Investments into education - Doing as the parents did*

Georg Kirchsteiger† and Alexander Sebald ‡

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Abstract

Empirical evidence suggests that parents with higher levels of education generally attach a higher importance to the education of their children. This implies an intergenerational chain transmitting the attitude towards the formation of human capital from one generation to the next. We incorporate this intergenerational chain into an OLG-model with endogenous human capital formation. In absence of any state intervention such an economy might be characterized by multiple steady states with low or high human capital levels. There are also steady states where the population is permanently divided into different groups with differing human capital and welfare levels. Depending on the parameters of the model, a temporary or permanent public investment into human capital formation is needed to overcome steady states with low human capital and welfare levels. Furthermore, even the best steady state is suboptimal when the human capital is privately provided. This inefficiency can be removed by a permanent public subsidy for education.

Keywords: Human Capital Formation, Education Subsidy, Indirect Reciprocity.

JEL Classification: H23, H52, I2.

1 Introduction

In modern economies human capital is one of the most important determinants of economic progress and welfare. In contrast to the investment into physical capital the formation of human capital is to a large extent not financed by its owner. Rather, parents and the state cover most of the expenditures on education. The parental engagement has traditionally been explained by credit market imperfections, parental altruism (see e.g. [4] and [5])
and/or an exchange between education expenditures for the children and old-age support for the parents (see e.g. [7] and [6]).

Parental altruism is traditionally assumed to be exogenously given in economic theory, neglecting its source and evolutionary development. Among biologists and social-psychologists, on the other hand, there exists by now a large consensus that preferences, norms and cultural attitudes are endogenous with respect to our socio-economic system (see also [3], [8], [11], [16] and [19]). It is argued that two main channels exist through which preferences are transmitted across generations. Preferences are passed on genetically and/or through a process of socialization whereby e.g. children adopt parental preferences by means of imitation.

One area where the transmission of preferences through socialisation / imitation has been found particularly important is "the attitude to education". According to the socio-psychological literature parents have a pervasive influence in shaping young people's attitudes to education (see e.g. [24], [9] and [10]). More precisely, parents with higher levels of education transmit a more positive attitude towards education to their children (see e.g. [24]).

This intergenerational transmission of attitudes can be viewed as an example of indirect reciprocity, which has been found to be particularly important within family relations (see e.g. [1], [2] and [20]). In contrast to direct reciprocity (see e.g. [13], [26]), we speak of indirect reciprocity when a person does not directly reciprocate to the behavior of another person, but rather reciprocates indirectly to a third party (see e.g. [1], [22], [23] and [15]). In the context of education financing this means people do not directly reciprocate for the education they have received from their own parents, but rather repay it by financing the education of their children. Hence, the more education parents have received themselves, the more they are willing to finance the education of their children. In this way investments into human capital do not only affect the immediate recipient, i.e. the next generation, but also future generations.

The intergenerational transmission of attitudes is in line with the empirical fact that for given family income, higher educated parents tend to spend more on the education of their children than parents with lower education (see [21]). Traditionally this has been explained by the so called "home environment externality" [17], which states that not only private and public investments into education, but also innate abilities and the "family environment" determine human capital formation. This strand of literature (see e.g. [5], [14], [17] and [18]) assumes that children’s ability to acquire human capital depends on parental levels of education. Higher levels of parental education are assumed to increase the marginal product of investments into the human capital of children. Hence, the higher the level of education of the parents, the more effective investments in human capital become. If parents care about their children, this "home environment externality" can explain the effect of parents’ human capital on the education expenditures. If such a "home environment externality" exists and parents only care about the educational level of their offsprings, Eckstein and Zilcha [14] also show that private investments into human capital are suboptimal. In their analysis the source of suboptimality is twofold. First, parents do not take into account the impact of their investment into the education of their children on their children’s wages. Second, 

\footnote{A similar intergenerational attitude transmission mechanism has been analysed in the context of arts education (see [12])}
they do not take into account their impact on the relative effectiveness of their children’s investments into the education of their grandchildren.

In contrast to the ‘home environment externality’, the intergenerational transmission of attitudes implies that parents directly affect children’s preferences, rather than their production of human capital. Parents’ preference for the human capital of their children depends on their own human capital, which was financed for by the grandparents. Our paper investigates the impact of this intergenerational chain on welfare and the optimal education policy. Using an OLG model we show that multiple steady states might exist. There always exists an illiterateness steady state, which is characterized by low incomes and no investments into formal education. Depending on the parameters of the model, a temporary or permanent public funding of education could be necessary to overcome this ‘bad’ steady state and to get the economy into a ‘good’ steady state with investments into formal education and higher welfare. Depending on the initial conditions there also exist steady states where the population is permanently split into a group with large human capital endowment and high welfare and a group with low human capital and welfare level. Again a temporary or permanent subsidy is necessary to overcome such a situation. Furthermore, even the best steady state is suboptimal, since the model investigated exhibits an externality. It is shown how a permanent, tax financed subsidy on human capital acquisition can internalize this inefficiency.

The paper is organized as follows. In the next section we describe the model, followed by a characterization of the economy with private investments into human capital. In section 4 we analyze the welfare properties of this economy. Finally, we draw conclusions. All the proofs are delegated to the appendix.

2 The model

We assume a competitive economy, in which the output in period $t$, $Y_t$, does not only depend on physical capital used in $t$, $K_t$, and on labour $L_t$, but also on human capital, $H_t$. The economy is endowed with a Cobb-Douglas production technology. The normalized production function is given by

$$y_t = k_t^\alpha h_t^{1-\alpha}$$

where $\alpha \in (0, 1)$ and $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $h_t = \frac{H_t}{L_t}$. $y_t$ denotes the output per worker in period $t$, and $k_t$ and $h_t$ are respectively physical and human capital per worker in $t$.

Every worker supplies inelastically one unit of labour, and for simplicity the number of workers is constant over time, i.e. $L_t = L$ for all $t$. Markets are assumed to be perfectly competitive, so that factors earn their marginal product:

$$r_t = f'_k(k_t, h_t) = \alpha \left( \frac{h_t}{k_t} \right)^{1-\alpha}$$

$$w_t = f(k_t, h_t) - k_t f'_k(k_t, h_t) = (1 - \alpha)k_t^\alpha h_t^{1-\alpha}$$

with $r_t$ being the interest rate and $w_t$ the wage.

The capital stock depreciates fully in one period, so that the savings in period $t-1$ equal the capital stock in period $t$. 

3
Human capital is produced by formal education, i.e. by schooling. We assume however, that even without any formal education everyone acquires some minimum human capital. We normalize human capital such that the minimum human capital is one. Human capital production is given by

\[ h_{t+1} = (e_t)^\beta + 1 \]  

with \( \beta \in (0, 1) \). \( e_t \geq 0 \) denotes the private expenditures into the formal education of a child born in \( t \). Of course, the resulting human capital becomes productive in period \( t + 1 \).

At each point in time three overlapping generations are alive in the economy.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t - 1 )</td>
</tr>
<tr>
<td>(1)</td>
<td>Education</td>
</tr>
<tr>
<td>(2)</td>
<td>Work</td>
</tr>
<tr>
<td>(3)</td>
<td>Retirement</td>
</tr>
</tbody>
</table>

Take a representative individual born at the beginning of period \( t - 1 \). In this period he belongs to the youngest generation 1 which gets educated. The amount of his education is decided upon by his parent. In the next period \( t \), the individual belongs to the working (parent) generation 2. In this period he works and has one child\(^2\). He divides his income between consumption in period \( t \), savings for consumption in \( t + 1 \) and spending for the education of his child. In period \( t + 1 \), the individual belongs to the retired generation 3 and consumes his savings. At the end of this period, the individual dies.

