Strategic Asymmetry in Dynamic Coordination Games with Learning

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Abstract

This paper studies a simple model of dynamic coordination with learning under incomplete information. We show that strategic asymmetry arises where the predecessor's strategy is a strategic substitute for the successor's while the latter is a strategic complement for the former. The role of dynamics and learning is identified by comparing the strategic interactions in the coordination games with different information and timing structures. We also demonstrate that dynamics with learning induces players to take more aggressive strategies.

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1 Introduction

There are many social and economic situations where we observe others' actions and these actions influence our view of the world and outcome of our decision making. For example, consider firms planning to adopt a new technology with network externality. The payoff to the adoption depends on the underlying fundamentals and the number of adopters. The underlying fundamentals are unknown and firms have private information about them. Then, some firm's adoption induces remaining firms (1) to update judgements about fundamentals since the adopter's private information is (partially) revealed by its decision and (2) to change their prospects of success from network effect.

These situations require consideration of dynamics, learning and coordination. The aim of this paper is to develop a simple model that can analyze this kind of dynamic game. Especially, it focuses on the following question: how does the existence of dynamics and learning change the outcome of the static coordination game?

Dasgupta (2001) also tries to answer a similar question. In contrast to this paper, it assumes the continuum of players in each period like the recent literature on dynamic coordination game including Burdzy, Frankel, and Pauzner (2001) and Chamley (2003). However, since observing actions from infinitely many players fully reveals the underlying fundamentals and makes the decision problem trivial, it works on the assumption of noisy observation of the history. Then dynamics, the essence of which is the observation of the past, is obscured and it leads to results very different from ours.

Focusing on single player's binary action in each period and learning, this paper follows the framework in the literature on herd behavior and informational cascade such as Banerjee (1992) and Bikchandani, Hirshleifer, and Welch (1992). However, they do not consider (positive) payoff externality characterizing coordination problem this paper is interested in.

We study a two-period, two-player game with one riskless action and one risky action. The payoff to the risky action depends on the underlying state and the number of players taking risky actions. Each player receives a noisy signal about the state and makes decisions in different periods. The player who moves in the second period, *player 2*, observes the action of the player who moves in the first period, *player 1*, and makes inferences about his signal.

We show that strategic asymmetry arises in this simple dynamic coordination game with learning: player 1's strategy is a strategic substitute[1] for player 2's strategy while player 2's strategy is a strategic complement to player 1's strategy. This result is noticeable since most of the literature following Bulow, Geanakoplos, and Klemperer (1985) and Fudenberg and Tirole (1984) pays attention to the strategically symmetric situations where both players regard others' strategies as either strategic complements or substitutes. We demonstrate that the strategic asymmetry is not a special but rather a general feature in dynamic coordination games with learning.

To establish a benchmark for comparison, we first consider a static version of the model and demonstrate that players' strategies are strategic complements to each other. Then to identify the role of dynamics alone, we analyze a version of the model where player 2 observes the signal as well as the action of player 1 and show that player 2's strategy is a strategic complement to player 1's while player 1's strategy does not influence player 2's.

Comparing these different strategic interactions, we identify the role of dynamics and learning. We then demonstrate that dynamics and learning induce players to take more aggressive strategy. Finally, we show that the differences in strategic interaction fundamentally change the results of comparative statics and emphasize the importance of identifying information and timing structure in real economic problem.

The paper is organized as follows. In Section 2, the model of two-player coordination game under incomplete information is presented with the basic assumptions. Section 3 analyzes three versions of coordination game in sequence to highlight the difference in strategic interaction and the role of dynamics and learning. The comparisons of equilibrium and comparative statics are given in Section 4. We conclude in Section 5.

[1]Player 1's strategy is a strategic substitute for (complement to) player 2's strategy if player 2's reaction function is decreasing (increasing) in player 1's strategy. This taxonomy of strategic interactions was introduced by Bulow, Geanakoplos, and Klemperer (1985) and Fudenberg and Tirole (1984).

2 The Model

There are two players $i = 1, 2$. Each player i chooses an action y_i from a binary action space, $\{0, 1\}$. Action 0 is riskless and guarantees the player zero payoff. On the other hand, action 1 is risky in the sense that the utility from action 1 is $\theta + v_i(n)$ where θ is an unknown state variable, $v_i(n)$ is a strictly increasing function in n, and n is the number of players choosing action 1. Hence two factors influence the payoff from the risky action. The first is how good the state of the world is, θ, and the second is how many agents coordinate on the risky action, $v_i(n)$. The additive separability of the utility is useful in distinguishing between uncertainties concerning the underlying fundamentals and the actions of players. We call the latter the strategic uncertainty.

The state variable, θ, is drawn from a set Θ and unknown to players. All players have a common non-degenerate prior distribution $G_0(\theta)$ with $\mathbb{E}_0(|\theta|) < \infty$, which has a strictly positive and smooth density function. Each player i observes a private signal $x_i \in X \equiv (\underline{x}, \overline{x})$ where $\underline{x}, \overline{x} \in \mathbb{R} \cup \{-\infty, \infty\}$. The signal x_i is drawn independently from the same conditional distribution $F(\cdot|\theta)$ with the density $f(\cdot|\theta)$. The density function, $f(\cdot|\theta)$, is assumed strictly positive and continuous for the whole domain, implying that no signal realization restricts the possible region of the state variable conditional on the observation of the signal and the posterior distribution is smooth.

We make the following standard assumption to simplify analysis.

Assumption 1 $f(x|\theta)$ *satisfies the strict monotone likelihood ratio property (strict MLRP).*

It is well known that under Assumption 1, the posterior distribution, $G(\theta|x)$, is ranked by the signal in the sense of first-order stochastic dominance: the posterior distribution on θ conditional on the signal draw x, $G(\theta|x)$, is decreasing in x for every prior distribution $G_0(\theta)$ (see Milgrom (1981)).

