Peer Group Effects and Optimal Education System*

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Abstract

The belief that peers’ characteristic in school influence the behaviour and outcomes of students has been important in shaping public policy. The type of group in which the individual can be depends on the educational system prevailing. This paper deals with the question of peer effects in education and ability tracking. I analyse two different educational systems: tracking and mixing. Several criteria are propose to compare them. I find that there is no educational system that yield unambiguously higher level of human capital than the other for all the individuals. I find that the average human capital across the population is maximized under tracking, and that the educational system that maximize college attendance depends on the wealth level in the population. Finally, I show that the minimum human capital required to attend college determines the system that guarantees equality of opportunities among the individuals.

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1 Introduction and Motivation

The belief that peers’ characteristics in school influence the behavior and outcomes of students has been important in shaping public policy. But, what are we exactly referring to when talking about “peer effects”? Arnott and Rouse (1987) used this term to refer to the effect on an individual’s performance of the ability distribution of their peers. In general there has been limited attention given to the mechanisms through which peer affects outcomes. The most common perspective is that peers, like families, are sources of motivation, aspirations, and direct interactions in learning.

Interest in social interactions, neighborhood effects, and social dynamics has seen recently a revival. A small literature has emerged in Economics that studies the generation of persistent inequality among a population due to neighborhood effects of various kinds. These effects all have the consequence of inducing sub-optimal levels of education for a group of the population, who then earn low incomes. These neighborhood effects consist mainly of three types: investment, role-model and peer-group influences. Investment refers to local public good provision. It occurs when the poor are segregated in a community: due to the low tax base, funding of local education is low, and hence children receive less education than in richer communities. Under the role-model effect, the behavior of one individual in a group is influenced by the characteristics of and earlier behaviors of older members of the group. Peer group influences refer to contemporaneous influences and so may be reciprocal.

The last two influences are usually understood to produce some sort of imitative behavior. In the context of education we will have that the relative desirability of staying at school is higher when adults in a community are college graduates or when one’s peers are also staying at school.\(^1\)

The peer group composition of schools is, therefore, undeniably important in the minds of parents as well as policy makers at the local and state level. Peer group effects have played an important role in a number of policy debates including ability tracking, anti-poverty programs in both rural areas and urban ghettos, and school desegregation.

There is a great deal of controversy regarding the practice of ability grouping

\(^1\)See Roemer and Wets (1994), Maski (1993) and Durlauf (2002). Roemer and Wets (1994) and Streufert (2000) show how economic segregation can lead to inaccurate assessments of the economic payoff to education. The basic idea in this type of analysis is that by depriving children in poor neighborhoods of successful role models (which is a necessary consequence of economics segregation), inferences on the benefits to education are made biased.
or tracking. On the one hand, the educational establishment appears to consider
tracking as unambiguously detrimental to student learning. In keeping with this view,
there has been considerable movement towards eliminating the practice of grouping
students according to ability. On the other hand, a number of recently published
articles have called into question this conventional wisdom. For example, Argys, Rees
and Brewer (1996) suggest that tracking creates winners and losers. Students placed
in the high track classes benefit in terms of achievement but students placed in the
low track classes are clearly hurt.

The aim of this paper is to study public intervention in education when the govern-
ment, taking into account the process of human capital production and in particular
the peer effect on students’ achievement, has to decide the optimal educational sys-
tem. I analyze two different educational systems. The first one tracking, consists on
grouping students based on their innate ability. The second one mixing, implies that
students are sorted into completely homogeneous groups.

Our model is a standard overlapping generations model in which each generation
lives for two periods. Individuals differ in two aspects: their innate ability, and the
human capital of their parents. In the first period of their lives individuals accumulate
human capital. The acquisition of human capital reflects the influence of family and
peers factors. At the beginning of this period they attend compulsory education,
and they are not allowed to work. I consider two different educational systems at
compulsory level: mixing and tracking. At some point of the first period, they also
have to decide whether to attend college or not. If they do, they spend the second part
of this period at college. If they do not, they enter immediately in the labour market
working as unskilled workers. By attending college they become skilled workers.
During the second period of their lives all individuals enter in the labour market and
work as educated if they decide to go to college or as uneducated in other case.

The objective of this paper is to evaluate these different educational system using
several criteria. First of all I check if there is an educational system strictly preferred
by all the individuals in the population, and with that aim I apply the concept of first
order stochastic dominance. As a second criteria I propose to find the system that
maximize the average human capital across the population. The third criteria that I
apply is second order stochastic dominance, that is I check if there is an educational
system preferred by a set of the population, the risk averse individuals. In addition

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2For example, data from the Schools and Staffing Survey suggest than 20% of school with pro-
grams for gifted children in 1990 had eliminated the programs by 1993 (Figlio and Page, 1998).
I compare both educational systems in terms of college attendance. I analyze also the intergenerational mobility that takes place under each educational system, and in particular I focus on the upward mobility. Finally I use the equality of opportunities criteria to compare the tracking and the mixing system.

First I provide some results about the characteristics of the distribution of human capital at compulsory level under both educational systems. I check if there are differences in the performance of both educational system for poor and rich societies, and for different weights of the peer effect on the students’ achievement. When evaluating both educational systems, I find that there is no educational system that yield unambiguously higher level of human capital than the other for all the individuals. Therefore, there is no unanimity among the individuals in the population when they have to choose between tracking and mixing. However, if we are interested in maximizing the average human capital at compulsory level across the population, then the optimal educational system is tracking. Using the concept of second order stochastic dominance I find also that we can not say that the distribution of human capital is more equitable under one educational system than the other.

The paper analyzes also which system provides the maximum proportion of college students. I show that it will depend on the wealth level of the population. In particular I find that as the minimum level of human capital needed to attend college increases, the wealth level required to maximize college attendance under mixing increasing too. With respect to the intergenerational mobility I find that when the minimum level of human capital required to attend college is very low, then the upward mobility is higher under mixing that under tracking. The reverse occurs for high levels of human capital required to attend college. Finally it is shown that the equitable system, that is, the one that guarantees equality of opportunities will depend on the minimum level of human capital required to attend college. Under certain circumstances mixing is equitable for intermediate level of human capital needed to attend college and tracking for higher levels.

The paper is organized as follows. In section 2 I describe the model and the main features of the human capital distribution under both educational systems. In section 3 I describe the individuals’ decision of whether to attend college or not. In section 4 I analyze the distribution of human capital under both educational systems. Section 5 analyzes the different criteria proposed to compare both educational systems. Finally, section 6 concludes.
2 Model

2.1 Individuals

I consider an overlapping generations economy in which individuals live for two periods. Each generation has constant size equal to 1. Individuals in each generation differ in two aspects: their innate ability, \( a_0 \), and their family background, denoted by \( x \).\(^3\) I will assume that \( a_0 \) is uniformly distributed on the interval \([0, 1]\). I also assume that \( x \) takes only two values, 1 and \( x > 1 \) with probabilities \( 1 - \lambda \) and \( \lambda \) respectively. I suppose that both characteristics are independently distributed.

In the first period of their lives, individuals accumulate human capital. At the beginning of this period they attend compulsory education, which is free of charge, and they are not allowed to work. At some point of this first period, they also have to decide whether to attend college or not. I denote by \( \gamma \), where \( \gamma \in [0, 1] \) the fraction of the first period that it is left after attending college. By attending college they become skilled workers.\(^4\)

During the second period of their lives all individuals have one unit of time. All of them enter the labor market and work as skilled workers if they decided to go to college, or as unskilled workers otherwise. The wage they receive is proportional to their own level of human capital.

2.2 Production of Human Capital

After attending compulsory education an individual with innate ability \( a_0 \) ends up having a level of human capital \( a_1 \):

\[
a_1 = a_0(1 + r),
\]

where \( r \) is the individual rate of return (see below).

The production of human capital at compulsory level depends on two factors. The first one, or “informal schooling”, refers to the level of parents’ income or their level of human capital.\(^5\)

\(^3\)Therefore \( x \) could be either the parents’ level of income or the parents’ human capital.

