Problem 1

Question 1

1.1. Consider one country which trades with the rest of the world and is described by the two-factor model with capital and labor. Keep the world price fixed. Suppose there is a positive immigration inflow but that these immigrants are wealthy and bring with them more capital per person than the native population. This will decrease production of the capital-intensive good but keep the wage and return on capital constant.

False: This is the Rybczynski theorem, but due to the fact that immigrants are bringing in a lot of capital the capital / labor ratio will increase. This will increase the production of the capital-intensive good.

1.2. The classical trade models (the Heckscher–Ohlin model and the Ricardian model) are well-suited to explain all three of the following facts: The increase in income inequality in the developed world, the decrease in the labor share in the developed world, the rise in income inequality in the developing world.

False. They can explain the first two, but not the latter.

1.3. Brexit is expected to disproportionately affect low-wage workers

False: It is expected to be proportional across income groups.

1.4. Imposing an import quota or imposing and import tariff are equivalent when markets are competitive and the home government sells the quota (and gets the revenue).

True

Problem 2.

Consider the Dornbusch, Fischer Samuelson (1977) model where home and foreign have labor supply of L and L^*, respectively and utility:

$$U = \int_0^1 ln(c(z))dz,$$

where $c(z)$ is consumption of variety $z \in [0, 1]$. The labor requirement in home of producing variety z is given by $a(z)$ with $a^*(z)$ the corresponding function for foreign. Let $A(z) = a^*(z)/a(z)$ be the function of comparative advantage with $A'(z) > 0$. Let w and w^* be the wages in home and foreign, respectively. Let $p(z)$ be the equilibrium price of variety z.

Question 1. Interpret the slope of $A(z)$

Answer: $A(z)$ represents the relative cost of production for foreign compared to home. Hence, foreign is relatively worse at producing higher z, i.e. they have a comparative advantage of lower goods.

Question 2. Show that if there exists an intermediate variety z' then home will produce all varieties $z > z'$ and that foreign will produce all varieties, $z < z'$.

Answer: An intermediate variety z' must require:

$$wa(z') = p(z') = w^*a^*(z') \Leftrightarrow$$

$$\frac{w}{w^*} = A(z')$$

and hence for $z < z'$ we must have:

$$wa(z) > w^* a(z) \Leftrightarrow$$

$$A(z) < \frac{w}{w^*} = A(z'),$$

which is met by the assumption of $A'(z) > 0$.

Question 3.

Show that the equilibrium can be described by the two endogenous variables: (w/w^*) and z' and write the two equations that determine them.

Answer: We already have:

$$\frac{w}{w^*} = A(z').$$

And then we need balanced trade (note, opposite of lecture notes)

$$(1 - z')(wL + w^* L^*) = wL \Leftrightarrow$$

$$(1 - z')w^* L^* = z' wL \Leftrightarrow$$

$$\frac{1 - z'}{z'} \frac{L^*}{L} = \frac{w}{w^*}$$

Question 4.

Draw the equilibrium in $(w/w^*, z')$ space.

Answer. $w/w^* = A(z')$ is upward sloping. $(1 - z')/z' \, (L^*/L) = w/w^*$ is downward sloping.

Question 5.

Show that there are gains from trade

Answer:

Utility in autarky:

$$U_A = -\int_0^1 ln(w/p(z))dz = -\int_0^1 ln(a(z))dz$$

Utility under trade:

$$U_T = \int_0^{z'} ln \left[\frac{w}{w^* a^*(z)} \right] dz - \int_{z'}^1 ln(a(z))dz.$$

Which we compare by asking whether:

$$U_T > U_A \Leftrightarrow$$

$$\int_0^{z'} ln \left[\frac{w}{w^* a^*(z)} \right] dz > \int_0^1 ln(\frac{1}{a(z)})dz \Leftrightarrow$$

$$\int_0^{z'} ln \left[\frac{wa(z)}{w^* a^*(z)} \right] dz > 0$$

which is met in equilibrium since $(w/w^*)(a(z)/a^*(z)) > 1$ for all $z < z'$ since these are the products where foreign has the comparative advantage.

Problem 3

Consider a Ricardian model with two countries, home and foreign (denoted $*$), each with a representative agent with identical preferences:

$$u = \left(c_T^{\frac{\sigma-1}{\sigma}} + c_W^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where c_T is consumption of "textiles" and c_W is consumption of "wine". Home has population L and foreign L^*. Labor is the only factor of production. Home has linear production described by:

$$y_T = a_T L_T,$$

$$y_W = a_W L_W,$$

where a_T is units of pieces of textiles per unit of labor and L_T is the number of workers employed in textile production and correspondingly for the production of wine, y_W, $a_T, a_W > 0$ (Note a_T represents productivity, whereas in Problem 2 a represented labor requirement, the inverse of productivity). Markets are perfectly competitive in both home and foreign. In foreign the production functions are:

$$y_T^* = a_T^* L_T^*,$$

$$y_W^* = a_W^* L_W^*,$$

which have analogous interpretation and $a_T^*, a_W^* > 0$.