We will consider three kinds of coordination game: static coordination game, dynamic coordination game without learning, and dynamic coordination game with learning. In these games, the player's strategy is a function of his own signal. We allow only monotonic strategy in the sense that players choose the risky action for all signal observation higher than a threshold.[2]

[2]We cannot obtain monotonic strategy automatically in the game since we cannot not apply the concept of rationalizability in the game with learning.

Players receive information about the state from their own signals. However, there are other ways of acquiring information in dynamic games. In the dynamic coordination game without learning, player 2 also observes player 1's signal. In the dynamic coordination game with learning, player 2 infers player 1's signal by observed action. Since the players follow the monotonic strategies, the information revealed by player 1's action takes the form of truncated interval of signal: the signal of player 1 is higher or equal to some cutoff point. Lemma 1 shows the basic properties of information updating in these situations.

Lemma 1 *Under Assumption 1, the following results hold: for $\theta \in \Theta$ and $x, x', k, l \in X$,*

1. $G(\theta|x)$ *is continuous and decreasing in* x.
2. $G(\theta|x < k)$ *and* $G(\theta|x \geq k)$ *are continuous and decreasing in* k.
3. $G(\theta|x, x')$ *is continuous and decreasing in* x *and* x'.
4. $G(\theta|x < k, x')$ *and* $G(\theta|x \geq k, x')$ *are continuous and decreasing in* k.
5. $\mathbb{P}(x' \leq l|x)$ *is continuous and decreasing in* x.
6. $\mathbb{P}(x' \leq l|x < k)$ *and* $\mathbb{P}(x' \leq l|x \geq k)$ *are continuous and decreasing in* k.
7. $\mathbb{E}(\theta|x)$ *is continuous and increasing in* x.
8. $\mathbb{E}(\theta|x, x')$ *is continuous and increasing in* x *and* x'.
9. $\mathbb{E}(\theta|x < k)$, $\mathbb{E}(\theta|x < k, x')$, $\mathbb{E}(\theta|x \geq k)$ *and* $\mathbb{E}(\theta|x \geq k, x')$ *are continuous and increasing in* k.

Proof. See Appendix. ■

The results in Lemma 1 are quite natural. By the strict MLRP, higher signal is good news while lower signal is bad news. If we do not observe the exact value of the signal and know only that it is either higher or lower than some cutoff point, then as the cutoff point increases, it is more likely that the unknown signal is high. Therefore, higher cutoff point is good news.

The following assumption is required to guarantee that the reaction functions and the equilibrium are well defined.

Assumption 2 *For every i,*

$$\sup_{x\in(\underline{x},\overline{x})} \lim_{x'\to\underline{x}} \mathbb{E}(\theta|x,x') + v_i(2) < 0 \quad \textit{and} \quad \inf_{x\in(\underline{x},\overline{x})} \lim_{x'\to\overline{x}} \mathbb{E}(\theta|x,x') + v_i(1) > 0. \tag{1}$$

Assumption 2 implies that the signal is always important for decision making and therefore should not be ignored: sufficiently low (high) signal makes the risky action inferior (superior) even if the coordination is successful (unsuccessful). It can be regarded as a version of "limit dominance" assumption which requires the extreme actions are strictly dominant for extreme signals in the global game literature (see Morris and Shin (2003)). The following lemma is immediate from the assumption.

Lemma 2 *Assumption 2 implies that for every i and k,*

$$\lim_{x'\to\underline{x}} \mathbb{E}(\theta|x\geq k,x') + v_i(2) < 0 \quad \textit{and} \quad \lim_{x'\to\overline{x}} \mathbb{E}(\theta|x<k,x') + v_i(1) > 0, \tag{2}$$

and

$$\lim_{x'\to\underline{x}} \mathbb{E}(\theta|x') + v_i(2) < 0 \quad \textit{and} \quad \lim_{x'\to\overline{x}} \mathbb{E}(\theta|x') + v_i(1) > 0. \tag{3}$$

Proof. See Appendix. ■

Example 1 illustrates that the common assumption of normal distribution in the literature satisfies Assumption 1 and 2.

Example 1 *Let θ be normally distributed with mean μ_0 and variance σ_0^2. The signal $x_i, i=1,2$ is determined by $x_i=\theta+\epsilon_i$ where ϵ_i is drawn independently from the identical normal distribution with zero mean and variance σ^2 for all i. Then $\Theta=X=\mathbb{R}$, $G(\theta)=\Phi(\frac{\theta-\mu_0}{\sigma_0})$ and $f(x|\theta)=\phi(\frac{x-\theta}{\sigma})$*

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the standard normal distribution and density function, respectively. Note that $\phi(\frac{x-\theta}{\sigma})$ is positive and continuous in $x \in X = (-\infty, \infty)$.

It is well known that $\phi(\frac{x-\theta}{\sigma})$ satisfies the strict MLRP. Additionally, it is a standard result in Bayesian updating for normal distribution that $\mathbb{E}(\theta|x) = \frac{\sigma^2\mu_0+\sigma_0^2 x}{\sigma^2+\sigma_0^2}$, $\mathbb{E}(\theta|x,x') = \frac{\sigma^2\mu_0+\sigma_0^2(x+x')}{\sigma^2+2\sigma_0^2}$ and $\mathbb{P}(x' \leq l|x) = \Phi((l - \frac{\sigma^2\mu_0+\sigma_0^2 x}{\sigma^2+\sigma_0^2})/\sqrt{\frac{\sigma^2(\sigma^2+2\sigma_0^2)}{\sigma^2+\sigma_0^2}})$. Then we obtain $\sup_x \lim_{x' \to -\infty} \mathbb{E}(\theta|x,x') + v_i(2) = -\infty$ and $\inf_x \lim_{x' \to \infty} \mathbb{E}(\theta|x,x') + v_i(1) = \infty$. ■

Generally, the symmetry of reaction functions in a game alone does not guarantee that the equilibrium is symmetric . The following proposition proves that the addition of strategic complementarity guarantees it. We present this simple and intuitive result here in a general form since we do not find it in the literature. Consider a N-player game in which each player's strategy set is partially ordered. Let s_i and s_{-i} respectively denote a player i's strategy and a profile of other players' strategies and let $b_i(s_{-i})$ denote player i's reaction function. Given an N-tuple s, define $T^{jk}s$ to be the N-tuple obtained from s by exchanging s_j and s_k.