Only the working generation has to make a decision. Individuals working in time \( t \) are assumed to maximize their utility function given by

\[ U(c_{2,t}, c_{3,t+1}, h_{t+1}) = \ln c_{2,t} + \gamma \ln c_{3,t+1} + \varphi(h_t) \ln h_{t+1}, \]  

where \( c_{2,t} \) denotes the immediate consumption of an individual working in period \( t \). \( c_{3,t+1} \) is the consumption in the next period \( t + 1 \) when the individual belongs to the retired generation 3. Since we assume full depreciation of the capital stock in one period, the savings in period \( t \) are the capital stock in period \( t + 1 \), and the old generation only consumes the interest on their savings. Therefore, \( c_{3,t+1} = k_{t+1} r_{t+1} \). \( h_{t+1} \) is the human capital of the child, which becomes effective in period \( t + 1 \). \( \gamma \) and \( \varphi \) measure the individual’s attitude towards future old-age consumption and towards the human capital of the child, respectively.

As explained in the introduction, there exists a lot of evidence that the importance parents attach to the education of their children is determined by indirect reciprocity. More precisely, the education a parent has received in his own childhood shapes his willingness to invest into the human capital of his own child. In order to capture this, we introduce an attitude function:

\[ \varphi : [1, \infty) \to \mathbb{R}_n^0, \]

\(^2\)For simplicity we assume that each adult has only one child, and each child has only one parent.
with \( \varphi(h_t) \) denoting the attitude of a parent with a human capital of \( h_t \).\(^3\) We assume that \( \varphi(h_t) \) is continuous and differentiable. If the parent has not received any formal education himself, he is not willing to finance any formal education of his child. Furthermore, his attitude towards his child’s education is positively correlated with his own human capital \( h_t \), which was financed for by his own parent. These considerations lead to

\[
\varphi(1) = 0
\]

and

\[
\varphi'(h_t) > 0.
\]

In the next section we characterize the economy with pure private investments into human capital.

### 3 Private investments into human capital

Agents working in period \( t \) have to decide how much of their wage income \( w_t \) they want to spend on instantaneous consumption and on the education of their child. Furthermore, they save in order to finance consumption when they are retired. Recall that due to full depreciation of the capital stock, \( c_{3,t+1} = k_{t+1}r_{t+1} \). Recall also that \( e_t = (h_{t+1} - 1)^\frac{1}{\beta} \).

The maximization problem of a representative agent working in \( t \) can be written as:

\[
\max_{c_{2,t},k_{t+1},h_{t+1}} U(c_{2,t}, k_{t+1}, h_{t+1}) = \ln c_{2,t} + \gamma \ln k_{t+1}r_{t+1} + \varphi(h_t) \ln h_{t+1}
\]

\[
\text{s.t. } w_t = c_{2,t} + c_{k_{t+1} + (h_{t+1} - 1)^\frac{1}{\beta}}
\]

\[
h_{t+1} \geq 1
\]

\[
c_{2,t}, k_{t+1} \geq 0
\]

Denote by \( \tilde{k}_{t+1}, \tilde{h}_{t+1} \) the utility maximizing choice of the agent working in period \( t \), when the human capital for the next generation is provided privately. The sequence of utility maximizing choices is denoted by \( \{\tilde{k}_t, \tilde{h}_t\}_{t=1}^\infty \), and \( k_1 \) and \( h_1 \) denote the initial endowments with physical and human capital. The solution is characterized by the following lemma.

**Lemma 1** If \( \tilde{k}_t > 0 \) it holds that:

i) The solution \((\tilde{k}_{t+1}, \tilde{h}_{t+1})\) fulfills the first order conditions

\[
\frac{\partial U}{\partial h_{t+1}} = \frac{\varphi(\tilde{h}_{t+1})}{\tilde{h}_{t+1}} - \frac{1}{\beta} \left( \frac{\tilde{h}_{t+1} - 1}{\tilde{w}_{t} - \tilde{k}_{t+1} - (\tilde{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right)^{\frac{1}{\beta}} = 0
\]

and

\[
\frac{\partial U}{\partial k_{t+1}} = \frac{\gamma}{k_{t+1}} - \frac{1}{\tilde{w}_{t} - \tilde{k}_{t+1} - (\tilde{h}_{t+1} - 1)^{\frac{1}{\beta}}} = 0
\]

\(^3\)Recall that even without formal education each individual is endowed with a minimum human capital normalized to 1. Hence, \( \varphi \) is defined for human capital levels not below 1.
\(\ddot{k}_{t+1} > 0.\)

\(\dddot{iii})\) If \(\tilde{h}_t = 1\), then \(\tilde{h}_{t+1} = 1.\)

\(\dddot{iv})\) If \(\tilde{h}_t > 1\), then \(\tilde{h}_{t+1} > 1.\)

Proof: see Appendix.

If \(k_1 = 0\), no production, no consumption, and no formal education is possible in any future period. Since this case is not interesting, we restrict the analysis from now on to \(k_1 > 0.\)

We show next that there exists no unlimited expansionary path.

**Proposition 2** There exists a triple \(h^m, k^m, w^m\) such that for any initial conditions \(\tilde{k}_1\) and \(\tilde{h}_1\) there exists a \(t^m\) such that:

\[
\begin{align*}
\tilde{h}_t &< h_m \text{ whenever } t > t^m \\
\tilde{k}_t &< k_m \text{ whenever } t > t^m \\
\tilde{w}_t &< w_m \text{ whenever } t > t^m
\end{align*}
\]

Proof: See Appendix

Next, we turn to the analysis of the existence and of the stability properties of steady states. We first analyze the benchmark case where the attitude towards the children’s education does not depend on parents’ education. Then we analyze the steady states for endogenous education attitudes.

### 3.1 Exogenous education attitude

As a benchmark we first analyze the situation where the attitude towards education is not determined by the attitude function \(\varphi(h_t)\), but exogenously determined at level \(\varphi > 0\). In this case, there exists a unique interior steady state with \(h^* > 1.\)

**Proposition 3** If the attitude towards education is exogenously fixed at level \(\varphi > 0\), there exists a unique steady state with formal education, i.e. with \(h^* > 1.\)

Proof: See Appendix

Simulations suggest that the steady state is globally stable. See for example Figure 1 in which we graphically report simulation results for an exogenous education attitude \(\varphi = 4\) and parameter values \(\alpha = 0.3, \beta = 0.7, \gamma = 0.5.\)

\[4\text{Further simulations with different initial conditions and different parameters were conducted showing the robustness of the results. These simulations are available from the authors upon request.}\]
Each line constitutes the optimal path of human and physical depending on the initial conditions $h_1$ and $k_1$. As one can easily see, for all initial conditions of physical and human capital the system converges towards $h^* = 1.18542$, and physical capital $k^* = 0.10281$. In other words, from any initial values of human and physical capital the system converges towards the steady state.

Repeating the same simulation exercise for different values of the attitude parameter $\varphi$ and other parameter values $\alpha$, $\beta$, $\gamma$ leads to different steady states $(h^*, k^*)$ with $h^* > 1$. In Table 1 we report the steady states $h^*$ and $k^*$ for $\alpha = 0.3$, $\beta = 0.7$, $\gamma = 0.5$ with varying levels of the exogenous education attitude parameter $\varphi$. Not surprisingly the steady state level of human capital increases in the exogenous education attitude $\varphi$.

<table>
<thead>
<tr>
<th>Exog. Attitude: $\varphi$</th>
<th>Human Capital: $h^*$</th>
<th>Physical Capital: $k^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.1</td>
<td>1.00007</td>
</tr>
<tr>
<td>2.</td>
<td>0.5</td>
<td>1.0033</td>
</tr>
<tr>
<td>3.</td>
<td>2</td>
<td>1.0707</td>
</tr>
<tr>
<td>4.</td>
<td>4</td>
<td>1.1854</td>
</tr>
</tbody>
</table>

Table 1: Steady state levels $h^*$ and $k^*$ for $\alpha=0.3$, $\beta=0.7$, $\gamma=0.5$ and varying degrees of $\varphi$.

Also for these parameter values we conducted simulations showing convergence to the steady state. In all the simulations with an exogenous attitude towards the education of the children the system converges towards the unique interior steady state. Hence simulations suggest that the steady state with agents investing into the formal education of their children is globally stable. As we will see in the next subsection, this result is in sharp contrast to the model with endogenous education attitudes.

