## Problem 1 (30 points)

True, false or unclear. Explain your answers. You can at most get half point for a correct answer without explanations.

1. A member country of the WTO has to treat all other member countries equally due to the ”Most Favored Nation” principle. Therefore regional agreements that lower tariffs for some but not all member countries are not permitted.

   *False. Regional agreements are permitted within the WTO provided that the duties with outside parties are not on the whole higher or more restrictive than the general incidence of the duties prior to the formation.*

2. Suppose there are two countries (Home and Foreign) and two goods (a freely traded good and a non-traded good) produced using only labor. Assume that Home is twice as productive as Foreign in both goods. Then the price of the non-traded good will be higher in Home.

   *False. The price of the traded good is* $p_t = w/a_t = p_t^* = w^*/a_t^*$ *which gives* $w/w^* = a_t/a_t^*$. *The non-traded goods prices are not equalized:* $p_{nt} = w/a_{nt}$ *and* $p_{nt}^* = w^*/a_{nt}^*$. *So the relative price of the non-traded good is* $p_{nt}/p_{nt}^* = \frac{a_t/ant}{a_t^*/a_{nt}^*}$. *As Home is twice as productive as foreign, the non-traded goods price is going to be equalized as well.*

3. In the Heckscher-Ohlin model, trade in goods is a perfect substitute for trade in factors.

   *True. This is due to the factor price equalization theorem.*

## Problem 2 (30 points)

Consider the Ricardian model where Home and Foreign have labor supply $L$ and $L^*$, respectively. There are two goods (1 and 2) and preferences over the goods are Cobb-Douglas: $U = c_1^{\alpha} c_2^{1-\alpha}$. Production technology in Home is linear: $Q_i = a_i L_i$ for $i = 1, 2$, and similar equations exist for Foreign (denoted with *). All markets are competitive.

Assume that

$$\frac{a_2}{a_1} > \frac{a_2^*}{a_1^*} \qquad (1)$$

Question 1. What is the economic interpretation of (1)?

*Answer: Home has a comparative advantege in producing good 2.*

Question 2. Set the price of good 2 to 1 and let $p^A$ be the relative price of good 1 in Home in autarky. Derive $p^A$.

*Answer: The utility function requires positive production of both goods. Consequently, an equilibrium requires that:*

$$\begin{aligned} p^A a_1 L_1 - w L_1 &= 0 \\ a_2 L_2 - w L_2 &= 0 \end{aligned}$$

*where $w$ is the wage in Home. From these we get:*

$$p^A a_1 = a_2 \Leftrightarrow p^A = \frac{a_2}{a_1}$$

Question 3. Show that there are gains from trade. A graphical argument only gives partial credit.

*Answer: There are many ways of doing this. Perhaps the simplest is to start out by asking the question of whether a representative agent under free trade would be able to afford what she purchased under autarky. Hence, consider the budget constraint under autarky:*

$$p^A c_1^A + c_2^A = wL = a_2 L$$

*where the zero profit condition ($a_2 = w$) implies the second equality. Now, consider the budget constraint under trade. Suppose, that price is $p > p^A$ which would imply that home would only produce good 1. This would imply a wage of $w = pa_1$ and a budget constraint of:*

$$pc_1 + c_2 = pa_1 L$$

*To see that this allows for the consumption $(c_1^A, c_2^A)$ note that*

$$pc_1^A + c_2^A = p^A c_1^A + (a_2 L - p^A c_1^A) = (p - p^A)c_1^A + a_2 L$$

*This has to be lower than $pa_1 L$ such that:*

$$\begin{aligned} &(p - p^A)c_1^A + (p^A a_1 - pa_1)L < 0 \Leftrightarrow \\ &(p - p^A)(c_1^A a_1 L) < 0 \end{aligned}$$

*This is negative by the assumption that $p > p^A$ and by the fact that consumption of both goods are positive such that $c_1^A < a_1 L$. Similar* *reasoning follows for the case of $p < p^A$ (although that case will not arise under (1)). Hence, Home cannot be hurt by trade but at most be left indifferent. All that remains is to make sure that both countries are not left indifferent , which (1) ensures.*

Question 4. To what extent does the Ricardian model explain the international trade that we observe in the data?

*Answer: The model does not explain intra-industry trade / trade between similar countries.*

## Problem 3 (40 points)

Consider a Krugman model of a closed economy with only one factor of production (labor) with a total stock of $L$. There is a representative agent with utility:

$$U = \left( \sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where $n$ is the number of varieties of a good, $c_i$ is consumption of good $i$ and $\sigma > 1$.