Proposition 1 *Suppose that reaction functions are symmetric in the sense that for all $i, j, k \in \{1, \ldots, N\}$, $b_i(s_{-i}) = b_i(T^{jk}s_{-i})$ and if $s_{-i} = s_{-j}$, $b_i(s_{-i}) = b_j(s_{-j})$. Then if players' reaction functions are nondecreasing, then all equilibria are symmetric.*

Proof. Suppose that an equilibrium $s^* = (s_1^*, \ldots, s_N^*)$ is not symmetric. Then there exist j and k such that $s_j^* \neq s_k^*$. Without loss of generality, assume $s_j^* > s_k^*$. Since reaction functions are symmetric, $T^{jk}s^* \neq s^*$ should also be an equilibrium. Since s^* and $T^{jk}s^*$ are equilibria, it follows that $s_j^* = b_j(s_{-j}^*)$ and $s_k^* = (T^{jk}s^*)_j = b_j((T^{jk}s^*)_{-j})$. Then, the only difference between s_{-j}^* and $T^{jk}s_{-j}^*$ is that player k's strategy increases from s_k^* to s_j^*. But player j's reaction decreases from s_j^* to s_k^* and this contradicts the assumption of nondecreasing reaction functions. Therefore, all equilibria should be symmetric. ■

3 Dynamics and Learning

We now analyze a sequence of games to highlight the role of dynamics and learning. We begin with the simultaneous action case and then extend to the sequential action cases with and without observing player 1's signal.

3.1 Simultaneous Action

Each player cannot observe the other's signal and action. Let $i, j \in \{1, 2\}$ and $i \neq j$. Given player j's cutoff point k_j^s, player i's optimal strategy is given by

$$s_i(x_i) = \begin{cases} 1 & \text{if } \mathbb{E}(\theta|x_i) + v_i(2)\mathbb{P}(x_j \geq k_j^s|x_i) + v_i(1)\mathbb{P}(x_j < k_j^s|x_i) \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, player i's optimal cutoff point k_i^s for given agent j's cutoff point k_j^s is determined by

$$\mathbb{E}(\theta|k_i^s) + (v_i(2) - v_i(1))\mathbb{P}(x_j \geq k_j^s|k_i^s) + v_i(1) = 0. \tag{4}$$

Proposition 2 *In the simultaneous action case, players' strategies are strategic complements to each other in the sense that one player's optimal cutoff point for the other's cutoff point is increasing in the other's cutoff point.*

Proof. Let k_j^s be given. Since $\mathbb{E}(\theta|k_i^s)$ and $\mathbb{P}(x_j \geq k_j^s|k_i^s)$ are continuous and increasing in k_i^s by Lemma 1, the left hand side of equation (4) is continuous and increasing in k_i^s. Also, it is less than 0 as $k_i^s \to \underline{x}$ and greater than 0 as $k_i^s \to \overline{x}$ by (3) in Lemma 2. Hence there exists a unique $k_i^s \in X$ satisfying equation (4) once k_j^s is given. Since the left hand side of equation (4) is obviously decreasing in k_j^s, k_i^s must increase as k_j^s increases to satisfy equation (4). ■

Higher cutoff point means more conservative strategy. The opponent's conservative strategy makes it difficult for the coordination to succeed. Therefore, the player takes a conservative strategy when he expects the other's conservative strategy. This is a typical coordination problem characterized by strategic complementarity.

Proposition 3 *There always exists an equilibrium in the simultaneous action case.*

Proof. Let $r_i^s(k_j^s) \in X,\ i, j = 1, 2, i \neq j$ denote the reaction function specifying player i's optimal cutoff point for each fixed cutoff point of player j. As shown in the proof of Proposition

2, these functions are well defined. Since the left hand side of equation (4) is continuous in k_i^s and k_j^s by Lemma 1, $r_i^s(k_j^s)$ is continuous in k_j^s. Equation (3) in Lemma 2 ensures there exist $\underline{k_1^s}, \overline{k_1^s} \in X$ such that $\mathbb{E}(\theta|\underline{k_1^s}) + v_1(2) = 0$ and $\mathbb{E}(\theta|\overline{k_1^s}) + v_1(1) = 0$, respectively. By continuity results in Lemma 1, it follows that $\lim_{k_2^s \to \underline{x}} r_1^s(k_2^s) = \underline{k_1^s}$ and $\lim_{k_2^s \to \overline{x}} r_1^s(k_2^s) = \overline{k_1^s}$. On the other hand, $r_2^s(\underline{k_1^s}) > \underline{x}$ and $r_2^s(\overline{k_1^s}) < \overline{x}$. Therefore, $r_1^s(k_2^s)$ and $r_2^s(k_1^s)$ intersect each other at least once in $k_1^s - k_2^s$ plane (Figure 1). ∎

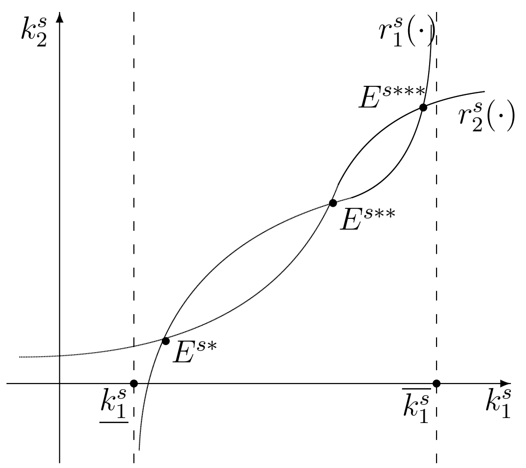

FIGURE 1. Since player 1's reaction curve is lower than player 2's reaction curve near $\underline{k_1^s}$ and higher near $\overline{k_1^s}$, they intersect at least once.

