quad(4)$$

Using (3) in (4) we get

$$\begin{aligned}2(2p_1-1)-p_1&=\frac{1+\gamma}{1-\gamma}\Leftrightarrow 3p_1=\frac{1+\gamma}{1-\gamma}+2\\&=\frac{3-\gamma}{1-\gamma}\Leftrightarrow p_1=p_{d|n}=\frac{3-\gamma}{3(1-\gamma)},\end{aligned}$$

which plugged back into (4) yields

$$\begin{aligned}&2p_2-\frac{3-\gamma}{3(1-\gamma)}=\frac{1+\gamma}{1-\gamma}\\&\Rightarrow p_2=p_{n|d}=\frac{6-2\gamma}{6(1-\gamma)}=\frac{3+\gamma}{3(1-\gamma)}.\end{aligned}$$

The difference between the prices is

$$p_{n|d}-p_{d|n}=\frac{3+\gamma}{3(1-\gamma)}-\frac{3-\gamma}{3(1-\gamma)}=\frac{2\gamma}{3(1-\gamma)},$$

Therefore firm 1's profit is

$$\begin{aligned}\pi_1\left(p_{d|n},p_{n|d}\right)\overset{\text{def}}{=}\pi_{d|n}&=(1-\gamma)p_{d|n}\left[\frac{p_{n|d}-p_{d|n}+1}{2}\right]\\&=(1-\gamma)\frac{3-\gamma}{3(1-\gamma)}\left[\frac{\frac{2\gamma}{3(1-\gamma)}+1}{2}\right]\\&=\frac{3-\gamma}{6}\left[\frac{2\gamma}{3(1-\gamma)}+\frac{3(1-\gamma)}{3(1-\gamma)}\right]=\frac{(3-\gamma)^2}{18(1-\gamma)}.\end{aligned}$$

Firm 2's profit is

$$\begin{aligned}&\pi_2\left(p_{d|n},p_{n|d}\right)\overset{\text{def}}{=}\pi_{n|d}\\&=p_{n|d}\left[1-\frac{(1-\gamma)\left(p_{n|d}-p_{d|n}+1\right)}{2}\right]\\&=\frac{3+\gamma}{3(1-\gamma)}\left[1-\frac{(1-\gamma)\left(\frac{2\gamma}{3(1-\gamma)}+1\right)}{2}\right]\\&=\frac{3+\gamma}{3(1-\gamma)}\left[1-\frac{\left(\frac{2\gamma+3(1-\gamma)}{3}\right)}{2}\right]\\&=\frac{3+\gamma}{3(1-\gamma)}\left[1-\frac{3-\gamma}{6}\right]=\frac{(3+\gamma)^2}{18(1-\gamma)}.\end{aligned}$$

The stage 1 game

Summing up, we thus have

$$\pi_{n|n}=\frac{1}{2},\qquad\pi_{d|d}=\frac{1-\gamma}{2},$$
$$\pi_{d|n}=\frac{(3-\gamma)^2}{18(1-\gamma)},\qquad\pi_{n|d}=\frac{(3+\gamma)^2}{18(1-\gamma)}.$$

It will be useful for the remaining analysis to relate the four profit levels to each other. First, by inspection it is obvious that $\pi_{d|d} < \pi_{n|n}$. Moreover, we have

$$\pi_{n|n} < \pi_{d|n} \Leftrightarrow \frac{1}{2} < \frac{(3-\gamma)^2}{18(1-\gamma)}$$
$$\Leftrightarrow 9(1-\gamma) < (3-\gamma)^2 = 9-6\gamma+\gamma^2 \Leftrightarrow 0 < 3\gamma+\gamma^2,$$

which always holds. Finally, for positive values of γ it is clear from inspection that $\pi_{d|n} < \pi_{n|d}$. Overall we therefore have the relationships

$$\boxed{\pi_{d|d} < \pi_{n|n} < \pi_{d|n} < \pi_{n|d}.} \tag{5}$$

We have now solved all the stage 2 subgames and derived expressions for the equilibrium profit levels in all of these. Using these profit levels we can illustrate the stage 1 interaction between the firms in a game matrix (where firm 1 is the row player and firm 2 is the column player):

	$x_2 = d$	$x_2 = n$
$x_1 = d$	$\pi_{d|d}$, $\pi_{d|d}$	$\pi_{d|n}$, $\pi_{n|d}$
$x_1 = n$	$\pi_{n|d}$, $\pi_{d|n}$	$\pi_{n|n}$, $\pi_{n|n}$

Inspecting the table, using (5), we see that there are two pure strategy Nash equilibria of this game, $(x_1, x_2) = (d, n)$ and $(x_1, x_2) = (n, d)$.