\footnote{Plugging in the attitude parameter $\varphi = 4$, the parameter values $\alpha = 0.3$, $\beta = 0.7$, $\gamma = 0.5$, $h^*$ and $k^*$ into condition (24) in Appendix 3 confirms that $h^* = 1.18542$ and $k^* = 0.10281$ constitutes the steady state.}
3.2 Endogenous education attitude

Going back to our model with endogenous education attitude, note that $\varphi(1) = 0$. This implies that conditions (6) and (7) are always fulfilled by:

$$
\begin{align*}
  h^* &= 1 \\
  k^* &= \left( \frac{\gamma(1 - \alpha)}{1 + \gamma} \right)^{\frac{1}{1-\alpha}}
\end{align*}
$$

In this steady state, no formal education takes place, and human capital is at its lowest possible level. This steady state, which we denote as illiterateness steady state, characterizes a situation where the economy is trapped in a vicious chain in which formal education is neglected: Since parents have no formal education, they are not willing to finance the formal education of their children, and hence the children are not interested in the education of the grandchildren, and so on.

Whether this illiterateness trap poses a severe problem depends crucially on the stability properties of this steady state and on the existence of other steady states. The same holds for the question whether temporary or permanent state intervention is necessary to avoid this steady state. The stability properties of the illiterateness steady state as well as the existence and the properties of other steady states depend on the form of the attitude function, $\varphi(h_t)$.

To illustrate the different possible outcomes, we use for the rest of this section a simple attitude function, namely

$$
\varphi(h_t) = \frac{1}{\delta} (h_t - 1). \quad (8)
$$

Using this attitude function, we get the following

**Proposition 4** In addition to the illiterateness steady state, the system exhibits the following steady states:

i) If $\beta < \frac{1}{2}$, there exists exactly one interior steady state with formal education.

ii) If $\beta > \frac{1}{2}$, the following holds: Except for non-generic values of the parameters of the model, there exist either two or no interior steady states with formal education.

Proof: See Appendix

Simulations show that for $\beta < \frac{1}{2}$ the interior steady state is globally stable, and hence the illiterateness steady state is unstable. In Figure 2 we represent simulation results for $\alpha = 0.3$, $\beta = 0.4$, $\gamma = 0.5$ and $\delta = 0.04$ for varying initial conditions of human and physical capital.
Figure 2: Endogenous education attitude with $\beta < \frac{1}{2}$

Again, each line constitutes the optimal path of human and physical capital depending on the initial conditions $h_1$ and $k_1$. As in Figure 1 one can easily see that also with the endogenous formation of attitudes and $\beta < \frac{1}{2}$ the system converges globally to the interior steady state, $h^* = 1.558$ and $k^* = 0.0581$. This suggests that for $\beta < \frac{1}{2}$ the interior steady state with $h^* > 1$ is globally stable, and the illiterateness steady state is unstable. In this case a slight perturbation is enough to overcome the illiterateness trap.

For $\beta > \frac{1}{2}$ the illiterateness steady state is globally stable when no interior steady state exists. In Figure 3 we report the simulation results for $\alpha = 0.3$, $\beta = 0.7$, $\gamma = 0.5$ and $\delta = 0.1$.

Figure 3: Endogenous education attitude with $\beta > \frac{1}{2}$ and no interior steady state

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6Further simulations with different initial conditions and different parameters were conducted showing the robustness of the results. These simulations are available from the authors upon request.
With these parameters, no interior steady state exists, and the simulation suggests that the illiterateness steady state with \( h^* = 1 \) and \( k^* = 0.125057 \) is globally stable. So in this case a permanent public intervention is necessary to overcome the illiterateness trap.

If two interior steady states exist, one of them and the illiterateness steady state are locally stable. In Figure 4 we report simulation results for \( \alpha = 0.3, \beta = 0.7, \gamma = 0.5, \delta = 0.04 \) and different initial conditions.

![Figure 4: Endogenous education attitude with \( \beta > \frac{1}{2} \) and two interior steady states](image)

One can easily see that depending on the initial level of human and physical capital the system either converges towards the illiterateness steady state \( h^* = 1 \) and \( k^* = 0.1250 \) with \( w^* = 0.377171 \) or to the stable interior steady state \( h^* = 1.2675 \) and \( k^* = 0.076934 \) with \( w^* = 0.382824 \). In this case a temporary public intervention is enough to make the transition from the ‘bad’ to the ‘good’ steady state. Note that the lack of disutility of labor implies that the wage is a measure of the welfare of the agents. Hence, agents are indeed worse off in the illiterateness steady state than in the other one.

Proposition 4 refers to economies with a homogenous population - each member of the first generation is endowed with the same human and physical capital, and hence all their offsprings are. So the possible multiplicity of stable steady states refers to whole economies: Depending on the initial conditions, otherwise identical societies might end up at different steady states (and connected welfare levels). One might wonder whether our model can produce a similar result within an economy: If the initial endowment with human capital is different for otherwise identical members of the first generation, will their descendents end up at different education levels and utility levels? In order to answer this question, we investigate an economy with a heterogenous population.
3.3 Heterogenous population

In this section we consider an economy with agents that are identical but for their initial endowment of human capital. So there are two different types of agents, $U$ and $O$, with initial endowment of human capital of $h^O_t$ and $h^U_t$. Since the initial human capital endowment of the two groups differ, the human capital of their offsprings might be different, too, leading different savings and physical capital levels.

Denote by $s$ the share of the $O$ types in the population. The average per capita production function of the economy is given by

$$y_t = (sk^O_t + (1-s)(k^U_t))^\alpha (sh^O_t + (1-s)(h^U_t))^{1-\alpha}.$$ 

From this we can derive the effective wage rate per unit of human capital that agents earn:

$$\frac{\partial y_t}{\partial (sh^O_t + (1-s)h^U_t)} = (1-\alpha) \left( \frac{sk^O_t + (1-s)k^U_t}{sh^O_t + (1-s)h^U_t} \right)^\alpha$$

Factor markets are competitive and agents receive the same effective wage rate and interest rate. They differ, however, in the wage that they earn as they differ in the amount of human capital. Wages are given by

$$w^O_t = h^O_t (1-\alpha) \left( \frac{sh^O_t + (1-s)k^U_t}{sh^O_t + (1-s)h^U_t} \right)^\alpha$$

$$w^U_t = h^U_t (1-\alpha) \left( \frac{sk^O_t + (1-s)k^U_t}{sk^O_t + (1-s)h^U_t} \right)^\alpha$$

Assuming for both types of agents the attitude function (8), the first order conditions for utility maximization are derived by inserting (8) and the wage of the respective type of agent into the FOCs as stated in Lemma 1:

$$\frac{\partial U^O}{\partial h^O_{t+1}} = \frac{1}{\beta} \left( \tilde{h}^O_{t+1} - 1 \right) - \frac{1}{\beta} \left( \tilde{h}^O_{t+1} - 1 \right)^{\frac{1}{\beta}-1} = 0$$

$$\frac{\partial U^O}{\partial k^O_{t+1}} = \gamma \frac{\tilde{h}^O_{t+1} - 1}{\tilde{k}^O_{t+1} - \tilde{h}^O_{t+1} - \tilde{h}^O_{t+1} - 1} = 0$$

$$\frac{\partial U^U}{\partial h^U_{t+1}} = \frac{1}{\beta} \left( \tilde{h}^U_{t+1} - 1 \right) - \frac{1}{\beta} \left( \tilde{h}^U_{t+1} - 1 \right)^{\frac{1}{\beta}-1} = 0$$

$$\frac{\partial U^U}{\partial k^U_{t+1}} = \gamma \frac{\tilde{h}^U_{t+1} - 1}{\tilde{k}^U_{t+1} - \tilde{h}^U_{t+1} - \tilde{h}^U_{t+1} - 1} = 0$$

Whenever $\tilde{k}^O_t = \tilde{k}^U_t$ and $\tilde{h}^O_t = \tilde{h}^U_t$ it is easy that these FOCs are equivalent to the one for the homogenous population as stated in Lemma 1. Hence, any steady state of the model
with a homogenous population constitutes also a steady state of the heterogenous population model. But for an initially heterogenous population, there may exist in addition steady states where the population remains split in two groups even in the long run. Take for example the model with the following parameter values: \( s = 0.5, \alpha = 0.3, \beta = 0.7, \gamma = 0.5, \) and \( \delta = 0.04. \) With these parameters, one of the steady states is given by \( h^{O^*} = 1.35307, k^{O^*} = 0.07008, \) \( h^{U^*} = 1, k^{U^*} = 0.10747, \) leading to wages of \( w^{O^*} = 0.43624 \) and \( w^{U^*} = 0.32241. \) In this steady state the \( O \)-types and their offspring have higher human capital, higher wages, and consequently a higher utility level than the \( U \)-types.