Proposition 3 proves only the existence of equilibrium, not the uniqueness. Static coordination game often has multiple equilibria. Indeed "coordination" matters because the better equilibrium is obtained if coordination succeeds. The global game literature such as Carlsson and Van Damme (1993) and Frankel, Morris and Pauzner (2003) shows that there exists a unique equilibrium in a large class of static coordination games of incomplete information.

However, for the purpose of this paper, it is enough to point out that the uniqueness is generally not guaranteed in the simultaneous action case. Before we demonstrate multiple equilibria in Example 2, it is convenient to assume the symmetry of players, that is, $v_1(\cdot) = v_2(\cdot)$.

Equation (4) with $v(\cdot) \equiv v_1(\cdot) = v_2(\cdot)$ makes players' reaction functions symmetric. Then Proposition 1 and 2 imply that equilibrium is always symmetric. Therefore equilibrium is obtained by (4) with $k^s \equiv k_i^s = k_j^s$, which is

$$\mathbb{E}(\theta|k^s) + (v(2) - v(1))\mathbb{P}(x \geq k^s|k^s) + v(1) = 0. \tag{5}$$

Example 2 *Assume the symmetry of players and consider the distributions of Example 1 with* $\mu = 0$ *and* $\sigma_0 = 1$*. Then,* $G(\theta) = \Phi(\theta)$*,* $f(x|\theta) = \phi(\frac{x-\theta}{\mu})$*,* $\mathbb{E}(\theta|x) = \frac{x}{1+\sigma^2}$ *and* $\mathbb{P}(x' \leq l|x) = \Phi((l - \frac{x}{1+\sigma^2})/(\sqrt{\frac{\sigma^2(2+\sigma^2)}{1+\sigma^2}}))$*. From (5), we obtain*

$$\frac{k^s}{1+\sigma^2} + (v(2) - v(1))(1 - \Phi((k^s - \frac{k^s}{1+\sigma^2})/\sqrt{\frac{\sigma^2(2+\sigma^2)}{1+\sigma^2}})) + v(1) = 0.$$

Rearrange the left hand side to define the function

$$h(k^s) \equiv \frac{k^s}{1+\sigma^2} + (v(2) - v(1))(1 - \Phi(\frac{\sigma k}{\sqrt{(1+\sigma^2)(2+\sigma^2)}})) + v(1).$$

Equilibrium is given by $h(k^s) = 0$*. Differentiating* $h(k^s)$ *with respect to* k^s *yields*

$$\begin{aligned} \tfrac{dh(k^s)}{dk^s} &= \tfrac{1}{1+\sigma^2} - (v(2) - v(1))\tfrac{\sigma}{\sqrt{(1+\sigma^2)(2+\sigma^2)}}\phi(\cdot) \\ &= \tfrac{\sigma}{\sqrt{1+\sigma^2}\sqrt{2+\sigma^2}}(\tfrac{\sqrt{2+\sigma^2}}{\sigma\sqrt{1+\sigma^2}} - (v(2) - v(1))\phi(\cdot)). \end{aligned}$$

Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$*,* $h(k^s)$ *is nondecreasing if*

$$\frac{\sqrt{2+\sigma^2}}{\sigma\sqrt{1+\sigma^2}} = \frac{1}{\sigma}\sqrt{1 + \frac{1}{1+\sigma^2}} \geq \frac{v(2) - v(1)}{\sqrt{2\pi}} \tag{6}$$

so that $h(k^s)$ *crosses 0 upward only once as* k^s *increases. Therefore (6) is the sufficient condition for the unique equilibrium.*

Now, suppose (6) does not hold. It follows that there exists some $\hat{k^s}$ *satisfying* $\frac{dh(\hat{k^s})}{dk^s} < 0$*. Since* $\frac{dh(k^s)}{dk^s}$ *depends only on* $v(2) - v(1)$*, we can find some* $v(\cdot)$ *with* $v(2) - v(1)$ *unchanged such that* $h(\hat{k^s}) = 0$ *and* $\frac{dh(\hat{k^s})}{dk^s} < 0$*. Then* $h(k^s)$ *crosses 0 downward at* $\hat{k^s}$ *and* $h(k^s) = 0$ *has at least two more roots since* $h(k^s)$ *is continuous in* k^s *and* $\lim_{k^s \to -\infty} h(k^s) = -\infty < 0$ *and* $\lim_{k^s \to \infty} h(k^s) = \infty > 0$.

Hence we show that (6) is the necessary and sufficient condition that there exists a unique equilibrium for every $v(\cdot)$. The left hand side of (6) is decreasing in σ and therefore a unique equilibrium is obtained as the noise of private signal vanishes. This is a standard result in the global game literature (see Morris and Shin (2003)). ■

3.2 Sequential Action with Observed Signal

Consider a model with two periods, $t = 1, 2$. Players 1 and 2 choose actions in period 1 and 2, respectively. Player 1 does not know player 2's signal and action. But player 2 observes agent 1's signal and action.

Player 2's cutoff point strategy maps from player 1's action and signal space into the signal space. Let $k_2^o(x_1; y_1)$ denote player 2's cutoff point responding to player 1's signal x_1 and action y_1.