\(^4\)The parameter \( \gamma \) can be interpreted as the cost of investment in human capital, or the fraction of earnings that would have been received in the absence of the investment.

\(^5\)Galor and Tsiddon (1994) call this factor home environment externality and distinguish it from global technological externality, by which the aggregate level of human capital of the parents’ generation is transferred to the children.
The second one is the “formal schooling” or “peer group effect”. At this educational level, individuals are separated into different groups. To simplify matters I will assume that there are only two groups. Thus, I suppose that the formal schooling will depend on the characteristics of the group in which the individual is placed. These characteristics can be summed up by the mean ability of the group $j$ or “peer” effect, denoted by $\overline{a_j}$. Thus, the rate of return of human capital will be as follows:

$$r = r(\overline{a_j}, x).$$

The main assumptions about $r$ are as follows. The importance of the parental education in the acquisition of human capital of the individual has been explored theoretically as well as empirically. I assume that the individuals’ level of human capital is an increasing function of the parental level of human capital ($r_2 > 0$). I will also assume that there exists diminishing returns to the parental human capital effect ($r_{22} < 0$). 6

The influence of peers ability on own educational achievement is well documented but still controversial. The most common perspective is that peers, like families are sources of motivation, aspirations and direct interactions in learning. Moreover, peers may affect the classroom process (aiding learning through questions and answers, contributing to the pace of instruction, or hindering learning through disruptive behavior à la Lazear (2002)). Most of these works focus on the average innate ability as the main characteristic of the student’s class mates which can affect his achievement. On the one hand, for example, Evans, Oates and Schwab (1992), and more recently Arcidiciano and Nicholson (2002) find a significant peer group effect that vanishes when they control for endogeneity. On the other hand, Henderson, Mieszkowski, and Sauvageau (1978), Summers and Wolfe (1977) and Zimmer and Toma (1997, 2000) report significant positive influences of higher achieving peers on achievement.

Once assumed that there exists a peer group effect, the question that arises now is related to the functional form of this effect. The majority of empirical works, among others, de Bartolome (1990), Henderson, Mieszkowski, and Sauvageau (1978), García-Díez (2000), Modrego and San Segundo (1988), Robertson and Symons (1996), Summers and Wolfe (1977), and Zimmer and Toma (1997) establish that the peer group

6 The empirical significance of the parental effect has been documented by Coleman et al (1966), Becker and Tomes (1986). More recently, among others, Feinstein and Symons (1999) found that parent interest is the principal via by which the attainments of each generation are passed to the next.

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effect is non-linear. The achievement of individual students rises with an improvement in the average quality of their classroom situations, but the increment in achievement decreases with the level of average class quality.

Following the empirical evidence previously commented, I assume that $r_1 > 0$ and $r_{11} < 0$. Feinstein and Symons (1999) also suggested the complementarity between parental interest and peer effect. Thus, I will assume that the production of human capital is characterized by complementarity between the parental human capital effect and the peer effect, $r_{12} > 0$.

To simplify the analysis, we assume the following human capital production function:

$$ r = (\pi_j)^{\alpha}(x)^{1-\alpha}. $$

The acquisition of human capital reflects the influence of family and peers factors, with respective weights $1 - \alpha$ and $\alpha$.

From Equations (1) and (2) we can observe that the peer effect becomes more effective in the production of human capital as the level of innate ability or parents’ income increases, that is $\frac{\partial^2 a_1}{\partial a_j \partial a_0} > 0$ and $\frac{\partial^2 a_1}{\partial a_j \partial x} > 0$.\(^8\)

In the second part of the first period, the individual decides whether to attend college or not. After attending college they will enjoy an increase in their level of human capital acquired during compulsory level. I denote such an increment by $\delta$ and, thus, those individuals who decide to attend college will end up with the following level of human capital:

$$ x_1 = a_1(1 + \delta) \quad (3) $$

The findings of recent empirical literature show that factors that take place in early stages of life are crucial determinants of children’s later success.\(^7\) In particular, Neal and Johnson (1996) find that differences in educational achievements by the time of high-school completion account for almost all the observed black-white wage gap. Therefore we assume that the acquisition of human capital at college is not

\(^7\)This technology of production of human capital is commonly used in this literature, see for example Benabou (1996), and Epple and Romano (1998 and 2002).

\(^8\)The empirical evidence regarding these properties is still mixed. Henderson et al. (1978) find no interaction between own ability and the benefits of an improved peer group, i.e. $\frac{\partial^2 a_1}{\partial a_j \partial a_0} = 0$ in this model. Argys et al. (1996) suggest $\frac{\partial^2 a_1}{\partial a_j \partial a_0} > 0$. Summers and Wolfe (1977) find some support for higher peer group benefits to lower ability students, that is, $\frac{\partial^2 a_1}{\partial a_j \partial a_0} < 0$.

\(^7\)See for example Keane and Wolpin (1997).
crucially affected by the family nor peers factors, and it just depends on the basis of the human capital previously acquired, $\delta = \delta(a_1)$. We assume that this increment, that reflects the efficacy of higher education, is an increasing function of the human capital acquired at compulsory level, but at a decreasing rate ($\delta_1 > 0, \delta_11 < 0$).

It is important to note that the characteristics of the group in which the individual is placed affect her final level of human capital $x_1$, through two different channels. First there is a direct effect since peers affects the human capital acquired at compulsory education. Second there is also an indirect effect since this level of human capital determines the efficacy of higher education and thus, as we will see below, the decision of the individual of whether to undertake college education.

It is important, therefore, to analyze the different composition of the groups at school (determined by the educational system prevailing). This composition is going to be crucial in determining the distribution of human capital across the population and, as we will see below in the individuals’ decision of whether to attend college or not.\footnote{Roemer and Wets (1994) distinguish different types of ‘neighborhood’ effects. All these effects have the consequence of inducing sub-optimal levels of education for a group in the population.} In the next section I will study the two different educational systems.

2.3 Educational Systems

I will consider two different educational systems at compulsory level: mixing and tracking. In this section I will describe them and I will also analyze the distribution of human capital across the population under both systems.

2.3.1 Mixing

Under this educational system students are sorted into completely homogeneous groups. Thus, the distribution of abilities within each track is exactly the same. In particular, it is uniform and with the same parameters than the innate ability distribution in the population. We denote the average ability in each group by $\overline{a_m}$.

It is the same in both groups and coincides with the average and the median ability in the population, i.e. $\overline{a_m} = m = 1/2$.

However, as individuals differ in their parents’ level of human capital, there will be two income groups in each classroom: the rich and the poor. With probability equal to $\lambda$, $a_1$ follows an uniform distribution on $(0,b)$, and with probability $(1-\lambda)$, $a_1$ follows a uniform distribution on $(0,a)$, where $b$ and $a$ denote the human capital
acquired by the “best” individual (more skilled) in the rich and the poor income
group, respectively:

\[ a' = 1 + (1/2)^\alpha \]
\[ b' = 1 + (1/2)^\alpha x^{1-\alpha}, \] (4) (5)

Under mixing, therefore, the C.D.F. (cumulative distribution function) of the
human capital acquired during compulsory level, denoted by \( F_M(a_1) \) is as follows:

\[
F_M(a_1) = \begin{cases} 
0 & \text{if } a_1 \leq 0 \\
\left( \frac{\lambda}{b'} + \frac{(1 - \lambda)}{a'} \right) a_1 & \text{if } 0 \leq a_1 \leq a' \\
(1 - \lambda) + \frac{\lambda}{b'} a_1 & \text{if } a' \leq a_1 \leq b' \\
1 & \text{if } a_1 > b'
\end{cases}
\] (6)

I denote by \( E_M(a_1) \) the expected value of \( a_1 \) under mixing. I have that:

\[ E_M(a_1) = (1 - \lambda) \frac{a'}{2} + \lambda \frac{b'}{2}. \]

or, using Equations (4) and (5):

\[ E_M(a_1) = \frac{(1 + (1/2)^\alpha)}{2} + \frac{\lambda}{2} (1/2)^\alpha [x^{1-\alpha} - 1]. \] (7)

Thus I have that \( E_M(a_1) \) is an average of the mean values of \( a_1 \) in the two income
groups, the poor and the rich income group under mixing with respective weights
\((1 - \lambda) \) and \( \lambda \). From the previous Equation we observe thus the expected value of \( a_1 \)
is a increasing function of both \( x \) and \( \lambda \).