Consider the simulations results in Table 2.

Table 2: see Appendix.

In Table 2 we report the results of 26 simulations for different initial conditions of human capital, \( h^1_O \) and \( h^1_U \) and the same parameter values: \( s = 0.5, \alpha = 0.3, \beta = 0.7, \gamma = 0.5, \) and \( \delta = 0.04. \) For each simulation we give the initial value (Initial Cond.) and the values for \( h^{O^*}, k^{O^*}, h^{U^*}, k^{U^*} \) as well as \( w^{O^*} \) and \( w^{U^*} \) that the system converges to (Sim. Results). The simulation results are sorted, first, by the initial level human capital of type-U and, second, by the absolute difference between the initial values of human capital of type-O and U. One can easily see that depending on the initial conditions the system will either converge towards an egalitarian steady state in which both types have the same human and physical capital (e.g. simulations 10, 11, 13 etc) or to an unegalitarian in which, as mentioned above, type-O converges towards \( h^{O^*} = 1.35307, k^{O^*} = 0.07008, \) and type-U converges towards \( h^{U^*} = 1, k^{U^*} = 0.10747 \) (e.g. simulations 1, 2, 3 and 4 etc). Furthermore, the lower the initial level of human capital of type-U and the higher the difference between the initial levels of human capital of type-U and O the more likely it is that differences remain even in the long run.

So depending on the initial conditions and on the parameter values of the model, it is possible that even in the long run the differences remain, irrespective of the fact that factor markets are perfectly competitive and that all agents face the same interest and wage rate. Illiterateness gets inherited from generation to generation, preventing convergence of the two population groups. Comparing the steady state wage of type-O, \( w^{O^*} = 0.43624, \) in the heterogenous case and the steady state wage, \( w^* = 0.382824, \) in the homogenous case one can see that \( w^{O^*} > w^*. \) The reason for this is twofold. First, the average level of human capital in the homogenous situation is higher implying a lower wage per efficiency unit. Secondly, type-O agents have a higher level of human capital in the heterogenous steady state compared to the homogenous situation leading to an additional effect on the wage, \( w^{O^*}. \) Consequently, \( O \)-types are better off in the heterogeneous steady state than in the homogeneous. This suggests that people with higher human capital might resist a special subsidy to overcome the illiterateness trap of the underdogs.

4 The optimal education subsidy

In this section we analyze the efficiency properties of all steady states of the model, and the possibilities to overcome inefficiencies. For tractability reasons, we restrict attention to the homogenous population case. We compare the private investments into human and physical capital with the investments a social planner would make if endowed with the same initial
capital levels. It turns out that this analysis can be carried out for a general attitude function with the properties as specified in section 2.

The social planner chooses the investment in human and physical capital such that he maximizes the weighted sum of utilities of all generations, subject to the resource constraint of the economy.

\[
\max_{c_{2,t}, c_{3,t+1}, h_{t+1}} W = \sum_{t=1}^{\infty} \omega_t \left[ \ln c_{2,t} + \gamma \ln c_{3,t+1} + \varphi(h_t) \ln h_{t+1} \right]
\]

s.t. \( k_{t+1}^{\alpha} h_t^{1-\alpha} = k_{t+1} + (h_{t+1} - 1)^{\frac{1}{\beta}} + c_{2,t} + c_{3,t} \) and

\[
\begin{align*}
& h_t \geq 1, \\
& k_t \geq 0
\end{align*}
\]

with \( \omega_t \) being larger than zero for all \( t \). Denote by \( \hat{k}_{t+1} \) and \( \hat{h}_{t+1} \) the optimal choice of the social planner. The sequence of optimal choices is denoted by \( \{\hat{k}_t, \hat{h}_t\}_{t=2}^{\infty} \), and \( k_1 > 0 \) and \( h_1 \geq 1 \) denote the initial endowments with physical and human capital.

Defining

\[
\xi_t = \frac{(1 - \alpha) \hat{k}_{t+1}^{\alpha} \hat{h}_{t+1}^{1-\alpha}}{\hat{k}_{t+1}^{\alpha} \hat{h}_{t+1}^{1-\alpha} - \hat{k}_{t+2} - (\hat{h}_{t+2} - 1)^{\frac{1}{\beta}} - \hat{k}_{t+1} \hat{r}_{t+1}}
\]

the socially optimal solution is characterized by the following lemma:

**Lemma 5** If \( \hat{k}_t > 0 \) it holds that:

i) The solution of the social planners problem fulfills the first order conditions

\[
\begin{align*}
\frac{\partial W}{\partial h_{t+1}} &= \omega_t \left( \frac{\varphi(\hat{h}_t)}{\hat{h}_{t+1}} - \frac{\frac{1}{\beta} (\hat{h}_{t+1} - 1)^{\frac{1}{\beta} - 1}}{\hat{k}_{t+1}^{\alpha} \hat{h}_{t+1}^{1-\alpha} - \hat{k}_{t+2} - (\hat{h}_{t+2} - 1)^{\frac{1}{\beta}} - \hat{k}_{t+1} \hat{r}_{t+1}} \right) \\
&+ \omega_{t+1} \left( \xi_t + \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \right) \\
&= 0 \\
\end{align*}
\]

and

\[
\begin{align*}
\frac{\partial W}{\partial k_{t+1}} &= \omega_t \left( \frac{\gamma}{\hat{k}_{t+1}^{\alpha} \hat{h}_{t+1}^{1-\alpha} - \hat{k}_{t+2} - (\hat{h}_{t+2} - 1)^{\frac{1}{\beta}} - \hat{k}_{t+1} \hat{r}_{t+1}} \right) = 0 \\
\end{align*}
\]

ii) \( \hat{k}_{t+1} > 0, \hat{h}_{t+1} > 1, \) and \( \hat{c}_{2,t} > 0 \).

Proof: see Appendix.

Comparing Lemma 1 with Lemma 5, one realizes that condition (7) of the private solution coincides with condition (10) of the optimal solution, but condition (9) differs from (6) by the
implies a second type of externality, which leads to the emergence of \( \phi \) of their children, but only about their human capital. This leads to an externality captured by the variable \( \xi \). Even if the attitude towards children’s education were independent of the own education, an externality would be present. The endogenous attitude towards education is not optimal. This result is not surprising, since parents do not care about the welfare of their children, but only about their human capital. This leads to an externality captured by the variable \( \xi \). Even if the attitude towards children’s education were independent of the own education, an externality would be present. The endogenous attitude towards education implies a second type of externality, which leads to the emergence of \( \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \) in (9). Both of these externalities are neglected when human capital is privately provided, leading to an underprovision of human capital. Recall that this inefficiency occurs even in the better, interior steady states.