In period 2, after observing player 1's signal and action, player 2's optimal cutoff point is determined by

$$\mathbb{E}(\theta | x_1, k_2^o(x_1; 1)) + v_2(2) = 0 \quad \text{and} \quad \mathbb{E}(\theta | x_1, k_2^o(x_1; 0)) + v_2(1) = 0,$$

$$\text{or equivalently, } \mathbb{E}(\theta | x_1, k_2^o(x_1; y_1)) + v_2(y_1 + 1) = 0. \tag{7}$$

Note that $k_2^o(\cdot; \cdot)$ is fully determined by equation 7 without considering player 1's cutoff point k_1^o.

Lemma 3 *Player 2's optimal cutoff point $k_2^o(x_1; y_1)$ is continuous in x_1 and decreasing in x_1 and y_1.*

Proof. Let x_1 and y_1 be given. The left hand side of equation 7 is continuous and increasing in $k_2^o(x_1; y_1)$ by Lemma 1. By Assumption 2, it follows that there exists a unique $k_2^o(x_1; y_1)$ satisfying equation 7 once x_1 and y_1 are given. Since the left side of equation 7 is continuous in x_1 and increasing in x_1 and y_1, the result follows. ■

Higher signal realization is good news about the state and higher observed action enhances coordination. Therefore, player 2 takes more aggressive strategy when he observes a higher signal or action.

Given player 2's cutoff point $k_2^o(\cdot;1)$, player 1's optimal cutoff point k_1^o in period 1 is determined by

$$\mathbb{E}(\theta|k_1^o) + (v_1(2) - v_1(1))\mathbb{P}(x_2 \geq k_2^o(k_1^o;1)|k_1^o) + v_1(1) = 0. \tag{8}$$

Note that the marginal Player 1 with signal k_1^o considers player 2's reaction to his signal k_1^o and action 1 as represented by $k_2^o(k_1^o;1)$.

We obtain the following result in contrast to Proposition 2.

Proposition 4 *In the sequential action with observed signal case, player 2's strategy is a strategic complement to player 1's strategy while player 1's strategy do not influence player 2's strategy.*

Proof. Since the left side of equation (8) is continuous and decreasing in $k_2^o(k_1^o;1)$, Lemma 1 and 3 implies that it is continuous and increasing in k_1^o. By (3) in Lemma 2, it follows that there exists a unique k_1^o once $k_2^o(\cdot;1)$ is given. Then since the left side of equation (8) is obviously decreasing in $k_2^o(\cdot;1)$, k_1^o must increase as $k_2^o(\cdot;1)$ increases. This establishes the first part of the result. The second part is straightforward in equation 7 which does not include k_1^o. ■

Therefore, strategic interaction is one-way in this sequential action case. Player 2's strategy matters to player 1 because it changes the prospect of coordination. However, strategic interaction in the reverse direction does not work. Note that player 2 makes a decision after he observes player 1's signal and action. Generally, the knowledge of the other's strategy does not provide more information than that of his signal and action. Player 2 bases his decision on the superior information from the observed signal and the action and therefore does not need to consider player 1's strategy. Player 1's strategy influences only the chances of player 2's observing action 1 in period 1 but not player 2's strategy itself.

This one-way strategic interaction implies a unique equilibrium as stated in Proposition 5.

Proposition 5 *There always exists a unique equilibrium* $(k_1^{o*}, k_2^{o*}(\cdot;\cdot))$ *under the sequential action with observed signal.*

Proof. Let $r_1^o(k_2^o(\cdot;1)) \in X$ denote the reaction function specifying player 1's optimal cutoff point for each cutoff point of player 2 observing the player 1's action 1. As shown in the proof of Proposition 4, this function is well defined so that player 1's equilibrium cutoff point $k_1^{o*} =$

$r_1^o(k_2^{o*}(\cdot;1))$ is obtained once player 2's equilibrium cutoff point $k_2^{o*}(\cdot;1)$ is given. Since $k_2^{o*}(\cdot;\cdot)$ is uniquely and independently determined in equation 7, the result follows. ■

By Proposition 4, player 1's strategy depends only on player 2's strategies. Then, the player in the last period does not need to consider others' strategies. Once the strategy in the last period is determined, the player in the period before the last period does not face strategic uncertainty and his strategy is determined. Therefore, dynamics alone removes strategic uncertainty completely in finite horizon games by backward induction and guarantees a unique equilibrium.

3.3 Sequential Action with Private Signal

Now suppose that player 2 cannot observe player 1's signal but can only infer it by observing player 1's action. Therefore player 2's cutoff point maps only from player 1's action space into the signal space. Let $k_2^p(y_1)$ denote player 2's cutoff point corresponding to player 1's action y_1.

Given player 1's cutoff point k_1^p, player 2's optimal cutoff point is determined by

$$\mathbb{E}(\theta|x_1 \geq k_1^p, k_2^p(1)) + v_2(2) = 0 \quad \text{and} \quad \mathbb{E}(\theta|x_1 < k_1^p, k_2^p(0)) + v_2(1) = 0. \tag{9}$$

Since player 1's action reveals information about whether his private signal is greater than his cutoff point or not, player 2 infers the state from this information as represented by $\mathbb{E}(\theta|x_1 \geq k_1^p, \cdot)$ and $\mathbb{E}(\theta|x_1 < k_1^p, \cdot)$.

Given player 2's cutoff point $k_2^p(1)$, player 1's optimal cutoff point in period 1 is determined by

$$\mathbb{E}(\theta|k_1^p) + (v_1(2) - v_1(1))\mathbb{P}(x_2 \geq k_2^p(1)|k_1^p) + v_1(1) = 0. \tag{10}$$

The following proposition is the main result of the paper.