2.3.2 Tracking

Under this system students are grouped based on their innate ability. For simplicity,
I permit at most two tracks . Thus, the median level of innate ability \( m \), is used as a
threshold ability to group students. Students are assigned to the high (low) track as
their own ability \( a_0 \geq (\leq) m \).

The distribution of human capital within each track is uniform but with different
parameters since the intervals are different. I denote by \( \bar{a}_h \) and \( \bar{a}_l \) the average ability
in the high and low track respectively. Thus, given the distributional assumption
on \( a_0 \), I have that \( \bar{a}_h = 3/4 \), whereas \( \bar{a}_l = 1/4 \). It is clear that for any distribution
function of \( a_0 \) the following condition is satisfied:

\[
\bar{a}_h > \bar{a}_m > \bar{a}_l.
\]  

(8)

However, there will be two income groups within each track. In the low track we have that, with probability equal to \( \lambda \), \( a_1 \) follows a uniform distribution on the interval \((0, c)\), and with probability \((1 - \lambda)\) it follows a uniform distribution on the interval \((0, a)\), where \( a \) and \( c \) denotes the human capital acquired by the “best” individual (more skilled) in the poor and the rich income group respectively, that is:

\[
a = \frac{1}{2} \left( 1 + \left( \frac{1}{4} \right)^\alpha \right)
\]

(9)

\[
c = \frac{1}{2} \left( 1 + \left( \frac{1}{4} \right)^\alpha \cdot x^{1-\alpha} \right).
\]

(10)

In the same way we have that, in the high track with probability equal to \( \lambda \), \( a_1 \) follows a uniform distribution on the interval \((b, e)\), and with probability \((1 - \lambda)\) it follows a uniform distribution on the interval \((d, f)\). We denote by \( b \) and \( d \), the human capital acquired by the “worst” individual (less skilled) in the poor and the rich income group respectively. We denote by \( e \) and \( f \) the human capital acquired by the “best” individual (more skilled) in the poor and the rich income group respectively, i.e.:

\[
b = \frac{1}{2} \left( 1 + \left( \frac{3}{4} \right)^\alpha \right)
\]

(11)

\[
e = \left( 1 + \left( \frac{3}{4} \right)^\alpha \right)
\]

(12)

\[
d = \frac{1}{2} \left( 1 + \left( \frac{3}{4} \right)^\alpha \cdot x^{1-\alpha} \right)
\]

(13)

\[
f = \left( 1 + \left( \frac{3}{4} \right)^\alpha \cdot x^{1-\alpha} \right).
\]

(14)

From the previous Equations (9) to (14) we have that the following two conditions over the extreme values of the intervals applies. First we have that \( a < c, b < d \) and \( e < f \). That is, independently of the ability group in which the individual is placed given two individuals with the same level of innate ability, the one whose parents have higher income level will always attain a higher level of human capital. The second one implies that \( a > 0, c > 0, e > b \) and \( f > d \). This means that, again independently of the ability group in which the individual is placed, given two individuals whose parents have the same income level, the one with higher level of innate ability will attain a higher level of human capital.
Now I need to introduce some assumptions to ensure that the support of the random variable $a_1$ has no empty intervals under tracking. The first assumption also ensures that I will not have a corner solution in the sense that no system is always clearly the best (under some particular criteria). The following assumptions refer to the relations between two intervals of the low track and high track, and between the two intervals of the high track and mixing.

**Assumption 1 (A.1):** $c > b$.

This assumption implies that the support of $a_1$ in the low track overlaps the support of $a_1$ in the high track. In other words, the “best” (the richest and more skilled) individual in the low track has greater human capital than the “worst” individual in the high track (the poorest and less skilled).

Assumption 1 implies that $x$ has to be above a threshold level: $x > \overline{x}(\alpha) = 3^{\frac{\alpha}{1-\alpha}}$.

**Assumption 2 (A.2):** $a' > d$.

It implies that the “best” individual within the poor group under mixing can achieve a higher level of human capital than the “worst” individual within the rich group in the high track. It implies that $x$ must be below a threshold level, in particular $x < \overline{x}(\alpha) = ((4/3)(1+2^{1-\alpha}))^{\frac{1}{1-\alpha}}$.

In addition A.2 implies that the two intervals within the high track overlap, as in the low track case. That is, the “best” individual in the low income group has greater human capital than the “worst” individual in the high income group.

From A.1, A.2, and Equations (4), (5), (12) and (14), I have that the following two conditions hold:

\[ e < b' < f, \quad (15) \]
\[ d < a' < e. \quad (16) \]

These conditions refer to the relationship between the intervals of $a_1$ under tracking and under mixing. In particular we have that, independently of the income group where the individual is placed, an individual with the highest level of innate ability will achieve a higher level of human capital under tracking than under mixing, that

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10 One could think that the mixing system represents the public education system whereas tracking represents a private system where only individuals with high levels of innate ability (and rich) are accepted. Thus, A.2 implies that the best student in the public school can achieve a higher level of human capital than the worst student in the private school.
is, \( a' < e \) and \( b' < f \). In addition, and due to the assumption on income differentials (A.1), an individual in the high income group will achieve under mixing a level of human capital higher than under tracking in the low income group, for the same level of innate ability, that is, \( e < b' \).

Therefore, from the previous two assumptions I have that the level of income for the rich individuals must belong to the following interval for any \( \alpha \in (0,1) \):

\[
\underline{x}(\alpha) < x < \overline{x}(\alpha).
\]  

(17)

In Figure 1 I illustrate the different intervals for \( a_1 \) and the relation among them, for both educational systems.

Now, under assumptions A.1 and A.2, the C.D.F. of \( a_1 \) under tracking, denoted by \( F_T(a_1) \) is as follows:

\[
F_T(a_1) = \begin{cases} 
  0 & \text{if } a_1 \leq 0 \\
  \left( \frac{\lambda}{2c} + \frac{(1-\lambda)}{2a} \right) a_1 & \text{if } 0 \leq a_1 \leq a \\
  \left( \frac{1-\lambda}{2} + \frac{\lambda}{e} a_1 \right) & \text{if } a \leq a_1 \leq b \\
  \left( \frac{\lambda}{2c} + \frac{(1-\lambda)}{e} \right) a_1 & \text{if } b \leq a_1 \leq c \\
  \left( \frac{\lambda}{2} + \frac{(1-\lambda)}{f} \right) a_1 & \text{if } c \leq a_1 \leq d \\
  \left( \frac{\lambda}{f} + \frac{(1-\lambda)}{e} \right) a_1 & \text{if } d \leq a_1 \leq e \\
  (1-\lambda) + \frac{\lambda}{f} a_1 & \text{if } e \leq a_1 \leq f \\
  1 & \text{if } a_1 > f 
\end{cases}
\]  

(18)

It can be checked that, as the weight of the peer effect on the human capital production function decreases, that is, \( \alpha \) tends to zero, the four intervals for \( a_1 \) under tracking are going to collapse in just two, the poor and the rich. In the first group \( a_1 \) follows a uniform distribution on the interval \([0,2]\), and in the second group \( a_1 \) follows a uniform distribution on the interval \([0,x+1]\). Under mixing we have that as \( \alpha \) tends to zero, \( a_1 \) will follow a uniform distribution in the same two intervals than in the tracking case.

