## Part 1:

(1A) The langrangian to the government problem writes

$$\mathcal{L}_G = V(q, Z) + \lambda \left[\sum_j t_j X_j(q, Z) - T\right]$$

The first-order condition for  $q_k$  equals

$$\frac{\partial \mathcal{L}_G}{\partial q_k} = \frac{\partial V}{\partial q_k} + \lambda [X_k + \sum_j t_j \partial X_j / \partial q_k] = 0$$

Use  $\alpha = \partial V / \partial Z$  and Roy's identity to rewrite as:

$$(\lambda - \alpha)X_k + \lambda \sum_j t_j \partial X_j / \partial q_k = 0$$

Now insert the Slutsky equation to obtain:

$$(\lambda - \alpha)X_k + \lambda \sum_j t_j (S_{jk} - X_k \partial X_j / \partial Z) = 0$$

Insert  $\mu \equiv \alpha + \lambda (\sum_j t_j \partial X_j / \partial Z)$  to obtain:

$$(\lambda - \mu)X_k + \lambda \sum_j t_j S_{jk} = 0$$

Rearrange as:

$$\frac{\lambda - \mu}{\lambda} = -\frac{\sum_j t_j S_{jk}}{X_k}$$

The numerator on the right-hand side is the revenue effect of the compensated behavioral responses to a small increase in the tax on good k. This can be interpreted as the marginal excess burden of the tax increase. The denominator is the mechanical revenue effect of a small increase in the tax on good k. The right hand side thus expresses the share of the potential revenue gain from a small tax increase that is lost. The result shows that this share should be equalized across all instruments. This ensures that the total excess burden is minimized given the revenue constraint. The optimal commodity tax system thus maximizes economic efficiency.

(1B) Doyle and Samphanthrak (2008) exploit that two U.S. states, Illinois and Indiana, first repealed and later reinstated, their gasoline taxes whereas the neighboring states left the gasoline taxes unchanged. The timing of these tax changes give rise to three natural experiments: (i) the simultaneous repeal of the gasoline tax by Illinois and Indiana; (ii) the reinstatement of the gasoline tax by Indiana; (iii) the reinstatement of the gasoline tax by Illinois. The causal impact on the consumer price is estimated by estimating the price change in the "treatment" state over and above the price change in the "control states" in a short time window around the tax change. The identifying assumption is that the average percentage change in gasoline prices across treatment and control states would have been identical in absence of the policy interventions in the treatment states (conditional on covariates).

(1C) An individual who has self-control problems in the way modeled by O'Donoghue and Rabin (2003) overconsumes goods that generate immediate utility and disutility in the future ("sin goods") according to his own long-term preferences. The optimal commodity tax system corrects this inefficiency by raising the tax on sin goods and lowering the tax on other goods. This is akin to a Pigouvian tax on goods that generate externalities. Here, the tax corrects for internalities, that is choices, which cause future harm on the individual itself and which is not fully taken into account because of the self-control problems. To the extent that.fatty foods tend to be subject to self-control problems, this framework provides a rationale for taxing such foods (e.g. potato chips) at a higher rate than other foods not subject to self-control problems(e.g. carrots).

## Part 2

(2A) Q: The mechanical revenue effect: Holding behavior constant, the tax revenue increases by:

$$\Delta M = \Delta \tau \Delta z [1 - H(z)]$$

The mass of people with income above z, that is 1 - H(z), each pay an increased tax bill of  $\Delta \tau \Delta z$  because the marginal tax is increased by  $\Delta \tau$  over an income interval of length  $\Delta z$ .