Proposition 6 *In the sequential action with private signal case, player 1's strategy is a strategic substitute for player 2's strategy while player 2's strategy is a strategic complement to player 1's strategy.*

Proof. Let k_1^p be given. The left hand side of equation (9) is continuous and increasing in $k_2^p(y_1)$

by Lemma 1. By equation (2) in Lemma 2, there exists a unique $k_2^p(y_1)$ satisfying equation (9) once k_1^p is given. Then, since the left side of equation (9) is increasing in k_1^p by Lemma 1, $k_2^p(y_1)$ must decrease as k_1^p increases to satisfy equation (9). This establishes the first part of the proposition. The second part can be proved in the same manner as Proposition 4 and is omitted. ■

As shown in Lemma 1, the information revealed by player 1's risky action choice becomes more favorable for the state as he takes a more conservative strategy with higher cutoff point . Therefore player 2 takes a more aggressive strategy. On the other hand, player 2's more conservative strategy decreases the possibility of coordination and induces player 1 to take a more conservative strategy in the same way as the simultaneous action and the sequential action with observed action cases.

Therefore, there is strategic asymmetry in this case: one player regards his opponent's strategy as a strategic complement while the other player regards his opponent's strategy as a strategic substitute. An example of strategic asymmetry was given in Bulow, Geanakoplos and Klemperer (1985): if industry marginal revenue is decreasing in total output and a firm produces more than half the total market output, then the large firm may regard the fringe firms' products as strategic complements while its competitive fringe regards the large firm's products as strategic substitutes. But strategic asymmetry has been regarded as an exception and most of the literature has paid little attention to it.

The contribution of this paper is to show that the strategic asymmetry is a general feature of the dynamic coordination and learning problem. Though it seems natural to think that the positive payoff externality in coordination game generates strategic complementarity directly, the addition of learning induces strategic substitution even in the coordination game.

To understand this seemingly surprising result, notice first that in the static coordination game, one player's lower cutoff point induces the other's lower cutoff point only by increasing the possibility of his taking action 1. Therefore, if his action is kept constant, then the change of his cutoff point does not make a difference in the prospect of coordination. The role of dynamics is to make player 2's strategy depend on player 1's action choice, that is, to control player 1's action when player 2 decides his reaction to player 1's different strategies. That's why player 1's strategy does not influence player 2's strategy even with positive payoff externality in sequential action with observed signal case.

Then, the addition of learning gives player 1's strategy a different channel through which it interacts with player 2's strategy. Player 1's different strategies change player 2's strategy by transmitting different information about his signal. Here player 1's lower cutoff point indicates that his signal is less favorable and induces player 2's higher cutoff point.

Though strategic uncertainty is preserved in this case contrary to the sequential action with observed signal, the strategic symmetry implies a unique equilibrium as stated in Proposition 7.

Proposition 7 *There always exists a unique equilibrium* $(k_1^{p*}, k_2^{p*}(1), k_2^{p*}(2))$ *in the sequential action with private signal case.*

Proof. Let $r_1^p(k_2^p(1)) \in X$ and $r_2^p(k_1^p) \in X$ denote the reaction functions specifying player 1's optimal cutoff point for each cutoff point of player 2 observing player 1's risky action choice and player 2's optimal cutoff point after observing player 1's risky action choice for each cutoff point of player 1, respectively. As shown in the proof of Proposition 6, these functions are well defined. They are also continuous since the left hand side of the first equation in (9) and equation (10) are continuous in k_1^p and $k_2^p(1)$. Equation (2) in Lemma 2 ensures that there exist $\underline{k_1^p}, \overline{k_1^p} \in X$ such that $\mathbb{E}(\theta|\underline{k_1^p}) + v_1(2) = 0$ and $\mathbb{E}(\theta|\overline{k_1^p}) + v_1(1) = 0$. By continuity in Lemma 1, it follows that $\lim_{k_2^p(1)\to \underline{x}} r_1^p(k_2^p(1)) = \underline{k_1^p}$ and $\lim_{k_2^p(1)\to \overline{x}} r_1^p(k_2^p(1)) = \overline{k_1^p}$. On the other hand, $r_2^p(\underline{k_1^p}) > \underline{x}$ and $r_2^p(\overline{k_1^p}) < \overline{x}$. Since $r_1^p(k_2^p(1))$ is upward sloping and $r_2^p(k_1^p)$ is downward sloping in $k_1^p - k_2^p(1)$ plane by Proposition 6, $r_1^p(k_2^p(1))$ and $r_2^p(k_1^p)$ intersect each other only once (Figure 2). Once k_1^{p*} and $k_2^{p*}(1)$ are determined, $k_2^{p*}(0)$ is uniquely determined by the second equation in (9)). ■

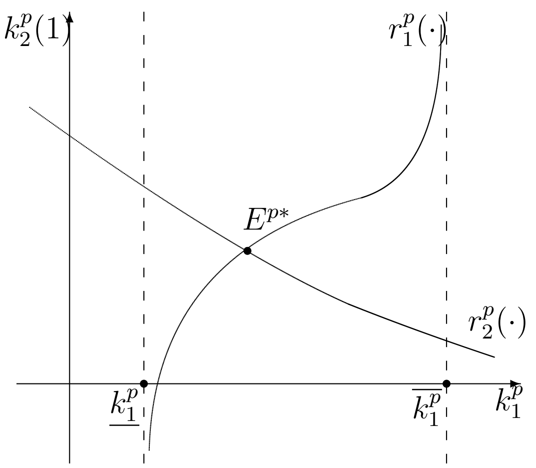

FIGURE 2. Since the slopes of players' reaction functions have the different signs, the difference between these reaction functions is monotone as we move along either axis. Therefore, they intersect only once.

The uniqueness result in Proposition 7 presents a striking contrast to Dasgupta (2001) where two-period dynamic coordination game with learning is not always guaranteed to have a unique equilibrium with switching strategies. This difference is caused by the assumption of the continuum players in his paper. Once the cutoff point is given, a player's action can be regarded as a binomial random variable with a distribution uniquely determined by the state variable. If players are symmetric and receive conditionally independent signals, players' actions are different realizations of the same random variable. Then the observation of realizations by an infinite number of players fully reveals the underlying state variable by the law of large numbers and makes trivial the decision-making in later periods. This is exactly how an informational cascade always occurs in Bikhchandani, Hirshleifer and Welch (1992).

