Suggested Solutions to:
Resit Exam, Spring 2018
Industrial Organization
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Question 1: Collusion in Hotelling's linear city model

Part (a)

In order to compute δ_0, we need to derive the firm profits given the optimal collusive price and given the optimal deviation price.

Let us first solve for the optimal collusive price. The firms are ex ante identical, so it seems natural to believe that the firms' joint profits are maximized if they charge the same price. To obtain further confirmation that this is true, write the firms' joint profits (given a covered market) as follows:

$$\begin{aligned}\pi = \pi_1 + \pi_2 &= (p_1 - c)\left(\frac{1}{2} - \frac{p_1 - p_2}{2\tau}\right) \\ &\quad + (p_2 - c)\left(\frac{1}{2} + \frac{p_1 - p_2}{2\tau}\right) \\ &= \frac{p_1 + p_2 - 2c}{2} - \frac{(p_1 - p_2)^2}{2\tau} \qquad (1)\end{aligned}$$

Here, the demand expressions used on the first two lines were obtained with the help of the standard derivation from the textbook and the lecture slides. The last term in (1) shows that making the two prices different only serves to lower the profits. We can conclude that at the optimum we have $p_1 = p_2 = p$. Therefore the problem can be written as $\max_p \{p - c\}$, and the optimal price is such that the consumers with the longest travel distance obtain no surplus. That is,

$$p^m = r - \frac{\tau}{2}. \qquad (2)$$

Given this price, each firm's profit is given by

$$\pi^m = \frac{p^m - c}{2} = \frac{r - c}{2} - \frac{\tau}{4}. \qquad (3)$$

Next, solve for the optimal deviation price. If firm 1 were to deviate while expecting firm 2 to choose p^m, the best deviation maximizes:

$$\max_{p_1} (p_1 - c)\left(\frac{1}{2} - \frac{p_1 - p^m}{2\tau}\right).$$

The FOC is given by

$$\frac{1}{2} - \frac{p_1 - p^m}{2\tau} - \frac{p_1 - c}{2\tau} = 0 \Leftrightarrow p^d = \frac{p^m + c + \tau}{2}. \qquad (4)$$

Plugging in the expression for p^m from (2) and then simplifying, we have

$$p^d = \frac{2r - \tau + 2(c + \tau)}{4} = \frac{2r + 2c + \tau}{4}.$$

In order to compute the profit that a firm earns by making the optimal deviation, first note that we can write

$$\begin{aligned}\frac{1}{2} - \frac{p^d - p^m}{2\tau} &= \frac{1}{2} - \frac{1}{2\tau}\left[\frac{2r + 2c + \tau}{4} - \frac{4r - 2\tau}{4}\right] \\ &= \frac{1}{2} + \frac{2r - 2c - 3\tau}{8\tau} \\ &= \frac{2r - 2c + \tau}{8\tau}\end{aligned}$$

and

$$p^d - c = \frac{2r + 2c + \tau}{4} - c = \frac{2r - 2c + \tau}{4}.$$

By combining the results above, we have that firm 1's optimal deviation profit is given by

$$\pi^d = \left(p^d - c\right)\left(\frac{1}{2} - \frac{p^d - p^m}{2\tau}\right) = \frac{(2r - 2c + \tau)^2}{32\tau}. \qquad (5)$$

In the question it is stated that collusion is possible if and only if[1]

$$\frac{\pi^m}{1 - \delta} \geq \pi^d + \delta\frac{\pi^n}{1 - \delta} \Leftrightarrow \delta \geq \frac{\pi^d - \pi^m}{\pi^d - \pi^n} \stackrel{\text{def}}{=} \delta_0,$$

[1] Note, however, that in the question the term π^n before the equivalence sign is incorrectly written as π^m. However, the expression after the equivalence sign (which is the one the students are asked to use) is correct.