Under tracking I have that the expected value of \( a_1 \), denoted by \( E_T(a_1) \) is:

\[
E_T(a_1) = (1-\lambda)\frac{a}{4} + (1-\lambda)\frac{3b}{4} + \lambda\frac{c}{4} + \lambda\frac{3d}{4}.
\]
and from Equations (11) to (14):

\[
E_T(a_1) = \left(\frac{4 + (1/4)^\alpha (1 + 3^{\alpha+1})}{8} + \frac{\lambda(1/4)^\alpha (1 + 3^{\alpha+1})}{8}\right) \left[\frac{x^{1-\alpha}}{\alpha} - 1\right].
\] (19)

Note that, as in the mixing case, the expected value of \(a_1\) is a weighted average of the mean value of \(a_1\) in the four income groups previously analyzed.

In the next section I provide a more detailed analysis of the human capital distribution function under both educational systems. I will analyze the effects of several parameters on the shape of this distribution.

### 2.4 Individuals’ decision

Individuals make decisions after attending compulsory schooling, when they decide whether to purchase higher education or not. Along the first period they accumulate human capital.

Then, I need to study how individuals decide whether to attend college or not. To do this, I assume that individuals maximize their consumption that is equal to their lifetime income. Thus, note that I assume that wages are a linear function of the level of human capital. In this paper I consider only the private returns to education, and I do not include the possible externalities created by human capital investments (e.g. low-skilled workers could benefit from high-skilled workers through a positive growth externalities). On the one hand, if an individual does not go to college, her lifetime income will be the unskilled wage, that is, the human capital acquired just after compulsory education \(a_1\), times the fraction of the first period that she works as an unskilled worker, \(\gamma\), plus the unskilled wage \(a_1\). That is, \(a_1\gamma + a_1\).

On the other hand, the lifetime income of those individuals who decide to go to college is the skilled wage, i.e. the increased level of human capital after attending college, \(a_1(1 + \delta(a_1))\).

Finally, individuals take as given the educational system. Then, for all individuals who decide to attend college, the following condition must hold\(^{11}\):

\[
a_1(1 + \delta(a_1)) \geq a_1(1 + \gamma),
\]

\(^{11}\)For some parameters value, educated workers may have lower lifetime income than the present value of the uneducated lifetime income, i.e. \(a_1(1 + \delta) < a_1(1 + \gamma)\). Thus, one may want to impose \(\gamma \leq \delta(f)\) to avoid this. In other case nobody will decide to attend college.
or,
\[ \delta(a_1) \geq \gamma. \]  \hspace{1cm} (20)

This condition determines a minimum level of human capital accumulated through the compulsory education \( a_1^* \), such that only individuals with a level of human capital above this threshold will attend college. That is, \( a_1^* \) is the value that satisfies the previous equation with equality.

We can analyze the composition of the students body under tracking, depending on the interval where \( a_1^* \) is placed. Thus, we can distinguish the following three cases. First, if \( a_1^* \in (0, b) \) then there are some individuals from the low track attending college whereas all the individuals placed in the high track attend college. Second, if \( a_1^* \in (b, c) \) there are some individuals from the low track and some others from the high track attending college. Finally, if \( a_1^* \in (c, f) \) only some individuals from the high track attend college.

The interval where \( a_1^* \) is placed depends on both the efficacy of higher education \( \delta(a_1) \), and the opportunity cost of attending college, \( \gamma \). In particular, for a given function \( \delta(a_1) \), an increase in the parameter \( \gamma \) implies an increase in the opportunity cost of acquiring additional education. As a result there will be a lower proportion of individuals willing to attend college. Following the same reasoning, for a fixed \( \gamma \), a change in the function \( \delta(a_1) \) such that for a given \( a_1 \) the efficacy of higher education increases, it implies that there will be a higher proportion of individuals willing to attend college.

3 The Human Capital Distribution

In this section I characterize the effects of the wealth level in the population, measured by the proportion of high income individuals in the population, and the impact of the peer effect on the distribution of \( a_1 \) under both educational systems. I will analyze if there is a significative change in the shape of both distribution functions when the level of income inequality increases in the population, and when the peer effect is reinforced in the human capital production.

3.1 Wealth and Income Inequality
We will see if there are differences in the performance of both educational systems for poor and rich societies. I analyze, therefore, for each educational system, the effect of the mean income level, on the human capital distribution. Note that in this model mean income is equal to \( \lambda x + (1 - \lambda) \). Thus, this effect can be decomposed into two components: the wealth level, measured by the parameter \( \lambda \), and the level of income inequality, measured by the parameter \( x \).

First I analyze for each educational system, the effect of the wealth level. On the one hand, under mixing we have that an increase in \( \lambda \) implies that there will be a lower proportion of individuals placed in the low income group. For given values of \( x \) and \( \alpha \), as \( \lambda \) increases the population moves to the high income group.

On the other hand, under the tracking system an increase in the wealth level implies that there will be a lower proportion of individuals placed in the low income groups for the low and the high track respectively. As in the mixing case, it implies that the population (other things equal) and so does the human capital distribution, will move to the high income groups for the low and the high track respectively.

Not surprisingly, the effect of the level of income inequality on the shape of the human capital distribution is quite similar to the effect of wealth level. Under mixing an increase in income inequality in the population implies a relatively lower proportion of individuals with levels of human capital below \( a' \). In addition, for a fixed value of \( \alpha \), there will be an increase in the level of human capital achieved by an individual whose parents have high income level. Therefore, the domain of \( a_1 \) increases and, other things equal, the human capital distribution becomes flatter.

If the educational system prevailing is tracking then, an increase in income inequality implies a relative decrease in the proportion of individuals with human capital below \( a \), or below \((b,e)\), i.e. from the low income group. As in the mixing case, the domain of \( a_1 \) increases and the human capital distribution becomes flatter.

In the following Proposition we describe both effects, of the wealth level and the income inequality, on the shape of the distribution function of human capital:

**Proposition 1** Let \( a_1 \geq 0 \), then the following two conditions hold for \( s = T, M \):

(i) \( F_s(a_1/\lambda') \leq F_s(a_1/\lambda) \) for every \( \lambda' > \lambda \),

(ii) \( F_s(a_1/x') \leq F_s(a_1/x) \) for every \( x' > x \).

**Proof.** It is immediate from \( F_M(a_1) \), in (6) and \( F_T(a_1) \), in (18). If we calculate the respective derivatives of both C.D.F. with respect to \( \lambda \) and \( x \), we can check that both
are strictly decreasing functions of the level of wealth, and income inequality in the population.

The previous Proposition implies that the mean value of \( a_1 \) increases with the wealth level and the level of income inequality in the population, under both educational systems. We can also check it from Equations (7) and (19). This result is very intuitive. A wealthier economy implies that there are either more individuals with high income level or, the same proportion of individuals but with a higher level of income. As a result, and from the properties of the human capital production, it implies an increase in the aggregated level of human capital for all the population.

3.2 Peer Effect

Now we analyze which is the effect of an increase in the role of peers \( \alpha \), in the human capital production. We assume throughout this section that \( \alpha \in (\underline{\alpha},\bar{\alpha}) \) where \( \underline{\alpha} \) and \( \bar{\alpha} \) are such that, for a given \( \tilde{x} \), \( x(\underline{\alpha})=\tilde{x} \), and \( x(\bar{\alpha})=\tilde{x} \).

First of all note that as \( \alpha \) tends to zero, the distribution of human capital under both educational systems will coincide, since it implies that peers have no effect on the human capital acquired at college, and as a result the difference in human capital across individuals is only due to differences in parents’ income level. However in this analysis we do not consider extreme values of \( \alpha \) since in that case A.1 and A.2 do not hold.

When the peer effect is reinforced in the human capital production there is a decrease in the level of human capital achieved by all individuals, independently of their parents’ income level, \( x \). It implies first that the different intervals for \( a_1 \) under both educational systems become smaller and second that both distributions of \( a_1 \) move to the left. As a result and since \( a_1 \) is uniformly distributed in each interval, there is an increase in the values of the C.D.F. in all the intervals under both educational systems.

**Proposition 2** Let \( a_1 \geq 0 \), then \( F_s(a_1/\alpha) \leq F_s(a_1/\alpha') \) for every \( \alpha' > \alpha \) for \( s = T, M \).