- **Conclusion:** the overall game has two SPNE (where the firms play pure at stage 1). In these equilibria, one of the restaurants discriminates whereas the other one does not.

Part (b)

To understand the logic, suppose (to start with) that firm 1 expects firm 2 *not* to discriminate. Given that, what would be the consequences for firm 1's profit if firm 1 discriminated? We should expect there to be two effects:

1. A direct negative effect on firm 1's demand and therefore on firm 1's profit. If firm 1 refuses to sell to the minority customers, then it cannot earn any profits on those customers.

2. An indirect, strategic effect, which is positive: If firm 1 refuses to sell to the minority customers, then (by assumption) this choice is observed by firm 2 before firm 2 chooses its price. Moreover, firm 2's demand will go up, because those customers who are not served by firm 1 will go to firm 2 instead. The optimal response to an increase in the demand is to charge a higher price,[2] so by choosing $x_1 = d$ firm 1 can make p_2 go up. The fact that firm 2 charges a relatively high price is good for firm 1's profits, for this makes it possible for firm 1 to charge a relatively high price itself without losing too many customers to firm 2.

The fact that firm 1 discriminates thus leads to a loss in sales for firm 1, which is bad for profits (the negative direct effect). However, it also leads to a higher price for firm 1, which is good for profits (the positive strategic effect). The algebra under a) shows that, perhaps surprisingly, the strategic effect is so strong that also the overall effect is positive.

Key to the result is thus the strategic effect. For that effect to be present it is clear from the above explanation that firm 2 must be able to *observe* firm 1's (irreversible) decision to discriminate. What's important for firm 1 is that firm 2 *believes* that firm 1 discriminates, so that firm 2 has an incentive to raise its price (if firm 1 could fool firm 2 by pretending to discriminate but then not actually doing it, then that would be ideal for firm 1). If firm 2 observes firm 1's decision to discriminate (and knows that it's irreversible), then firm 2 will of course (correctly) believe that firm 1 discriminates.

In the stage 2 game the firms' choice variables (i.e., the prices) are strategic complements. This is crucial for the strategic effect to work in the right direction (i.e., for the effect to have a *positive* impact on firm 1's profit). To see this, note that for discrimination to have any chance of being profitable for firm 1, it must be that firm 1 optimally is charging a higher price with discrimination than without.[3] For that to be the case, the choice variables must be strategic complements, as illustrated

[2] Saying that firm 2's demand goes up and that it is this that makes firm 2 charge a higher price is a slight simplification. In fact it is not only that firm 2's demand goes up, but also that the own price elasticity of firm 2's demand goes down (because some of the customers can only buy from firm 2). It is the lower elasticity that makes firm 2 charge a higher price. If the demand increased but the elasticity remained the same, then this would not change firm 2's optimal price.

[3] At least that must be the case as long as the net demand effect for firm 1 is negative. In principle one could imagine that, even though firm 1 loses demand by not serving the minority customers, the fact that firm 2 raises its price could lead to a gain in demand for firm 1 that exceeds the loss it made by discriminating. However, it seems very unlikely that the indirect price effect can be that strong (and one can probably show that this is indeed impossible, due to the stability assumptions that will be satisfied in a standard Hotelling model like this one).

by the following chain of reactions:

a) Firm 2's demand goes up.

b) Firm 2's price goes up.

c) As firm 1's and firm 2's price are strategic complements, firm 1's price also goes up.

As long as the effects a) and b) work in the directions indicated above—which we should expect to be the case under quite general assumptions—strategic complements are required for firm 1's price to increase.

Finally we can also consider the possibility that firm 1 expects firm 2 to indeed discriminate itself. In this case, if firm 1 also discriminated then this would as before lead to a loss in sales for firm 1, which is bad for profits. Moreover, given that firm 2 also is not serving the minority customers, there would not be any demand increase for firm 2 and therefore no strategic effect that could boost firm 1's profits. Therefore, if firm 1 expects firm 2 to discriminate, then we should not expect firm 1 to have an incentive to discriminate too. This observation, together with the ones we made above, help us understand why there can be an equilibrium where one firm discriminates, but not one where both firms do it simultaneously.