In the question it is also stated that the one-shot Nash equilibrium profits that a firm earns are given by $\pi^n = \tau/2$. Plugging in this profit expression and the ones in (3) and (5) into the definition of δ_0, we obtain

$$\begin{aligned}
\delta_0 &= \frac{\pi^d - \pi^m}{\pi^d - \pi^n} = \frac{\frac{(2r-2c+\tau)^2}{32\tau} - \left(\frac{r-c}{2} - \frac{\tau}{4}\right)}{\frac{(2r-2c+\tau)^2}{32\tau} - \frac{\tau}{2}} \\
&= \frac{\frac{(2r-2c+\tau)^2}{32\tau} - 8\tau\left(\frac{2r-2c-\tau}{32\tau}\right)}{\frac{(2r-2c+\tau)^2}{32\tau} - \frac{16\tau^2}{32\tau}} \\
&= \frac{(2r-2c+\tau)^2 - 8\tau(2r-2c-\tau)}{(2r-2c+\tau)^2 - 16\tau^2} \\
&= \frac{(2\widehat{r}+\tau)^2 - 8\tau(2\widehat{r}-\tau)}{(2\widehat{r}+\tau)^2 - 16\tau^2}.
\end{aligned}$$

Even though it is not required to answer the question, we can note that the last expression can be simplified further:

$$\begin{aligned}
\delta_0 &= \frac{(2\widehat{r}+\tau)^2 - 8\tau(2\widehat{r}-\tau)}{(2\widehat{r}+\tau)^2 - 16\tau^2} \\
&= \frac{(2\widehat{r}-3\tau)^2}{(2\widehat{r}-3\tau)(2\widehat{r}+5\tau)} = \frac{2\widehat{r}-3\tau}{2\widehat{r}+5\tau},
\end{aligned}$$

where the shorthand notation $\widehat{r} \stackrel{\text{def}}{=} r - c$ was introduced.

In order to compute the limit value $\lim_{\tau\to 0}\delta_0$, it does not matter if we use the simplified expression for δ_0 or one of the earlier expressions. For example, we can write

$$\begin{aligned}
\lim_{\tau\to 0}\delta_0 &= \lim_{\tau\to 0}\frac{(2\widehat{r}+\tau)^2 - 8\tau(2\widehat{r}-\tau)}{(2\widehat{r}+\tau)^2 - 16\tau^2} \\
&= \frac{(2\widehat{r}-0)^2 - 0}{(2\widehat{r}+0)^2 - 0} = 1.
\end{aligned}$$

That is, as the transportation cost τ approaches zero, the critical value δ_0 approaches one.

The interpretation of this (which is not asked about) is that, since $\delta < 1$, collusion becomes impossible in the limit as the transportation cost approaches zero.

Part (b)

- A key observation is that the market is covered, which means that the amount of trade is not affected by the introduction of competition.

- Moreover, we should expect a symmetric equilibrium, both with and without collusion. This means that the transportation costs for the consumers is also the same, with and without collusion.

- Hence, there is no impact on total surplus of the introduction of competition (only the distribution of the surplus between consumers and firms would be affected).

Question 2: A market with vertically related firms

Part (a)

Solve for the subgame-perfect equilibrium values of p, w and λ.

We solve the game by backward induction, first studying the downstream firm's problem at stage 2 and then the upstream firm's problem at stage 1.

The downstream firm's profits are:

$$\pi^D = \lambda (1-p)(p-w) - \frac{1}{2}\lambda^2. \tag{6}$$

The first-order conditions to the problem of maximizing these profits with respect to p and λ can be written as

$$\frac{\partial \pi^D}{\partial p} = \lambda [-(p-w) + (1-p)] = 0, \tag{7}$$

$$\frac{\partial \pi^D}{\partial \lambda} = (1-p)(p-w) - \lambda = 0. \tag{8}$$

Notice that $\lambda = 0$ cannot be profit-maximizing, for this would yield zero profits whereas setting both p and λ positive but small yields positive profits. Equation (7) therefore implies that

$$-(p-w) + (1-p) = 0 \Rightarrow \boxed{p^*(w) = \frac{1+w}{2}.} \tag{9}$$

And then equation (8) gives us

$$\lambda^*(w) = (1 - p^*(w))(p^*(w) - w) = \boxed{\frac{(1-w)^2}{4}}. \tag{10}$$