**Proof.** It is immediate from \( F_M(a_1) \), in (6) and \( F_T(a_1) \), in (18). If we calculate the derivative of both C.D.F. with respect to \( \alpha \), we can check that both are strictly increasing functions with the parameter of the peer effect in the human capital production function.
The previous Proposition shows that the mean of $a_1$ for $\alpha$, $E_s(a_1/\alpha)$ is greater than its mean for any $\alpha' > \alpha$, $E_s(a_1/\alpha')$ independently of the educational system prevailing. We can also check it from Equations (7) and (19). As previously commented this result is due to the negative effect that an increase in the impact of the peer effect has on the human capital achievement.

4 Evaluating educational systems

In section 2 I presented a comparative static analysis of the performance of both educational systems for poor and rich societies, within the same educational system. In this section I analyze the distribution of $a_1$ under both two educational systems: tracking and mixing. We propose several criteria to establish a comparison between both distributions.

First of all we check if there is an educational system strictly preferred by all the individuals in the population, and with that aim we apply the concept of first order stochastic dominance. As a second criteria we propose to find the system that maximizes the average human capital across the population. The third criteria that we apply is second order stochastic dominance, that is we check if there is an educational system that is unanimously preferred by all risk averse individuals. In addition we compare both educational systems in terms of college attendance. We analyze also the intergenerational mobility that takes place under each educational system, in particular we focus on the upward mobility. Finally we use the equality of opportunities criteria to compare the tracking and the mixing system.

4.1 First Order Stochastic Dominance

In a previous section I have checked that the human capital achieved by the individual is a random outcome. In addition the final level of human capital achieved by any individual depends also on the educational system prevailing: tracking and mixing. There are two natural ways in which both distributions of human capital can be compared: according to the level of human capital and according to the dispersion of this human capital.

In this section I analyze the first idea. I will check, therefore, if there is a distribution $F_s(a_1)$ under educational system $s$, that yields unambiguously higher level of
human capital than the other. With this aim I use the concept of first order stochastic dominance.\textsuperscript{12}

**Definition 3** Given two distribution functions $F(\cdot)$ and $G(\cdot)$, $F(\cdot)$ first order stochastically dominates $G(\cdot)$, $F(\cdot) \succ_{\text{FOSD}} G(\cdot)$, if:

(i) $F(z) \leq G(z)$ for all $z \in \mathbb{R}$ and

(ii) there exists $\tilde{z}$ such that $F(\tilde{z}) < G(\tilde{z})$.

We will check if there is a system under which the probability of attaining at least any level of human capital is higher than under the other system. In the following Proposition I show that there is no distribution of human capital that FOSD the other.

**Proposition 4** $F_r(a_1) \not\succ_{\text{FOSD}} F_s(a_1)$ for $r, s = M, T$ and $r \neq s$.

**Proof.** (i) $F_T(a_1) \not\succ_{\text{FOSD}} F_M(a_1)$. From $F_T(a_1)$ in Equation (18) and $F_M(a_1)$ in Equation (6) we can check that for any $a_1 \in [0, a]$, $(F_T(a_1) - F_M(a_1)) > 0$ for every $\lambda, \alpha$ and $x$. (ii) $F_M(a_1) \not\succ_{\text{FOSD}} F_T(a_1)$. Using again Equation (18) and (6) we can check that for any $a_1 \in [d, f]$, $(F_T(a_1) - F_M(a_1)) < 0$ for every $\lambda, \alpha$ and $x$.

In the following remark I derive an implication of the previous Proposition.

**Remark 5** $F_M(a_1)$ cuts $F_T(a_1)$ from below for all $\lambda, \alpha$ and $x$.

We illustrate the previous result in Figure 2. We can conclude, therefore, that given the choice between both educational systems, some individuals prefer tracking and some others prefer mixing. If we use this criteria, therefore, there is no unanimity among all the individuals in the population in the choice of the educational system. In the next sections I propose several comparisons between tracking and mixing based on different criteria.

### 4.2 Average Human Capital

In the previous section I have shown that there is no unanimity among the individuals in the population when they have to choose between tracking and mixing. Thus, we

\textsuperscript{12}Both concepts of first and second order stochastic dominance were introduced in economics in Rothschild and Stiglitz (1970).
have to look for a different criteria to compare them. In this section I use the average human capital attained under each educational system.

The empirical evidence suggest that the growth level in the economy is positively correlated to the average level of human capital in the population. Thus, I think that a crucial criteria to make a choice between both educational system should be the average human capital achieved under each educational system. In the next Proposition I show that the average human capital is maximized under tracking.

**Proposition 6** Let \( \alpha > 0 \). Then, \( E_T(a_1) > E_M(a_1) \) for all \( x \) and \( \lambda \).

**Proof.** From Equations (7) and (19), and rearranging terms we have that the difference between the expected value of \( a_1 \) under tracking and mixing, \( E_T(a_1) - E_M(a_1) \) is positive if and only if the following condition holds:

\[
\lambda x^{1-\alpha} \left( \frac{1}{2} \right)^\alpha p(\alpha) + (1 - \lambda) \left( \frac{1}{2} \right)^\alpha p(\alpha) > 0,
\]

where \( p(\alpha) = \left( \frac{1}{3} \right)^\alpha (1 + 3^{\alpha+1} - \frac{1}{2}) \). Note that this expression is equivalent to the following one:

\[
p(\alpha) \left( \frac{1}{2} \right)^\alpha (1 + \lambda(x^{1-\alpha} - 1)) > 0.
\]

The previous expression is positive if and only if \( p(\alpha) > 0 \) for all \( \alpha \). But the function \( p(\alpha) \) is strictly increasing with \( \alpha \) and is equal to zero if \( \alpha = 0 \). This proves the claim.

Therefore, for any wealth level in the population, if the government is interested in maximizing the average human capital across the population at compulsory level it should choose the educational system that groups students based on their innate ability. It is important to stress the fact the previous result applies to the compulsory level. When analyzing the average of human capital at college level it has to be taken into account that after compulsory education some individuals have dropped off the educational system, and thus, the average human capital is not that straight forward.

From the previous Proposition it can be checked also that a ranking of the means of the two distributions under each educational system does not imply that one FOSD the other. In the next section I try to check if this ranking of the educational system

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13Galor and Tissdon (1994) and Galor and Zeira (1997) called it “global technological externality”. This term has been used by several other studies (eg. Benabou (1996), Iyigun and Owen (1996), Mountford (1996) and Tamura (1996)).
based on their respective average human capital is robust to other criteria: second order stochastic dominance.

4.3 Second Order Stochastic Dominance

In a previous section I proposed two natural ways to compare two distribution functions. The first one, that lead to the concept of first order stochastic dominance, was already analyzed. I showed that there is no unanimity in the choice of one educational system among all the individuals in the population. In this section I focus on a particular set of individuals, the risk averse ones. I will check if there is a distribution of human capital $F_s(a_1)$ under educational system $s$, unambiguously less risky than the other one. With this aim I use the concept of second order stochastic dominance.

**Definition 7** Given two distribution functions $F(\cdot)$ and $G(\cdot)$, $F(\cdot) \succ_{SOSD} G(\cdot)$, if

(i) \[ \int_{-\infty}^{y} F(z)dz \leq \int_{-\infty}^{y} G(z)dz \] for all $y \in \mathbb{R}$ and

(ii) there exists $\tilde{y}$ such that \[ \int_{-\infty}^{\tilde{y}} F(z)dz < \int_{-\infty}^{\tilde{y}} G(z)dz. \]

In the next Proposition I show that there is no distribution that SOSD the other.

**Proposition 8** $F_r(a_1) \not\succ_{SOSD} F_s(a_1)$ for $r, s = M, T$ and $r \neq s$.

**Proof.**

(i) $F_T(a_1) \not\succ_{SOSD} F_M(a_1)$. Using $F_T(a_1)$ in Equation (18) and $F_M(a_1)$, in Equation (6) we can check that, $\int_{0}^{b} (F_T(a_1) - F_M(a_1)) > 0$, for every $\lambda, \alpha$ and $x$.