Can this inefficiency be overcome by public expenditures on human capital formation? Think of a situation where the public finances schools and universities. Even if schools and universities are fully financed by the state, parents still have to take care of the children’s costs of living, the costs of supplementary education, the costs of teaching material and other things indirectly connected to the human capital formation of children. Hence, parts of the education expenditures are always paid by parents. Furthermore, in such a system of mixed financing a better education of the children requires higher expenditures of parents as well as of the state. Finally, the education level of the children is largely influenced by the parent’s willingness to cover the children’s costs of living, even in a system where the state finances schools and universities. To model such a situation where human capital formation is partly privately, partly publicly financed, assume that in each period \( t \) private education expenditures are subsidized by the state at a rate \( s_t \). To finance this subsidy, wage income is taxed at a rate \( \tau_t \). The balanced budget condition for the state for period \( t \) is given by:

\[
s_t (h_{t+1} - 1)^{\frac{1}{\gamma}} = \tau_t w_t. \tag{11}
\]

We assume that an individual agent takes the tax rate and the subsidy scheme as given when he maximizes his utility. This implies that he does not take into account the balanced budget condition of the state. This assumption seems plausible for a large economy with many agents. With this simplification, the decision problem of a representative agent working in period \( t \) can be written as:

\[
\max_{c_{2,t}, c_{3,t+1}, h_{t+1}} U(c_{2,t}, c_{3,t+1}, h_{t+1}) = \ln c_{2,t} + \gamma \ln c_{3,t+1} + \varphi(h_t) \ln h_{t+1}
\]

s.t. \((1 - \tau_t)w_t = c_{2,t} + \frac{c_{3,t+1}}{r_{t+1}} + (1 - s_t) (h_{t+1} - 1)^{\frac{1}{\gamma}}\) and

\[
h_{t+1} \geq 1, \\
c_{2,t} \geq 0, \\
c_{3,t+1} \geq 0.
\]

Denote by \( \bar{k}_{t+1} \) and \( \bar{h}_{t+1} \) the utility maximizing choice of the agent working in period \( t \), when the human capital formation is subsidized. The sequence of utility maximizing choices is denoted by \( \{\bar{k}_t, \bar{h}_t\}_{t=2}^\infty \), and \( k_1 > 0 \) and \( h_1 \geq 1 \) denote the initial endowments of the economy.
with physical and human capital. Using the budget constraint to insert for $c_{2,t}$ the first order
conditions are:

$$
\frac{\partial U}{\partial h_{t+1}} = \frac{\varphi(h_{t+1})}{h_{t+1}} - \frac{(1 - s_t)\frac{1}{\beta} (h_{t+1} - 1)^{\frac{1}{\beta} - 1} - 1}{(1 - \tau_t)\bar{w}_t - \bar{k}_{t+1} - (1 - s_t) (h_{t+1} - 1)^{\frac{1}{\beta}}} = 0 \quad (12)
$$

$$
\frac{\partial U}{\partial k_{t+1}} = \frac{\gamma}{k_{t+1}} - \frac{1}{(1 - \tau_t)\bar{w}_t - \bar{k}_{t+1} - (1 - s_t) (h_{t+1} - 1)^{\frac{1}{\beta}}} = 0. \quad (13)
$$

Applying the same reasoning as in the proof of lemma 1 it is easy to see that the first
order conditions characterize the solution.

Is it possible to find a sequence of subsidy schemes $\{s_t, \tau_t\}_{t=1}^{\infty}$ such that the sequence of
socially optimal choices is induced? For given initial endowment with physical and human
capital such a sequence would have to induce a sequence of individual choices $\{k_{t+1}, h_{t+1}\}_{t=1}^{\infty}$
such hat $\bar{k}_{t+1} = k_{t+1}$ and $\bar{h}_{t+1} = h_{t+1}$ for all periods. Furthermore, the sequence of schemes
would have to respect the balanced budget condition (11) in all periods.

The following proposition shows that there exists indeed a sequence of subsidy schemes
that induces an optimal outcome.

**Proposition 6** For $\hat{k}_t > 0$ it holds that:

i) The sequence of subsidy schemes $\{s_t, \tau_t\}_{t=1}^{\infty}$ defined by

$$
\begin{align*}
\text{ } & s_t = \frac{\beta}{(\gamma + 1)} \left( \frac{\bar{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}}{\hat{h}_{t+1} - 1} \right)^{\frac{1}{\beta} - 1} \omega_{t+1} \left( \xi_t + \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \right) \\
\text{ } & \tau_t = s_t \frac{(\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}}{\bar{w}_t} 
\end{align*}
$$

induces a sequence of choices $\{\bar{k}_{t+1}, \bar{h}_{t+1}\}_{t=1}^{\infty}$ such that $\bar{k}_{t+1} = k_{t+1}$ and $\bar{h}_{t+1} = h_{t+1}$ in all $t$.

ii) $\{s_t, \tau_t\}_{t=1}^{\infty}$ respects the balanced budget condition in all periods.

iii) For all $t$, $0 < s_t < 1$.

Proof: see Appendix

The above proposition shows that an appropriate subsidy scheme can ensure efficiency.
The optimal subsidy rate is always strictly larger than zero, so a permanent subsidy is
necessary to achieve efficiency. The optimal rate in period $t$, however, depends on the
optimal values of human and physical capital in periods $t$, $t+1$, and $t+2$. Since nothing
guarantees that these optimal human and physical capital values are constant over time, $s_t$
might vary over time accordingly.
5 Conclusions

We have shown that the private allocation of resources leads to inefficient human capital formation. If parent’s attitude towards education of the children depends on their own education, the economy might get trapped in an illiterateness steady state where a low education level of the parents leads to negligence of the children’s education, reproducing the low education level in the next generation. To overcome such a steady state, temporary or permanent state intervention is necessary, depending on the stability properties of the illiterateness steady state. Because of the intergenerational transmission of education attitudes the population of an economy might also be split in the long run into different education groups, even if the agents are identical in all respects but for their initial endowment with human capital.

When the economy is not trapped in such an illiterateness steady state, the purely private financing of the education system also leads to inefficiencies. These inefficiencies can be overcome by a permanent public support for the education of children. This conclusion requires some qualifications. First, a similar result would occur if the parents’ attitude toward the education of their children were independent of their own education. Second, if the economy is not in a steady state, the efficient tax and subsidy rates might change from period to period. For political reasons as well as for lack of information, it may be difficult to make these necessary adjustments. Third, the optimal subsidy rate depends on the weight the social planner puts on the different generations. Hence, there is room for intergenerational conflicts. Finally, our model is based on the assumption that labor supply is fixed. Hence, the taxation of wage income does not create any excess burden on the labor market. If labor supply is elastic and if a non-distortive tax is not available, a trade-off exists between the inefficiency created by the tax system and the inefficiency due to the externalities in the human capital formation.

Notwithstanding these qualifications, it can be concluded that the broadening of the model of human behavior to allow for more complex intergenerational relations leads to inefficiencies that have been neglected so far. The analysis thus gives further support for government intervention to support an optimal investment into the education of our children in order to achieve a maximum amount of welfare.

References


6 Appendix

6.1 Proof of Lemma 1

Note first that $\tilde{h}_{t+1}$ and $\tilde{k}_{t+1}$ have to be finite for all finite values of $(\tilde{k}_t, \tilde{h}_t)$. The utility function is strictly quasiconcave, implying a unique solution, which might be either interior (in which case the first order conditions hold) or at the lower bounds. By (3) $\bar{w}_t > 0$ whenever $\tilde{k}_t > 0$. Furthermore, $\bar{w}_t - \tilde{k}_{t+1} = \tilde{c}_{2,t} + \left( \tilde{h}_{t+1} - 1 \right)^{\frac{1}{\beta}} \geq \tilde{c}_{2,t} > 0$ due to the INADA condition of the utility function with respect to the consumption levels. This implies that $\frac{\partial U}{\partial \tilde{k}_{t+1}} = \infty$ at $k_{t+1} = 0$. This requires that the condition (7) as well as ii) must hold.

As for the solution for the human capital, note first that for $\tilde{h}_t = 1$, $\frac{\partial U}{\partial \tilde{h}_{t+1}} = 0$ at $h_{t+1} = 1$. This gives iii) and that condition (6) holds in this case.

If $\tilde{h}_t > 1$, $\frac{\partial U}{\partial \tilde{h}_{t+1}} = \infty$ at $h_{t+1} = 1$, implying $\tilde{h}_{t+1} > 1$. This gives iv) and that condition (6) holds also for $\tilde{h}_t > 1$, which completes the proof. ■

6.2 Proof of Proposition 2

We first introduce the following dynamic system, denoted as upper bound economy and by superscript $b$, which will be useful for the proof:

$$ h_{t+1}^b = (1 - \alpha)^3(k_t^b)^{\alpha^3}(h_t^b)^{(1-\alpha)^3} + 1 \quad (16) $$

$$ k_{t+1}^b = (1 - \alpha)(k_t^b)^{\alpha}(h_t^b)^{(1-\alpha)} \quad (17) $$
The proof now proceeds in three steps. In the first step, we will show that for the same initial conditions for human and physical capital the path of the upper bound economy provides an upper bound for the path of the economy we analyze. In the second step, we will show that the upper bound economy exhibits a globally stable steady state, to which the system converges from any initial conditions. In the third step we will use this steady state to finalize the proof.