The upstream firm's profits, given that it anticipates the downstream firm's optimal response, are:

$$\begin{aligned} \pi^U &= \lambda^*(w)[1-p^*(w)](w-c) \\ &= \frac{(1-w)^2}{4}\left[1 - \frac{1+w}{2}\right](w-c) \\ &= \frac{(1-w)^3(w-c)}{8}. \end{aligned}$$

The first-order condition is:

$$\frac{\partial \pi^U}{\partial w} = \frac{-3(1-w)^2(w-c) + (1-w)^3}{8} = 0. \tag{11}$$

Notice that $w = 1$ cannot be profit-maximizing, for this would yield zero profits whereas setting w positive but small yields positive profits. Equation (11) therefore implies that

$$3(w-c) = 1 - w \Rightarrow w^* = \frac{1+3c}{4}.$$

This in turn yields

$$p^*(w^*) = \frac{1+w^*}{2} = \frac{1 + \frac{1+3c}{4}}{2} = \frac{5+3c}{8}$$

and

$$\begin{aligned} \lambda^*(w^*) = \frac{(1-w^*)^2}{4} = \frac{\left(1 - \frac{1+3c}{4}\right)^2}{4} &= \frac{(3-3c)^2}{64} \\ &= \frac{9(1-c)^2}{64}. \end{aligned}$$

Summing up, we have that the subgame-perfect equilibrium values of p, w and λ are:

$$p^* = \frac{5+3c}{8}, \quad w^* = \frac{1+3c}{4}, \quad \lambda^* = \frac{9(1-c)^2}{64}.$$

Part (b)

Suppose the firms integrate and become one single firm. Calculate again the subgame-perfect equilibrium values of p and λ.

The integrated firm must incur the production cost c for every unit that it is selling. It must also incur the advertising costs. The wholesale price w, however, does not matter at all under integration. The integrated firm's profits can therefore be written as:

$$\pi^I = \lambda(1-p)(p-c) - \frac{1}{2}\lambda^2.$$

Notice that this expression is identical to (6) above, except that the w in (6) is here replaced by c. That means that we can use the results in equations (9) and (10) above, only substituting c for w. We thus have that the subgame-perfect equilibrium values of p and λ are given by

$$p^I = \frac{1+c}{2}, \qquad \lambda^I = \frac{(1-c)^2}{4}.$$

Part (c)

Would you expect aggregate consumer surplus to be largest under integration or under non-integration? Spell out your reasons and the logic. Answer verbally only.

- We should expect aggregate consumer surplus to be larger under integration than under non-integration.

- The reason is that the actions taken by the non-integrated downstream firm influences also the upstream firm's profits. Moreover, internalizing those external effects (which the firms would do after integration) helps also the consumers, not only the upstream firm's profits. In particular, the integrated firm will have a stronger incentive to lower the price and to do advertising, since both the downstream and upstream profits are positively affected by that. Also, both activities help consumers and the consumer surplus (because consumers gain from a lower price and from the opportunity to buy the good).

- Because of the logic discussed above, we should expect that the retail price is lower under integration and the advertising level is higher under integration (one can confirm that they indeed are).

Part (d)

Suppose now that, as under (a), the firms are not integrated. Moreover, the retail price p is now chosen not by Firm D at stage 2, but by Firm U at stage 1 (we can interpret this as resale price maintenance, RPM). Everything else in the model is unchanged. Would you expect RPM, modeled like this, to give rise to the same outcome (i.e., the same equilibrium values of p and λ) as under integration? Spell out your reasons and the logic. Answer verbally only.

- No, we should not expect RPM (in this sense) to give rise to the same outcome as under integration.

- The reason is that, in the original model, there are *two* variables that are chosen by the downstream firm, which both involve an externality. Under RPM the upstream firm can exert full control over one of these (the retail price), but the advertising level is still chosen by the downstream firm. It is not clear how the upstream firm, with the help of a *single* instrument, would ensure that the downstream firm behaves correctly in *two* independent dimensions.