(ii) $F_M(a_1) \not\succ_{SOSD} F_T(a_1)$. Note now that the expected value of any random variable defined on the interval $[0, \bar{z}]$ can be calculated in the following way: $E[z] = \bar{z} - \int_{0}^{\bar{z}} F(z)dz$. Thus, if $F_M(a_1) \not\succ_{SOSD} F_T(a_1)$ then the following inequality should hold: $f - E_M(a_1) \leq f - E_T(a_1)$. The final result is immediate from Proposition 6.

We can conclude, therefore, that given the choice to the risk averse individuals in the population, between the distribution of human capital under both educational systems there is no unanimity. Some of them will choose mixing whereas some other will choose tracking. The concept of second order stochastic dominance is related
to equity. If the risk averse individuals prefer one distribution to another this is because such distribution is more equitable than the other. However from the previous Proposition we can not say that the distribution of human capital is more equitable under one educational system than under the other.

4.4 Proportion of college students

In the previous section I have shown that there is no unanimity among the risk averse individuals in the population when they have to choose between tracking and mixing. Thus again, we have to look for a different criteria to compare both educational systems. In this section I use the proportion of individuals that decide to attend college under each educational system.

One could think of higher education as positive for the individual himself and his well-being, but also one could think about the positive externalities generated by more highly educated people for the entire society. Therefore we want to analyze which system, tracking or mixing, provides higher education to the highest number of individuals. I will provide a result that says that whether tracking or mixing is the best depends on how wealthy society is.

In section 3 I analyzed the effect of income level within each distribution of human capital. I analyze now how the wealth level in the economy can affect the relationship between the distributions of human capital under both educational systems. First of all I need to introduce some notation. I denote by $\tilde{a}_1$ the level of human capital such that $F_M(\tilde{a}_1) = F_T(\tilde{a}_1)$, that is, the value of $a_1$ where both C.D.F., $F_T$ and $F_M$, cross each other. By $\hat{\lambda}$ I denote the proportion of rich individuals in the economy such that for a given $\alpha_0 \in (\alpha, \alpha')$ and $x_0 \in (x, x')$ then, $\frac{\lambda}{1-\lambda} = G(\alpha', x')$, where $G(\alpha, x) = \frac{2x^2y(e-a')}{(e-2a')e}$.

In Remark (?) it was established that $F_M$ cuts $F_T$ from below for any $\lambda, \alpha$ and $x$. The next Proposition shows that both C.D.F. can cross each other in two different interval of $a_1$ depending on the proportion of rich individuals in the population, $\lambda$.

**Proposition 9** There is a unique $\tilde{a}_1 \neq 0$ that belongs to either $(a, b)$ if $\lambda < \hat{\lambda}$ or to $(c, d)$ if $\lambda > \hat{\lambda}$.

**Proof.** In the proof of Proposition 4 it was shown that $F_T(a_1) - F_M(a_1) > 0$ for all $a_1 \in (0, a)$, whereas $F_T(a_1) - F_M(a_1) < 0$ for all $a_1 \in (d, f)$. It is easy to verify also that $F_T(b) - F_M(b) < (>)0$ if and only if $F_T(c) - F_M(c) < (>)0$. If we evaluate the two C.D.F., under mixing and tracking for $a_1 = c$, we can check that, $F_M(c) =$
and from Equation (18) the wealth level will attain very low values of the human capital distribution at a tale of rich individuals in the economy, independently of the interval where $\tilde{a}_1$ belongs to.\footnote{Note from Equation (18) that if $\tilde{a}_1 \in (a, b)$ then, $\tilde{a}_1 = \frac{(1-\lambda)\alpha' \lambda' e^b}{\lambda' + (1-\lambda)2eb}$. If $\tilde{a}_1 \in (c, d)$ then, $\tilde{a}_1 = \frac{(1-\lambda)\alpha' \lambda' e^b}{2(\lambda' e + (1-\lambda)b' e - a' \lambda')}$.}

In addition we can easily check that $\tilde{a}_1$ is an increasing function of the proportion of rich individuals in the economy, independently of the interval where $\tilde{a}_1$ belongs to.

From the previous Proposition we have that, when the proportion of high income level in the population is low enough then, $F_M$ becomes soon above $F_T$, that is, for very low values of the human capital distribution. The intuition behind this result could be as follows. Suppose for example that $\lambda = 0$. In that case, as we have seen, under mixing $a_1$ follows a uniform distribution on the interval $[0, a']$, whereas under tracking exactly one half of the population has a level of human capital $a_1$ uniformly distributed on the interval $[0, a]$ and the other half has a level of $a_1$ uniformly distributed on the interval $[b, c]$. As a result $F_T(a) - F_M(a) > 0$ but $F_T(b) - F_M(b) < 0$. Following the same reasoning, if $\lambda = 1$ we have that $F_T(c) - F_M(c) > 0$ but $F_T(d) - F_M(d) < 0$. It could be interpreted in the following way. As the population becomes wealthier, the average human capital increases as we have checked in section 3. Therefore, the level of human capital such that there is the same proportion of individuals under both educational systems with a lower or equal level of human capital, i.e. $\tilde{a}_1$, increases. In other words, the C.D.F. under mixing is going to "dominate" the C.D.F. under tracking for a larger set of $a_1$.

Thus from the previous Proposition and Remark (?), we have that if $a_1 \in (0, \tilde{a}_1)$ then $F_T(a_1) - F_M(a_1) > 0$, and if $a_1 \in (\tilde{a}_1, f)$ then $F_T(a_1) - F_M(a_1) < 0$. In other words, there is always a higher proportion of individuals accumulated in the left tale of $a_1$, that is, for low values of $a_1$ under tracking, than under mixing. In the right tale of $a_1$, for high values of $a_1$ the reverse occurs.

We have analyzed the effect of the wealth level in the population on the shape of both distributions of human capital, and in particular, on the value of $a_1$ for which both distribution cross each other, that is $\tilde{a}_1$. It is important now to consider how the wealth level will affect the relationship between $\tilde{a}_1$ and $a_1^*$ since this relation, as we will see below, determines the system that maximizes the proportion of college students.
In the following Proposition I show that there is a threshold value of $\lambda$ such that, when $\lambda$ is above that threshold, the optimal educational system in maximizing college attendance is mixing, whereas for lower values the optimal educational system is tracking. I show also that this threshold is increasing with the opportunity cost of attending college. Note from Equation (??) that the minimum level of human capital required to attend college is increasing with the cost of opportunity of attending college, $\gamma$. I denote by $\gamma$ the opportunity cost such that $a_1(\gamma) = a$, and by $\overline{\gamma}$ the opportunity cost such that $a_1(\overline{\gamma}) = d$.

**Proposition 10** As the opportunity cost of attending college increases, the proportion of rich individuals required to maximize college attendance under mixing increases too.