The behavioral revenue effect: Behavioral responses to the tax reduce tax payments by:

$$\Delta B = -h(z)\frac{\partial z}{dT'(z)}\Delta\tau T'(z)$$

The mass of people who become subject to a higher marginal tax, that is h(z), each reduce their taxable income by  $-(\partial z/dT'(z)) \cdot \Delta \tau$  and each dollar of reduction in taxable income generates a revenue loss of T'(z). This equation can be rewritten in the following way using the elasticity of taxable income e(z):

$$\Delta B = h(z)e(z)z\frac{T'(z)}{1-T'(z)}\Delta\tau\Delta z$$

The *social welfare cost*: The increased tax payment is associated with utility costs with a social value of:

$$\Delta W = \Delta \tau \Delta z [1 - H(z)] G(z)$$

This is simply the increased tax payment (absent behavioral responses) for persons with income exceeding z multiplied by the average social welfare weight for persons with income exceeding z, that is G(z). As usual, behavioral responses to a small tax change have no first-order effect on individual utilities.

(2B) **Q**: In an optimum, it must hold that the social gain of a small tax increase equals the social cost of a small tax increase:

$$\Delta M = \Delta W + \Delta B$$

Insert the expressions derived under the previous question:

$$\Delta \tau \Delta z [1 - H(z)] = \Delta \tau \Delta z [1 - H(z)] G(z) + h(z) e(z) z \frac{T'(z)}{1 - T'(z)} \Delta \tau \Delta z$$

Reduce and rearrange to obtain:

$$\frac{T'(z)}{1 - T'(z)} = \frac{1 - G(z)}{e(z)} \cdot \frac{1 - H(z)}{zh(z)}$$

**Q**: A larger value of e(z) implies that those who face increased marginal tax rates respond more strongly to taxation. This in turn implies a larger marginal efficiency loss and a lower optimal marginal tax rate. A larger value of 1 - G(z) implies lower welfare weight on those facing increased tax bills. This in turn implies a higher optimal marginal tax rate. A larger value of 1 - H(z) implies larger mass of people who pay higher tax bills after reform but face no change in marginal tax rates and hence do not change behavior. This in turn implies a larger mechanical revenue gain relative to the behavioral revenue loss

and therefore a higher optimal marginal tax rate. A larger value of h(z) implies a larger mass of people who face an increase in the marginal tax rate. This in turn implies a larger marginal efficiency loss and a lower optimal marginal tax rate.

(2C) At the income level of the highest earning individual in the economy, H(z) is precisely one since there is no income mass above that income level. This implies that the optimal marginal tax rate at this income level is zero. Intuitively, the marginal tax at the highest income level raises zero revenue (no mechanical effect), but distorts the labor supply decision of the top earning individual (negative behavioral revenue effect). Thus, the marginal tax should be zero. This result has no implications for practical policymaking since it applies only at the highest income level and not at the income level marginally below it.

## Part 3: Shorter questions

(3A) Under the "new view" firms are cash-rich and finance the marginal investment with retained earnings. Hence, cash is trapped inside the corporation - it will be hit by the dividend tax regardless of whether it is paid out to the shareholders now or later - and the dividend tax rate (provided that it is expected to remain constant in the future) does therefore not affect the choice between dividend payments and re-investment. This prediction is at odds with the empirical provided by Chetty and Saez (2005) showing a large and sharp increase in dividend payments around the time of the U.S. tax reform in 2003, which significantly lowered the dividend tax rate. The excellent answer describes the theoretical explanation for this outcome provided by Chetty and Saez (2010) in a model of the firm with agency problems.

(3B) Corporate tax rates have fallen steadily in the OECD countries since the early 1980s from an average level close to 50% to today's level around 25%. The theory of international tax competition explains the phenomenon with a higher capital mobility, which has made investments more tax sensitive. The capital outflow following a tax increase is considered a social cost from the perspective of the individual country, hence a larger tax rate sensitivity of the capital stock implies a lower optimal corporate tax rate from the perspective of individual countries. However, the theory has the normative implication that tax rates are too low from a global point because individual countries do not account for the positive externality of corporate taxation in the form of capital inflows in neighboring countries. The theory therefore suggests that corporate taxes should be coordinated (and raised) at the international level.