Each variety is produced by a single monopolist. The labor cost of producing $q > 0$ units is

$$l = f + \frac{1}{\varphi} q$$

where $f, \varphi > 0$ and unit labor costs of producing zero units is zero. Let $w$ denote the wage, i.e. factor payments to one unit of labor. The labor costs of production is identical for all firms.

Question 1. Derive the demand function for a single variety, $i$, taking total income $wL$ as given. Show that it equals

$$c_i = \frac{wL}{P} \left( \frac{p_i}{P} \right)^{-\sigma}$$

where $P$ is the ideal price index defined as

$$P^{1-\sigma} = \sum_{i=1}^{n} p_i^{1-\sigma}$$

Hint: Utility is ordinal. Hence, it is possible (though not required) to maximize $\sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}}$ instead.

*Answer: Standard derivation of CES demand function. Many ways to do it. One way is to set up the problem*

$$\sum_{i=1}^{n} c_i^{\frac{\sigma-1}{\sigma}} - \lambda \left( \sum_{i=1}^{n} p_i c_i - wL \right)$$

*from which we get f.o.c.'s for two varieties $i$ and $j$*

$$\frac{\sigma-1}{\sigma} c_i^{-\frac{1}{\sigma}} p_i^{-1} = \lambda$$
$$\frac{\sigma-1}{\sigma} c_j^{-\frac{1}{\sigma}} p_j^{-1} = \lambda$$

*Equating these we get*

$$c_i p_i^{\sigma} = c_j p_j^{\sigma} = c_j p_j p_j^{\sigma-1}$$

*Summing both sides over all $j$ we get*

$$c_i p_i^{\sigma} = \sum_{j=1}^{n} c_j p_j p_j^{\sigma-1} = wL \sum_{j=1}^{n} p_j^{\sigma-1} = wLP^{\sigma-1} \Leftrightarrow$$
$$c_i = \frac{wL}{P} \left( \frac{p_i}{P} \right)^{-\sigma}$$

*Which is what we set out to show.*

Question 2. Each firm chooses $p_i$ to maximize profits, ignoring the effects of its choice on the prices set by other firms. Solve for the firm's optimal price.

*Answer: The price is going to be a mark-up over marginal costs, where the mark-up is determined by the elasticity of substitution:*

$$p_i = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}$$

Question 3. Let there be free entry into the market such that the profits of every firm equals zero. What is the equilibrium production of an individual firm?

*Answer: Equilibrium production can be obtained from the zero-profit condition:*

$$\left(\frac{\sigma}{\sigma-1}\frac{w}{\varphi}-\frac{\sigma-1}{\sigma-1}\frac{w}{\varphi}\right)q-fw=0\Leftrightarrow$$
$$\frac{1}{\sigma-1}\frac{q}{\varphi}=f\Leftrightarrow$$
$$q=(\sigma-1)f\varphi$$

Question 4. What is the equilibrium number of varieties?

*Answer: Use the labor market clearing condition:*

$$L=(f+q/\varphi)n=(f+f(\sigma-1))n\Leftrightarrow$$
$$n=\frac{L}{\sigma f}$$

Question 5. Show that the total income of the country is $Y=npq$

*Answer: Start from the equilibrium number of varieties and multiply both sides of the equation by $w/\varphi$ and $\sigma/(\sigma-1)$:*

$$n=\frac{L}{\sigma f}\Leftrightarrow n\frac{w}{\varphi}\frac{\sigma}{\sigma-1}=\frac{wL}{\sigma\varphi f}\frac{\sigma}{\sigma-1}\Leftrightarrow$$
$$np=\frac{wL}{q}\Leftrightarrow wL=Y=npq$$

*where we have used the price equation and the equilibrium production equation together with $Y=wL$.*

Question 6. Suppose the closed economy opens up to free trade with a similar sized country. Show that there are gains from trade. What are the sources of these gains?

*Answer: The utility function, using the symmetry of the model, is:*

$$U=\left(\sum_{i=1}^{n}c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}=n^{\frac{\sigma}{\sigma-1}}c$$

*where c is identical consumption per person of each variety. Production per firm is q and therefore consumption per person is:*

$$c=\frac{q}{L}=\frac{f\varphi}{L}(\sigma-1)$$

*such that*

$$U = \left(\frac{L}{\sigma f}\right)^{\frac{\sigma}{\sigma-1}} \frac{f\varphi}{L}(\sigma - 1)$$

*where* $\frac{\partial U}{\partial L} > 0$. *Hence we have shown that there are gains from trade. In this model the sources of gains from trade are from increased variety and only that.*