**Proof.** From Proposition 9 and Remark 10 we have that a necessary and sufficient condition to ensure that $1 - F_M(a_1^*) > (<) 1 - F_T(a_1^*)$ is that $\tilde{a}_1 > (<) a_1^*$. If $\gamma < \overline{\gamma}$ then, from Proposition 9 we have that $\tilde{a}_1 > a_1^*$ for all $\lambda$. Now let $\gamma$ belong to $(\underline{\gamma}, \overline{\gamma})$. If $\gamma$ is such that $a_1^* \in (b, c)$ then, from Proposition 9 we have that if $\lambda < \tilde{\lambda}$ then, $\tilde{a}_1 < a_1^*$ and if $\lambda > \tilde{\lambda}$ then, $\tilde{a}_1 > a_1^*$. Now let $\gamma$ be such that $a_1^* \in (a, b)$ or $a_1^* \in (c, d)$. Then, for each $a_1^*$ there is one $\lambda$, denoted by $\tilde{\lambda}$, such that $\tilde{a}_1(\tilde{\lambda}) = a_1^*$. Thus, since $\tilde{a}_1$ is increasing with $\lambda$ we have that, if $\lambda < \tilde{\lambda}$ then $\tilde{a}_1 < a_1^*$ and if $\lambda > \tilde{\lambda}$ then, $\tilde{a}_1 > a_1^*$. From Proposition 9 we have that $\tilde{\lambda} < \overline{\lambda}$. Finally, if $\gamma > \overline{\gamma}$ then, from Proposition 9 we have that $\tilde{a}_1 < a_1^*$ for all $\lambda$. ■

I illustrate the previous result in Figure 3. We can interpret it in the following way. First note that for extreme values of $a_1^*$, the proportion of high income individuals in the population plays no role in choice of the educational system that maximize the proportion of college students. If the minimum level of human capital required to attend college is very low, the proportion of college students is maximized under mixing. The intuition could be that under mixing there is a high proportion of individuals with intermediate levels of $a_1$, whereas under tracking the proportion of individuals with both very low and very high levels of $a_1^*$ is higher than under mixing. For the same reason, if the minimum level of income required to attend college is very high, the proportion of college students is maximized under tracking. In this case the proportion of individuals with greater levels of $a_1$ is higher under tracking.

We can observe also that for intermediate values of $a_1^*$, the educational system that maximizes college attendance depends on the wealth level in the population. In particular, as the proportion of rich or educated individuals increases the optimal
educational system changes from tracking to mixing. The intuition could be as follows. Take as given a minimum level of human capital required to attend college, \( a_1^* \). In poor societies there is a higher proportion of individuals with low income level so, in order to increase the proportion of individuals with a level of human capital above \( a_1^* \) the optimal educational system is tracking since in that case we let the peer group input play a more important role in the human capital production. However, in wealthier societies where there is a higher proportion of individuals with high income level, the optimal educational system is mixing since in that case the peer group effect is not crucial in order to increase the proportion of individuals with a level of human capital above \( a_1^* \).

In addition, the previous Proposition implies that as the minimum level of human capital required to attend college increases, the wealth level required to maximize college attendance under mixing increases too. In other words, it is needed a higher proportion of rich individuals, whose children will achieve high levels of human capital, if the peer group effect plays a less important role in the production of human capital.

In the following section we analyze a different criteria to compare both educational systems. We will focus now on the intergenerational mobility among individuals with different levels of education.

4.5 Intergenerational mobility

In the previous sections we have not focused on particular sets of individuals among the population. Now we assume that the government is interested in maximizing the welfare of the worse-off. In particular we assume now that the government is interested in maximizing the probability of attending college of those individuals whose parents have low income level or low levels of human capital.

Thus, in this section I analyze the upward mobility in this economy under both educational systems.

I denote by \( \pi_{1,s} \) the probability of attending college for any individual given that their parents have low income or educational level \( x = 1 \), under educational system \( s \), for \( s = M, T \), that is \( \pi_{1,s} = 1 - F_s(a_1^*/x = 1) \). In the next Proposition we show that when the minimum level of human capital required to attend college is very low then, mixing maximizes the proportion of individuals attending college whose parents have low income or education level. We show also that for high values of \( a_1^* \) the reverse occurs.
Proposition 11 Let $a^*_1 \in (0,e)$. The following statements hold:

(a) If $a^*_1 < (1/2)a'$, then $\pi_{1,M} > \pi_{1,T}$.
(b) If $a^*_1 > (1/2)a'$, then $\pi_{1,M} < \pi_{1,T}$.

Proof. In a previous section we have shown that, under both educational systems $a_1$ is uniformly distributed on the different intervals determined by each income and ability group. Thus under mixing, if $a^*_1 \in (0,a_0)$ then $\pi_{1,M} = 1 - \frac{a^*_1}{a'}$ whereas if $a^*_1 \in (a',b')$ then $\pi_{1,M} = 0$. These probabilities under tracking are as follows. If $a^*_1 \in (0,a)$ then $\pi_{1,T} = \frac{2a-a^*_1}{2a'}$, for any $a^*_1 \in (a,b)$ we have that $\pi_{1,T} = \frac{1}{2}$. Following the same reasoning, if $a^*_1 \in (b,e)$ then $\pi_{1,T} = \left(\frac{1}{2}\right) - \frac{1}{2} \frac{(a^*_1-b)}{(e-b)}$. The proof follows just by comparing $\pi_{1,M}$ and $\pi_{1,T}$ for the different intervals. ■

From the human capital production function we have that, for those individuals with innate ability below the median level, the final human capital acquired under mixing is higher than under tracking. In particular, among those with poor parents, those individuals with innate ability equal to the median level ($a_0 = 1/2$) have a final human capital under mixing equal to $(1/2)a'$. Under tracking some of them will be placed in the high track and thus their final human capital is equal to $b$. However, some of those individuals will be placed in the low track, and their final human capital is equal to $a$.

Therefore, if the minimum level of human capital required to attend college is such that $a^*_1 < (1/2)a'$, the proportion of individuals with an equal or higher level of human capital under tracking is higher than under mixing. The reverse occurs when $a^*_1$ takes intermediate values, in particular when $a^*_1 \in ((1/2)a',e)$, and the upward mobility under tracking is higher than under mixing.

Finally, note that when $a^*_1 > e$ then, the probability of attending college is equal to zero for those individuals born of less educated parents, independently of the educational system prevailing.

4.6 Equality of Opportunities

In this section I will use a different criteria to compare the distribution of human capital under both educational systems. In particular I assume now that the government chooses the educational system that guarantees that the individuals’ decision of whether to attend college or not is taken independently of parents’ income.

I denote by $\pi_{x,s}$ the probability of attending college of an individual with parents’ income $x > 1$, under educational system $s$, where $s = M, T$. Thus $\pi_{x,s} = 1$–
$F_p(a_1^*/x > 1)$. Note that the probabilities of college attendance depend on the minimum level of human capital required to attend college, $a_1^*$.

For every educational system we define the income premium $p_s(a_1^*)$ or income gap as the difference in the probability of attending college between the rich and the poor:

$$p_s(a_1^*) = \pi_{x,s} - \pi_{1,s}$$

(21)

Note that the income premium is defined only for strictly positive values of both $\pi_{x,s}$ and $\pi_{1,s}$, that is, when the minimum level of human capital required to attend college is such that there are individuals for both income groups attending college under both systems. It is equivalent to say that the income premium is defined for every $a_1^* \leq a'$.

An educational system is called “equitable” if it minimizes this income premium. As we will see below, which system is more equitable depends on the minimum level of human capital required to attend college.

The next proposition shows that, when comparing the income premium of those individuals less able, we have to consider also the level of income inequality in the economy to determine the equitable system. Thus, I denote by $\tilde{x}(\alpha)$ the threshold level of income inequality that determines the equitable system in each case, where $\tilde{x}(\alpha) = 2^\alpha(\frac{1}{\alpha-1})$ and $\tilde{x}(\alpha) \in [\bar{x}(\alpha), \underline{x}(\alpha)]$

**Proposition 12** Let $a_1^* \leq b$. The following statements hold:

(a) If $x \leq \tilde{x}(\alpha)$ then, $p_T(a_1^*) \leq p_M(a_1^*)$.

(b) If $x \geq \tilde{x}(\alpha)$ then, there is a $\eta_1 \in (a,b)$ such that $p_T(a_1^*) \geq p_M(a_1^*)$ if $a_1^* < \eta_1$ and $p_T(a_1^*) \leq p_M(a_1^*)$ if $a_1^* > \eta_1$.