We impose:

$$\frac{a_T}{a_T^*} > \frac{a_W}{a_W^*} \tag{1}$$

Question 1. What is the economic interpretation of (1)

Answer: Home has a comparative advantage in textile production.

Question 3. Set the price of wine=1 and let p^A be the price of textiles (relative to wine) in home in autarky. Find the equilibrium price in autarky in home.

Answer: The utility function requires positive production of each consumption good. Consequently, an equilibrium requires that:

$$p^A a_T L_T - w L_T = 0,$$

$$a_W L_W - w L_W = 0,$$

where w is the wage in home. This requires:

$$p^A a_T = a_W \Leftrightarrow$$

$$p^A = \frac{a_W}{a_T}.$$

Question 3. Show that trade constitutes a Pareto improvement (i.e. there are gains from trade). A graphical argument only gives partial credit

Answer: There are many ways of doing this. Perhaps the simplest is to start out by asking the question of whether a representative agent under free trade would be able to afford what she purchased under autarky.

Hence, consider the budget constraint under autarky:

$$p^A c_T^A + c_W^A = wL = a_W L$$

where w is the wage and zero profit - $a_W = w$ - implies the second equality.

Now, consider the budget constraint under trade. Suppose, that price is $p > p^A$ which would imply that home would only produce textiles. This would imply a wage of $w = pa_T$ and a budget constraint of:

$$pc_T + c_W = pa_T L,$$

where c_T and c_W are the consumption of textiles and wine in home under trade. To see that this allows for the consumption of (c_T^A, c_W^A) note that:

$$pc_T^A + c_W^A = pc_T^A + (a_W L - p^A c_T^A) = (p - p^A)c_T^A + a_W L.$$

This has to be lower than $pa_T L$ such that:

$$(p - p^A)c_T^A + \left(p^A a_T - pa_T\right) L < 0 \Leftrightarrow$$

$$(p - p^A)(c_T^A - a_T L) < 0.$$

This is negative by the assumption that $p > p^A$ and by the fact that consumption of both goods are positive such that $c_T^A < a_T L$. Similar reasoning follows for the case of $p < p^A$ (although, that case will not arise under (1). Hence, home cannot be hurt by trade but at most be left indifferent. All that remains is to make that not both countries are left indifferent which equation (1) ensures.

Question 4.

Consider an equilibrium in which there is perfect specialization: Suppose that home has a comparative advantage in textile production and that $a_T L = a_W^* L^*$. Suppose the productivity of home in textile production, a_T, increases: Under what conditions will that i) benefit home, ii) benefit foreign.

Answer: First, we need the equilibrium price which requires:

- the relative supply:

$$\frac{a_T L}{a_W^* L^*}$$

- the relative demand (from the utility function):

$$\frac{c_T}{c_W} = p^{-\sigma},$$

which is the same for both countries. Hence, equilibrium requires:

$$\frac{a_T L}{a_W^* L^*} = \frac{c_T + c_T^*}{c_W + c_W^*} = p^{-\sigma},$$

that is:

$$p = \left(\frac{a_T L}{a_W^* L^*}\right)^{-\frac{1}{\sigma}}.$$

and then:

$$w = p a_T = a_T^{\frac{\sigma-1}{\sigma}} \left(\frac{L}{a_W^* L^*}\right)^{-\frac{1}{\sigma}}$$

$$V(p, w),$$

where standard results give:

$$\frac{\partial V}{\partial p} = -\frac{\partial V}{\partial I} c_T$$

$$\frac{\partial V}{\partial w} = \frac{\partial V}{\partial I} L.$$

So for home we get:

$$\frac{dV}{da_T} = -\frac{\partial V}{\partial I} c_T \frac{\partial p}{\partial a_T} + \frac{\partial V}{\partial I} L \frac{\partial w}{\partial a_T}$$

$$= \frac{\partial V}{\partial I}\left[L\frac{\sigma-1}{\sigma}\left(\frac{a_T L}{a_W^* L^*}\right)^{-\frac{1}{\sigma}} + \frac{1}{\sigma} c_T \frac{1}{a_T}\left(\frac{a_T L}{a_W^* L^*}\right)^{-\frac{1}{\sigma}}\right]$$

$$= \frac{\partial V}{\partial I}\frac{1}{a_T \sigma}\left(\frac{a_T L}{a_W^* L^*}\right)^{-\frac{1}{\sigma}}\left[a_T L(\sigma-1) + c_T\right].$$

And then we need the final endogenous variable, c_T which comes from the initial equilibrium value of $p = 1$ which gives:

$$c_T = \frac{1}{2} a_T L.$$

And then we have that productivity improves conditions for home if:

$$(\sigma - 1) + \frac{1}{2} > 0 \Leftrightarrow$$

$$\sigma > 1/2.$$

That is as long as the two goods are not "too complements". Analogous reasoning shows that for foreign there is always an improvement in welfare.

Question 5. Give an economic interpretation of your result from question 4.

Answer: When production of a good increases the relative price declines. This price decline is large if the two goods are complements. Hence, if this is the case the price decline that home suffers when it becomes more productive can outweigh the direct productivity effect and leave home worse off. This is sometimes referred to as Immiserizing growth.