Step 1: If the indirect reciprocity economy and the upper bound economy start at the same initial conditions \(k_1^b = \tilde{k}_1 > 0\) and \(h_1^b = \tilde{h}_1\), it holds that:

\[
\begin{align*}
    k_t^b &\geq \tilde{k}_t \text{ for all } t > 1 \\
    h_t^b &\geq \tilde{h}_t \text{ for all } t > 1.
\end{align*}
\]

The proof is made by induction. For the same initial conditions \(k_1^b = \tilde{k}_1\) and \(h_1^b = \tilde{h}_1\), the definition of the upper bound economy, (3), and (4) give

\[
k_2^b = (1 - \alpha)(k_1^b)^\alpha (h_1^b)^{(1 - \alpha)} = (1 - \alpha)(\tilde{k}_1)^\alpha (\tilde{h}_1)^{(1 - \alpha)} = \tilde{w}_1 \geq \tilde{k}_2
\]

and

\[
h_2^b = (1 - \alpha)^\beta (k_1^b)^{\alpha \beta} (h_1^b)^{(1 - \alpha) \beta} + 1 = (\tilde{w}_1)^\beta + 1 \geq (\tilde{e}_1)^\beta + 1 = \tilde{h}_2.
\]

So \(k_2^b \geq \tilde{k}_2\) and \(h_2^b \geq \tilde{h}_2\). It is obvious that \(k_{t+1}^b\) and \(k_{t+1}^b\) are monotonically increasing in \(h_t^b\) and \(k_t^b\). This implies that \(k_{t+1}^b \geq \tilde{k}_{t+1}\) and \(h_{t+1}^b \geq \tilde{h}_{t+1}\) whenever \(k_t^b \geq \tilde{k}_t\) and \(h_t^b \geq \tilde{h}_t\), which completes the proof of Step 1.

Step 2: For any initial condition \(k_1^b > 0\) the upper bound economy converges to a unique stable state \(k^{bs}, h^{bs}\) with \(k^{bs} > 0\) and \(h^{bs} > 1\).

To show this, note first that \(k_1^b > 0\) implies that \(k_t^b > 0\) and \(h_t^b > 1\) for all \(t > 1\). From the definition of the upper bound economy we get

\[
h_{t+1}^b = (k_{t+1}^b)^\beta + 1,
\]

implying that

\[
h_t^b = (k_t^b)^\beta + 1.
\]

Hence, equation of motion of the upper bound economy is characterized by:

\[
k_{t+1}^b = (1 - \alpha) (k_t^b)^\alpha \left((k_t^b)^\beta + 1\right)^{(1 - \alpha)}.
\]

Differentiating we get

\[
\frac{\partial k_{t+1}^b}{\partial k_t^b} = \left[(1 - \alpha) (k_t^b)^{(\alpha - 1)} \left((k_t^b)^\beta + 1\right)^{(-\alpha)}\right] \left[\left((1 - \beta) + \beta\right) (k_t^b)^\beta + 1\right] > 0
\]

\[\text{Recall that we restrict our analysis to the nontrivial case of } \tilde{k}_1 > 0, \text{ which of course implies that } k_1^b > 0.\]
and
\[
\frac{\partial^2 k_{t+1}^{b}}{\partial k_t^{b} \partial k_t^{b}} = \left[ - (1 - \alpha)^2 (k_t^{b})^{\alpha - 2} \left( (k_t^{b})^{\beta} + 1 \right)^{(-1-\alpha)} \right] \\
\left[ \alpha (1 - \beta) + \beta (1 - \beta) (k_t^{b})^{2\beta} + (1 - \beta) (2\alpha + \beta) (k_t^{b})^{\beta} + \alpha \right] < 0
\]
since
\[
- (1 - \alpha)^2 (k_t^{b})^{\alpha - 2} \left( (k_t^{b})^{\beta} + 1 \right)^{(-1-\alpha)} < 0 \\
\alpha (1 - \beta) + \beta (1 - \beta) (k_t^{b})^{2\beta} > 0 \\
(1 - \beta) (2\alpha + \beta) (k_t^{b})^{\beta} > 0 \\
\alpha > 0.
\]

Hence, the equation of motion (18) is strictly monotone and concave in $k_t$. This and the fact that the system has a steady state at $k_t = 0$ implies that there is at most one other steady state with $k_t > 0$.

To investigate the possibility of steady states with $k^{bs} > 0$, we set $k_t^{b} = k_{t+1}^{b} = k^{bs}$ in (18) and get:
\[
k^{bs} = (1 - \alpha) (k^{bs})^{\alpha} [(k^{bs})^{\beta} + 1]^{(1-\alpha)},
\]

implying:
\[
(k^{bs})^{(\frac{1}{\alpha - 1})} = (1 - \alpha) (k^{bs})^{\alpha} [(k^{bs})^{\beta} + 1].
\]

Dividing by $(k^{bs})^{(\frac{1}{\alpha - 1})}$ leads to
\[
1 = (1 - \alpha) (k^{bs})^{(-1)} [(k^{bs})^{\beta - 1} + 1] \\
1 = (1 - \alpha) [(k^{bs})^{(\beta - 1)} + (k^{bs})^{(-1)}].
\]

The right hand side of equation (19) is continuous and strictly monotonically decreasing in $k^{bs}$. Furthermore,
\[
\lim_{k^{bs} \to 0} (1 - \alpha) [(k^{bs})^{(\beta - 1)} + (k^{bs})^{(-1)}] = \infty \\
\lim_{k^{bs} \to \infty} (1 - \alpha) [(k^{bs})^{(\beta - 1)} + (k^{bs})^{(-1)}] = 0.
\]

Hence, there exists a unique $k^{bs} > 0$ fulfilling (19) characterizing the second steady state of the upper bound economy. The steady state value of $h$ is given by
\[
h^{bs} = (k^{bs})^{\beta} + 1 > 1
\]

Recall that the equation of motion (19) is strictly monotone and concave. Hence, $k_{t+1}^{b} > k_t^{b}$ whenever $k_t^{b} < k^{bs}$ and $k_{t+1}^{b} < k_t^{b}$ whenever $k_t^{b} > k^{bs}$. Therefore, the upper bound economy converges to the steady state $k^{bs}, h^{bs}$ whenever $k_t^{b} > 0$.

Step 3: For any initial conditions $\tilde{k}_1 \geq 0$ and $\tilde{h}_1 \geq 0$ there exists a $t^m$ such that:
\[
\tilde{h}_t < h^{bs} + 1 \text{ whenever } t > t^m \\
\tilde{k}_t < k^{bs} + 1 \text{ whenever } t > t^m \\
\tilde{w}_t < (1 - \alpha)(k^{bs} + 1)^{\alpha}(h^{bs} + 1)^{1-\alpha} \text{ whenever } t > t^m
\]

Step 3) follows immediately from Step 1), Step 2), and the wage equation 3).
6.3 Proof of Proposition 3

Education attitude is exogenously given at $\varphi$. Taking this into account and inserting 3 and 2 into 6 and 7 gives

\[
(1 - \alpha) \tilde{k}_t^{\alpha} \tilde{h}_t^{1-\alpha} = \left( \left( \frac{1 + \gamma}{\beta \varphi} \right) \frac{\tilde{h}_{t+1}}{\tilde{h}_{t+1} - 1} + 1 \right) \left( \tilde{h}_{t+1} - 1 \right)^{\frac{1}{\beta}} \tag{20}
\]

\[
(1 - \alpha) \tilde{k}_t^{\alpha} \tilde{h}_t^{1-\alpha} = \left( \frac{1 + \gamma}{\gamma} + \frac{\beta \varphi}{\gamma} \frac{\tilde{h}_{t+1} - 1}{\tilde{h}_{t+1}} \right) \tilde{k}_{t+1}. \tag{21}
\]