To avoid this problem, Dasgupta (2001) assumes that player 2 observes the proportion of player 1s taking risky actions only with noise. It may be a reasonable assumption in the model with a large number of players but may not be a good one for identifying the role of dynamics. Then player 2's strategy depends not only on player 1's action but also on the noisy signal on top of it. The role of dynamics to control the history in decision making does not work any more. The strategic complementarity from the past to the future revives since player 1's aggressive action makes it more likely that player 2 is in history better for risky action. Hence his model lies somewhere between the static and the dynamic coordination allowing multiple equilibria in two-period setting.

4 Discussion

We have presented three different information and timing structures to demonstrate how strategic interaction between players changes fundamentally. We compare equilibria and comparative statics for these different models.

4.1 Equilibrium

We first compare the equilibrium cutoff points of the three models. It is useful to collect equations satisfied by the equilibrium cutoff points for the three models.

In the simultaneous action case, the equilibrium cutoff point k_1^{s*} and k_2^{s*} are determined by the following equations:

$$\mathbb{E}(\theta|k_1^{s*}) + (v_1(2) - v_1(1))\mathbb{P}(x \geq k_2^{s*}|k_1^{s*}) + v_1(1) = 0, \tag{11}$$

$$\mathbb{E}(\theta|k_2^{s*}) + (v_2(2) - v_2(1))\mathbb{P}(x \geq k_1^{s*}|k_2^{s*}) + v_2(1) = 0. \tag{12}$$

In the sequential action with observed signal case, the equilibrium cutoff points k_1^{o*} and $k_2^{o*}(k_1^{o*};1)$ satisfy the following equations:

$$\mathbb{E}(\theta|k_1^{o*}) + (v_1(2) - v_1(1))\mathbb{P}(x_2 \geq k_2^{o*}(k_1^{o*};1)|k_1^{o*}) + v_1(1) = 0, \tag{13}$$

$$\mathbb{E}(\theta|k_1^{o*}, k_2^{o*}(k_1^{o*};1)) + v_2(2) = 0. \tag{14}$$

In the sequential action with private signal case, the equilibrium cutoff points k_1^{p*} and $k_2^{p*}(1)$ satisfy the following equations:

$$\mathbb{E}(\theta|k_1^{p*}) + (v_1(2) - v_1(1))\mathbb{P}(x_2 \geq k_2^{p*}(1)|k_1^{p*}) + v_1(1) = 0, \tag{15}$$

$$\mathbb{E}(\theta|x_1 \geq k_1^{p*}, k_2^{p*}(1)) + v_2(2) = 0. \tag{16}$$

Since player 2's cutoff points in these cases are functions on the different domains, they are not directly comparable. Hence we focus only on player 1's cutoff point. We find that dynamic and learning make player 1 more aggressive as stated in Proposition 8.

Proposition 8 *1. Player 1 in the sequential action with private signal case takes the most aggressive strategy in three cases*

2. Player 1 takes a more aggressive strategy in the sequential action with observed signal case *than in the simultaneous action case if*

$$\mathbb{E}(\theta|k_1^{s*}, k_2^{s*}) + v_2(2) > 0. \tag{17}$$

Proof. 1. Since the left hand side of equation (15) is increasing in k_1^{p*} and decreasing in $k_2^{p*}(1)$, comparing equation (15) with equation (11) shows either $k_1^{p*} < k_1^{s*}$ and $k_2^{p*}(1) < k_2^{s*}$ or $k_1^{p*} \geq k_1^{s*}$ and $k_2^{p*}(1) \geq k_2^{s*}$ hold. Suppose $k_1^{p*} \geq k_1^{s*}$ and $k_2^{p*}(1) \geq k_2^{s*}$. Since $\mathbb{E}(\theta|\cdot,\cdot)$ is increasing, it follows from equation (16) that $\mathbb{E}(\theta|x_1 \geq k_1^{s*}, k_2^{s*}) + v_2(2) \leq 0$. However, $\mathbb{E}(\theta|x_1 \geq k_1^{s*}, k_2^{s*}) + v_2(2) > \mathbb{E}(\theta|k_2^{s*}) + (v_2(2) - v_2(1))\mathbb{P}(x \geq k_1^{s*}|k_2^{s*}) + v_2(1) = 0$ by $\mathbb{E}(x \geq k, x') \geq \lim_{k \to \underline{x}} \mathbb{E}(x \geq k, x') = \mathbb{E}(x')$, which is a contradiction. Therefore, $k_1^{p*} < k_1^{s*}$.

Now, suppose $k_1^{p*} \geq k_1^{o*}$. Note that $\mathbb{E}(\theta|x \geq k, x') = \frac{\int_k^{\overline{x}} \mathbb{E}(\theta|x,x')f(x,x')dx}{\int_k^{\overline{x}} f(x,x')dx} > \mathbb{E}(\theta|k, x')$. Then comparing equation (14) with equation (16) shows $k_2^{p*}(1) < k_2^{o*}(k_1^{o*}; 1)$. Then $\mathbb{E}(\theta|k_1^{p*}) + (v_1(2) - v_1(1))\mathbb{P}(x_2 \geq k_2^{p*}(1)|k_1^{p*}) + v_1(1) > \mathbb{E}(\theta|k_1^{o*}) + (v_1(2) - v_1(1))\mathbb{P}(x_2 \geq k_2^{o*}(k_1^{o*}; 1)|k_1^{o*}) + v_1(1) = 0$ which contradicts equation (15). Hence $k_1^{p*} < k_1^{o*}$.