**Proof.** From Equations (6) and (21) we have that the income premium under mixing when $a_1^* \in (0,a')$ is $p_M(a_1^*) = a_1^* \left( \frac{b-a'}{a-a'} \right)$. Take first any $a_1^* \in (0,a)$ and the resulting income premium under tracking from Equations (18) and (21) is $p_T(a_1^*) = a_1^* \left( \frac{\bar{x} - \underline{x}}{2\bar{x} - \underline{x}} \right)$. By A.2 we have that $a < a'$. Thus, just comparing both income premium and from Equations (4), (5) for $p_M(a_1^*)$ and Equations (9) and (10) for $p_T(a_1^*)$ we can check that $p_T(a_1^*) \geq (\leq) p_M(a_1^*)$ if and only if $x \geq (\leq) \tilde{x}(\alpha)$. Now take any $a_1^* \in (a,b)$, we can check that the income premium under tracking is $p_T(a_1^*) = \frac{1}{2} \left( 1 - \frac{a_1^*}{\bar{x}} \right)$. Let $\eta_1 = \frac{a_1^*}{2(a-b)}$ be a level of human capital strictly lower than $b$. Then, if we compare again the income premium under tracking and mixing we find that a sufficient condition to ensure that $p_T(a_1^*) \leq p_M(a_1^*)$ is $x \leq \tilde{x}(\alpha)$. If $x > \tilde{x}(\alpha)$ then,
just by comparing both income premium it can be checked that \( p_T(a_1^*) \geq (\leq) p_M(a_1^*) \) if and only if \( a_1^* \leq (\geq) \eta_1 \). ■

I illustrate the previous result in Figure 4. First of all note that if \( a_1^* \leq b \) then, every individual placed in the high track will attend college, and thus both individuals with high and low income parents share the same probability of attending college, and equal to (1/2). This implies that the income premium under tracking is measuring just the income gap for those individuals placed in the low track.

The proposition above shows that when income inequality is low, tracking is the most equitable system, independently of the level of human capital required to attend college. If it is not the case and there is a high level of income inequality, as \( a_1^* \) increases, tracking becomes the equitable educational system.

The next proposition shows that equitable educational system for the most able individuals does not depend on the income inequality prevailing in the economy, as for the less able one, and it just depends on the minimum level of human capital required to attend college. It can be shown that when there are individuals from both income groups attending college, under both educational systems, tracking is equitable. If it is not the case and there is a high level of income inequality, as \( a_1^* \) increases, tracking becomes the equitable educational system.

**Proposition 13** Let \( a_1^* > b \) then \( p_M(a_1^*) > p_T(a_1^*) \) independently of the level of income inequality in the economy.

**Proof.** Take any \( a_1^* \) that belongs to \((b, c)\). From Equations (18) and (21) we obtain
\[
p_T(a_1^*) = a_1^* \left( \frac{\alpha}{\alpha_2} \right) .
\]
As \( a' > b \) we that the income premium under mixing is \( p_M(a_1^*) = a_1^* \left( \frac{\nu-a'}{a' - c} \right) . \) From Equations (4) and (5) for \( p_M(a_1^*) \) and Equations (10) and (11) we can check that \( p_M(a_1^*) > p_T(a_1^*) \) is equivalent to the following expression:
\[
\left( x^{1-\alpha} - 1 \right) (1 + \left( \frac{\alpha}{\alpha_2} \right))(1 + \frac{\alpha}{\alpha_2} x^{1-\alpha}) > \left( \frac{\alpha}{\alpha_2} \right) \left( x^{1-\alpha} - 3^\alpha \right) (1 + \frac{\alpha}{\alpha_2} + \frac{\alpha}{\alpha_2} x^{1-\alpha})
\]
and this expression is positive for every \( x \) and \( \alpha \). Now take any \( a_1^* \) that belongs to \((c, d)\). From Equations (18) and (21) we obtain
\[
p_T(a_1^*) = \frac{1}{2} \left( \frac{a_1^*}{b} - 1 \right).
\]
From A.2 we have that \( d < a' \), and thus, \( p_M(a_1^*) > p_T(a_1^*) \) implies that the following inequality must hold: \( a_1^* < \frac{\nu-a'}{a' - c} \). A sufficient condition to ensure it is \( \frac{\nu-a'}{a' - c} \leq d \). The last inequality is equivalent to the following one: \( \frac{\nu-a'}{a' - c} \geq \frac{d-b}{2bd} \). From Equations (4) and (5) for \( p_M(a_1^*) \) and Equations (11) and (13) for \( p_T(a_1^*) \) we can check that the last inequality holds if and only if \( x \geq (2^\alpha(4/3)^\alpha)^{\frac{1}{1-\alpha}} \). But this is always true since
\( \bar{\alpha} > (2^{\alpha}(4/3)^{\alpha})^{\frac{1}{\alpha}} \). Now, if \( a^*_t \) belongs to \((d,a')\) from Equations (18) and (21) we have that \( p_T(a^*_t) = a^*_t \left( \frac{d-b}{2\sigma d} \right) \). But we have just seen that \( \frac{b_0-a_0}{a_0} \geq \frac{d-b}{2\sigma d} \), and thus \( p_M(a^*_t) > p_T(a^*_t) \) in this interval. 

The intuition behind this result could be as follows. First note that when the minimum human capital required to attend college is above \( b \) there are no individuals from the low track and the low income group attending college. Since the positive effect of the peer variable is higher for the most skilled individuals, under tracking, and in particular, for those students placed in the high track, the parents’ income is not a crucial factor to determine their final level of human capital. However, under mixing and for those individuals with the same level of innate ability their parents income has a higher relative weight. As a result the difference in the probability of attending college is crucially affected by the income variable. I illustrate this proposition in Figure 4.

5 Conclusions

In this paper we analyze public intervention in education when the government, taking into account the process of human capital production and in particular the peer effect on students’ achievement, has to decide the optimal educational system. I analyze two different educational systems. The first one tracking, consists on grouping students according to their innate ability. The second one mixing implies that students are sorted into completely homogeneous groups. I provide some results about the characteristics of the distribution of human capital at compulsory level under both educational systems.

Therefore, the objective of this paper was to evaluate both educational systems using several criteria. First of all I checked if there is an educational system strictly preferred by all the individuals in the population, and with that aim applied the concept of first order stochastic dominance. I find that there is no educational system that yield unambiguously higher level of human capital than the other for all the individuals. Therefore, there is no unanimity among the individuals in the population when they have to choose between tracking and mixing.

However, if we are interested in maximizing the average human capital at compulsory level across the population, then the optimal educational system is tracking.

The third criteria that I used is second order stochastic dominance, that is I check if there is an educational system preferred by a set of the population, the risk averse
individuals. It was checked that we can not say that the distribution of human capital is more equitable under one educational system than the other, or is riskier than the other.

In addition I compare both educational systems in terms of college attendance. I show that it will depend on the wealth level of the population. In particular I find that as the minimum level of human capital needed to attend college increases, the wealth level required to maximize college attendance under mixing increasing too.

I analyze also the intergenerational mobility that takes place under each educational system, and in particular I focus on the upward mobility. With respect to the intergenerational mobility I find that when the minimum level of human capital required to attend college is very low, then the upward mobility is higher under mixing that under tracking. The reverse occurs for high levels of human capital required to attend college. Finally I use the equality of opportunities criteria to compare the tracking and the mixing system. There it is shown that the equitable system, that is, the one that guarantees equality of opportunities will depend on the minimum level of human capital required to attend college. Under certain circumstances mixing is equitable for intermediate level of human capital needed to attend college and tracking for higher levels.

The model allows for some extensions. On the one hand it could be interesting to check the robustness of the main results of the paper to particular features of the model. For example we could introduce different measures of the so called “peer effect”, in particular some measure of the level of heterogeneity in the group. In addition is could be interesting to consider another innate ability distributions. It could be important to consider also the possibility that students are placed in tracked classes for only a subset of subjects. In addition to adding realism, incorporating this possibility will be helpful for determining the optimal design of a optimal educational system. On the other hand it could be also interesting to compare both educational systems in a dynamic set up.
References


1.a. MIXING

Poor: \( x = 1 \)

Rich: \( x > 1 \)

1.b. TRACKING

FIGURE 1.- EDUCATIONAL SYSTEMS
FIGURE 2.- HUMAN CAPITAL DISTRIBUTION
FIGURE 3.- PROPORTION OF COLLEGE STUDENTS
$p_M < p_T$

$p_M > p_T$