Substituting 21 into 20 for $k_t$ and rearranging terms gives

\[
(1 - \alpha) \left( \frac{\gamma}{\beta \varphi} \right)^\alpha = \frac{1}{\tilde{h}_t \left( \tilde{h}_t - 1 \right)^{(\frac{1}{\beta} - 1)\alpha}} \left( \frac{1 + \gamma}{\beta \varphi} \right) \tilde{h}_{t+1} \left( \tilde{h}_{t+1} - 1 \right)^{\frac{1}{\beta} - 1} + \frac{1}{\tilde{h}_t \left( \tilde{h}_t - 1 \right)^{(\frac{1}{\beta} - 1)\alpha}} \left( \tilde{h}_{t+1} - 1 \right)^{\frac{1}{\beta}}. \tag{22}
\]

Since we investigate the steady state, we ignore the indices. Rearranging terms one gets

\[
(1 - \alpha) \left( \frac{\gamma}{\beta \varphi} \right)^\alpha = \left( \frac{1 + \gamma}{\beta \varphi} \right) (h^* - 1)^{(\frac{1}{\beta} - 1)(1 - \alpha)} + \frac{1}{h^*} (h^* - 1)^{\frac{1}{\beta} - (\frac{1}{\beta} - 1)\alpha}. \tag{24}
\]

Obviously, the left hand side of 24 is positive. For the right hand side, notice that $\frac{1}{\beta} - (\frac{1}{\beta} - 1)\alpha > 0$ and $(\frac{1}{\beta} - 1)(1 - \alpha) > 0$. This implies that the right hand side is strictly increasing in $h^*$, that it is zero for $h^* = 1$, and that it goes to infinity for $h^*$ going to infinity. Hence there exists exactly one value $h^*$ that fulfills 24, and this value is strictly larger than 1. \textbf{■}

6.4 Proof of Proposition 4

Inserting the attitude function, 3 and 2 into 6 and 7 gives

\[
(1 - \alpha) \tilde{k}_t^{\alpha} \tilde{h}_t^{1-\alpha} = \left( \left( \frac{1 + \gamma}{\beta \varphi} \right) \frac{\tilde{h}_{t+1}}{\tilde{h}_{t+1} - 1} + 1 \right) \left( \tilde{h}_{t+1} - 1 \right)^{\frac{1}{\beta}} \tag{25}
\]

\[
(1 - \alpha) \tilde{k}_t^{\alpha} \tilde{h}_t^{1-\alpha} = \left( \frac{1 + \gamma}{\gamma} + \frac{\beta \varphi}{\gamma} \frac{\tilde{h}_{t+1} - 1}{\tilde{h}_{t+1}} \right) \tilde{k}_{t+1}. \tag{26}
\]

Substituting 25 into 26 for $k_t$, ignoring the indices, and rearranging terms leads to the following condition for an interior steady state:
\[(1 - \alpha) \left(\frac{\delta \gamma}{\beta}\right)^\alpha = \left(\frac{\delta (1 + \gamma)}{\beta}\right) (h^* - 1)^{(\frac{1}{\beta} - 2)(1-\alpha)} + \frac{1}{h^*} (h^* - 1)^{\frac{1}{\beta} - (\frac{1}{\beta} - 2)\alpha}. \quad (27)\]

Define
\[
\text{lhs} : = (1 - \alpha) \left(\frac{\delta \gamma}{\beta}\right)^\alpha \\
rhs(h^*) : = \left(\frac{\delta (1 + \gamma)}{\beta}\right) (h^* - 1)^{(\frac{1}{\beta} - 2)(1-\alpha)} + \frac{1}{h^*} (h^* - 1)^{\frac{1}{\beta} - (\frac{1}{\beta} - 2)\alpha} \\
a(h^*) : = \left(\frac{\delta (1 + \gamma)}{\beta}\right) (h^* - 1)^{(\frac{1}{\beta} - 2)(1-\alpha)} \\
b(h^*) : = \frac{1}{h^*} (h^* - 1)^{\frac{1}{\beta} - (\frac{1}{\beta} - 2)\alpha}.
\]

Proof of i) \(\text{lhs}\) strictly positive. If \(\beta < \frac{1}{2}, \left(\frac{1}{\beta} - 2\right)(1-\alpha) < 0\) and \(\frac{1}{\beta} - \left(\frac{1}{\beta} - 2\right) \alpha > 2\). This implies that \(\frac{\partial a}{\partial h^*} > 0\) and \(\frac{\partial b}{\partial h^*} > 0\). \(\text{rhs}\) is strictly increasing in \(h^*\). Furthermore, \(\text{rhs}(h^* = 1) = 0\), and \(\lim_{h^* \to \infty} a(h^*) = \infty\). Therefore there exists exactly one \(h^* > 1\) such that \(\text{lhs} = \text{rhs}(h^*)\).

Proof of ii) Again, \(\text{lhs}\) strictly positive. If \(\frac{1}{2} < \beta < 1, \left(\frac{1}{\beta} - 2\right)(1-\alpha) < 0\) and \(1 < \frac{1}{\beta} - \left(\frac{1}{\beta} - 2\right) \alpha\). This implies that \(\lim_{h^* \to 1} a(h^*) = \infty, \lim_{h^* \to \infty} a(h^*) = 0, \lim_{h^* \to \infty} b(h^*) = \infty\) and \(b(h^* = 1) = 0\). This gives \(\lim_{h^* \to 1} \text{rhs}(h^*) = \lim_{h^* \to \infty} \text{rhs}(h^*) = \infty\) and finite values of \(\text{rhs}\) for all other values of \(h^*\).

Next we show that \(\text{rhs}(h^*)\) has a unique local extremum in the interior. Because of \(\lim_{h^* \to 1} \text{rhs}(h^*) = \lim_{h^* \to \infty} \text{rhs}(h^*) = \infty\), a unique interior local extremum must be a unique local minimum of \(\text{rhs}(h^*)\). Uniqueness of the local minimum implies that \(\frac{\partial \text{rhs}(h^*)}{\partial h^*} < 0\) for all values of \(h^*\) below this minimum and \(\frac{\partial \text{rhs}(h^*)}{\partial h^*} > 0\) for all values of \(h^*\) above this minimum. In the interior, any local extremum is characterized by the condition
\[
\frac{\partial \text{rhs}(h^*)}{\partial h^*} = 0
\]
leading to
\[
\frac{(h^* - 1)}{h^*} - \left(\frac{1}{\beta} - 2\right)(1-\alpha) \left(\frac{\delta (1 + \gamma)}{\beta}\right) \frac{h^*}{(h^* - 1)^2} = 0
\]
\[
\Rightarrow \quad \frac{1}{\beta} - \left(\frac{1}{\beta} - 2\right) \alpha.
\]

For the left hand side of equation 28, we have
\[
\lim_{h^* \to 1} \left(\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^* - 1)^2}\right) = \infty,
\]
with \(z = -\left(\frac{1}{\beta} - 2\right)(1-\alpha) \left(\frac{\delta (1+\gamma)}{\beta}\right) > 0\), and
\[
\lim_{h^* \to \infty} \left(\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^* - 1)^2}\right) = 1.
\]
This implies that there is at least one local extremum in the interior. To check uniqueness, we will show \( \frac{(h^*-1)}{h^*} + z \frac{h^*}{(h^*-1)^2} \) is strictly decreasing in in \( h^* \) for the relevant values of \( h^* \). The first derivative of the left hand side is given by

\[
\frac{\partial}{\partial h^*} \left( \frac{(h^*-1)}{h^*} + z \frac{h^*}{(h^*-1)^2} \right) = -zh^2 (h^* + 1) + (h^* - 1)^3 \over h^2 (h^* - 1)^3
\]

To see that this derivative is strictly negative for the relevant values of \( h^* \), note first that \( \frac{1}{\beta} - \left( \frac{1}{\beta} - 2 \right) \alpha > 1 \). All solutions to equation 28 must satisfy the condition

\[
\frac{(h^* - 1)}{h^*} + z \frac{h^*}{(h^*-1)^2} > 1
\]

which is equivalent to

\[
zh^2 > h^* (h^* - 1)^2 - (h^* - 1)^3.
\]

Inserting into 29 implies that

\[
\frac{\partial}{\partial h^*} \left( \frac{(h^*-1)}{h^*} + z \frac{h^*}{(h^*-1)^2} \right) < \frac{-z(h^* (h^* - 1)^2 - (h^* - 1)^3)(h^* + 1) + (h^* - 1)^3}{h^2 (h^* - 1)^3} = \frac{-2}{h^2 (h^* - 1)} < 0
\]

So the left hand side of 28 is strictly decreasing in the relevant area, and hence equation 28 has a unique solution. This implies that \( rhs(h^*) \) has a unique local minimum in the interior whenever \( \beta > \frac{1}{2} \), and that \( \frac{\partial rhs(h^*)}{\partial h^*} < 0 \) for all values of \( h^* \) below this minimum and \( \frac{\partial rhs(h^*)}{\partial h^*} > 0 \) for all values of \( h^* \) above this minimum. Furthermore, recall that \( \lim_{h^* \to -1} rhs(h^*) = \lim_{h^* \to -\infty} rhs(h^*) = \infty \). So for generic parameter values there are two possibilities: Either there exist two different \( h^* \) such that \( lhs = rhs(h^*) \). In this case there are two interior steady states. On the other hand, it is possible that \( lhs < rhs(h^*) \) for all \( h^* > 1 \) which implies that there is no interior steady state.

By example we show that both possibilities are indeed feasible. Take first the case \( \alpha = 0.3, \beta = 0.7, \gamma = 0.5, \delta = 0.01 \) and then the case. \( \alpha = 0.3, \beta = 0.7, \gamma = 0.5, \delta = 0.04 \). In the first case condition 27 can be written as:

\[
0 = (1 - 0.3) \left( \frac{(0.1)(0.5)}{0.7} \right)^{0.3} - \left( \frac{(0.1)(1+0.5)}{0.7} \right)(h^* - 1)^{\frac{1}{\pi^* - 2}(1-0.3)} - \frac{1}{h^*}(h^* - 1)^{\frac{1}{\pi^* - 2}} = F(h^*).
\]

and in the second case it can be written as:

\[
0 = (1 - 0.3) \left( \frac{(0.04)(0.5)}{0.7} \right)^{0.3} - \left( \frac{(0.04)(1+0.5)}{0.7} \right)(h^* - 1)^{\frac{1}{\pi^* - 2}(1-0.3)} - \frac{1}{h^*}(h^* - 1)^{\frac{1}{\pi^* - 2}} = G(h^*).
\]
When trying to solve $F(h^*) = 0$ for $h^*$ one finds no solution, whereas solving $G(h^*) = 0$ gives exactly two solutions: $h^*_1 = 1.09496$ and $h^*_2 = 1.2675$.
These results are illustrated by Figure 5.

![Figure 5](image)

**Figure 5:** Endogenous education attitude with $\beta = 0.7$, $\delta = 0.1$ and $\delta = 0.04$

As one can easily see in the first case there is no $h^*$ such that $F(h^*) = 0$, whereas in the second case there are two $h^*$ such that $G(h^*) = 0$: $h^*_1 = 1.09496$ and $h^*_2 = 1.2675$.

### 6.5 Proof of Lemma 5

Note first that $\hat{h}_{t+1}$ and $\hat{k}_{t+1}$ have to be finite for all finite values of $(\hat{k}_t, \hat{h}_t)$. The solution might be either interior (in which case the first order conditions hold) or at the lower bounds. Since $\hat{k}_t > 0$, production takes place in period $t$. Combining this fact with the INADA condition of the individual utility functions implies that $\hat{k}_t^{\alpha} \hat{h}_t^{1-\alpha} - \hat{k}_{t+1} - \left(\hat{h}_{t+1} - 1\right)^{\frac{1}{\beta}} - \hat{k}_t \hat{r}_t = c_{2,t} > 0$.
Therefore, $\frac{\partial W}{\partial k_{t+1}} = \infty$ at $k_{t+1} = 0$. Hence, $\hat{k}_{t+1} > 0$, and condition (10) holds.

As for the solution for the human capital, note again that since $\hat{k}_t > 0$, production takes place in period $t$. Again, combining this fact with the Inada condition of the individual utility functions implies that $\hat{k}_t^{\alpha} \hat{h}_{t+1}^{1-\alpha} - \hat{k}_{t+2} - \left(\hat{h}_{t+2} - 1\right)^{\frac{1}{\beta}} - \hat{k}_{t+1} \hat{r}_{t+1} = c_{2,t+1} > 0$. Hence $\frac{\partial W}{\partial h_{t+1}} = \infty$ at $h_{t+1} = 1$, implying $\hat{h}_{t+1} > 1$. This gives condition (9), which completes the proof. ■
6.6 Proof of Proposition 6

i) By combining (12), (13) and the balanced budget condition of the government (11) one gets:

\[
\frac{\varphi(\hat{h}_t)}{\hat{h}_{t+1}} - (1 - s_t) \frac{(\gamma + 1)}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) = 0. \tag{30}
\]

Furthermore combining (9) and (10) gives:

\[
\frac{\varphi(\hat{h}_t)}{\hat{h}_{t+1}} - \frac{(\gamma + 1)}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) + \frac{\omega_{t+1}}{\omega_t} \left( \xi_t + \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \right) = 0. \tag{31}
\]

In order to establish the optimal subsidy rate which ensures that \( \hat{h}_{t+1} = \hat{h}_{t+1} \) we set (30) equal to (31), furthermore set \( \hat{h}_{t+1} = \hat{h}_{t+1} \) and solve for \( s_t \).

\[
\frac{\varphi(\hat{h}_t)}{\hat{h}_{t+1}} - \frac{(\gamma + 1)}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) + \frac{s_t}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) = \frac{\varphi(\hat{h}_t)}{\hat{h}_{t+1}} - \frac{(\gamma + 1)}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) + \frac{\omega_{t+1}}{\omega_t} \left( \xi_t + \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \right).
\]

This can be written as:

\[
\frac{\varphi(\hat{h}_t)}{\hat{h}_{t+1}} - \frac{(\gamma + 1)}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) + \frac{s_t}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) = \frac{\varphi(\hat{h}_t)}{\hat{h}_{t+1}} - \frac{(\gamma + 1)}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) + \frac{\omega_{t+1}}{\omega_t} \left( \xi_t + \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \right).
\]

From which it follows:

\[
s_t \frac{(\gamma + 1)}{\beta} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) = \frac{\omega_{t+1}}{\omega_t} \left( \xi_t + \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \right).
\]

Solving for \( s_t \) gives:

\[
s_t = \frac{\beta}{(\gamma + 1)} \left( \frac{\hat{h}_{t+1} - 1}{\hat{w}_t - (\hat{h}_{t+1} - 1)^{\frac{1}{\beta}}} \right) \frac{\omega_{t+1}}{\omega_t} \left( \xi_t + \varphi'(\hat{h}_{t+1}) \ln \hat{h}_{t+2} \right).
\]
ii) It is obvious that (15) implies that the balanced budget condition is fulfilled whenever \( \bar{h}_{t+1} = \hat{h}_{t+1} \).

iii) Since \( \xi_t > 0 \), \( \varphi'(\hat{h}_{t+1}) > 0 \), \( \hat{h}_{t+2} > 1 \), and \( \hat{w}_t - \left( \hat{h}_{t+1} - 1 \right)^\frac{1}{3} = c_{2,t} + k_{t+1} > 0 \), the optimal subsidy rate \( s_t > 0 \).

On the other hand, if \( s_t = 1 \) the price parents have to pay for the human capital of the children would be zero. Therefore, demand for education would be infinite, which is of course not feasible. Hence, the optimal subsidy rate must fulfill \( 0 < s_t < 1 \). ■
Table 2: Simulations results for a population of heterogeneous agents with different initial values of human capital and $s = 0.5$, $\alpha = 0.3$, $\beta = 0.7$, $\gamma = 0.5$, $\delta = 0.04$.

<table>
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<th>Initial Cond</th>
<th>Type O</th>
<th>Sim. Results</th>
<th>Type U</th>
<th>Sim. Results</th>
</tr>
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<td>Physical Capital</td>
<td>Wage</td>
<td>Human Capital</td>
<td>Physical Capital</td>
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</table>

Note, simulation results are sorted, first, by the initial level human capital of type-U and, second, by the absolute difference between the initial values of human capital of type-O and U.