2. Since the left hand side of equation (13) is increasing in k_1^{o*} and decreasing in $k_2^{o*}(k_1^{o*}; 1)$, comparing equation (13) with equation (11) shows that either $k_1^{o*} < k_1^{s*}$ and $k_2^{o*}(k_1^{o*}; 1) < k_2^{s*}$ or $k_1^{o*} \geq k_1^{s*}$ and $k_2^{o*}(k_1^{o*}; 1) \geq k_2^{s*}$ hold. Suppose $k_1^{o*} \geq k_1^{s*}$ and $k_2^{o*}(k_1^{o*}; 1) \geq k_2^{s*}$. Then it follows from equation (14) that $\mathbb{E}(\theta|k_1^{s*}, k_2^{s*}) + v_2(2) \leq 0$ which contradicts the assumption of $\mathbb{E}(\theta|k_1^{s*}, k_2^{s*}) + v_2(2) > 0$. Therefore, $k_1^{o*} < k_1^{s*}$. ■

Consider the marginal player 1 who receives a signal equal to his cutoff point. In the sequential action with private signal case, his risky action makes player 2 think his signal is much more favorable than his actual signal while his signal is precisely revealed in the sequential action with observed signal case. Therefore, player 1's risky action induces player 2 to take a more aggressive strategy in the private signal case than in the observed signal case and player 1 takes a more aggressive strategy in the private signal case based on the expectation of such response of player 2.

To compare the dynamic cases with the static case, note that players take more aggressive strategies as they are more certain of the success of coordination. The observation of player 1's risky action assures player 2 of the success of coordination and induces him to take a more aggressive strategy. Considering this effect of his risky action on player 2, player 1 in dynamic cases is more certain of the success of coordination than players in the static case. Therefore, if players' information about the state in the static case is the same as the dynamic cases, player 1 would take a more aggressive strategy in dynamic cases.

However, player 2 receives additional information about the state in dynamic cases. Consider first the sequential action with private signal case. In this case, player 1's risky action gives player 2 the information that player 1's signal is higher than some cutoff point. Note that regardless of the value of the cutoff point, this information is better than no information in the simultaneous action case since no information is equivalent to the information that player 1's signal is higher than the lowest possible signal. Therefore, player 1 in sequential action with private signal case takes a more aggressive strategy than players in simultaneous action case.

Now, consider the marginal player 1 again. If his signal is sufficiently low, then player 2 could take a more conservative strategy after observing his signal even if player 2 observes his risky action. Then player 1 would take a more conservative strategy in sequential action with observed signal case than in simultaneous action case. It matters whether the marginal player 1's signal, that is, the equilibrium cutoff point of player 1 is high or low. That is what the condition (17)) is about.

Inserting $v_2(2) = -\mathbb{E}(\theta|k_2^{s*}) + (v_2(2) - v_2(1))\mathbb{P}(x < k_1^{s*}|k_2^{s*})$ from equation (12) into equation (17), we obtain the equivalent condition

$$\mathbb{E}(\theta|k_1^{s*}, k_2^{s*}) > \mathbb{E}(\theta|k_2^{s*}) - (v_2(2) - v_2(1))\mathbb{P}(x < k_1^{s*}|k_2^{s*}).$$

This requires exactly the observation of the marginal player 1's signal not to be too bad news to the marginal player 2 in the simultaneous action case.

Assuming the symmetry of players by $k^{s*} \equiv k_1^{s*} = k_2^{s*}$, we can examine the above condition further. Recalling the results in Example 1, we obtain $\mathbb{E}(\theta|k^{s*}) = \frac{\sigma^2\mu_0 + \sigma_0^2 k^{s*}}{\sigma^2 + \sigma_0^2}$ and $\mathbb{E}(\theta|k^{s*}, k^{s*}) = \frac{\sigma^2\mu_0 + 2\sigma_0^2 k^{s*}}{\sigma^2 + 2\sigma_0^2}$ under the assumption of normal distribution. Then $\mathbb{E}(\theta|k^{s*}, k^{s*}) \geq \mathbb{E}(\theta|k^{s*})$ if and only if $k^{s*} \geq \mu_0$. If the equilibrium cutoff point is higher than the prior mean, then the observation of an additional signal equal to the cutoff point in the simultaneous action case is good news to the marginal player 2 and the introduction of dynamics makes player 1 more aggressive. Note also that if the prior distribution is sufficiently vague which corresponds to $\sigma_0^2 \to \infty$ in the above normal distribution case, then $\mathbb{E}(\theta|k^{s*}, k^{s*}) \geq \mathbb{E}(\theta|k^{s*})$ is always satisfied by $\mathbb{E}(\theta|k^{s*}, k^{s*}) \simeq \mathbb{E}(\theta|k^{s*}) \simeq k^{s*}$.

In summary, dynamics and learning lead player 1 to take a more aggressive strategy. Dynamics alone requires the equilibrium cutoff point not to be very low in order to make player 1 aggressive.

4.2 Comparative Statics

Next we consider an idiosyncratic shock shifting $v_i(\cdot), i = 1, 2$. For the analysis, it is useful to focus on the reaction functions.

We have shown in Section 3 that the following reaction functions are well defined and continuous: $r_1^s(k_2^s)$ and $r_2^s(k_1^s)$ in the simultaneous action case; $r_1^o(k_2^o(\cdot; 1))$ in the sequential action with observed signal case; $r_1^p(k_2^p(1))$ and $r_2^p(k_1^p)$ in the sequential action with private signal case. Propositions 2, 4 and 6 ensures that $r_1^s(k_2^s), r_2^s(k_1^s), r_1^o(k_2^o(\cdot; 1))$ and $r_1^p(k_2^p(1))$ are upward sloping and $r_2^p(k_1^p)$ is downward sloping.

By Propositions 3,5 and 7, equilibrium always exists and is determined by the intersection of these reaction functions in the corresponding planes except for the sequential action with observed signal case where $k_2^{o*}(\cdot; 1)$ is first determined independently of k_1^{o*} and then k_1^{o*} is determined by $k_1^{o*} = r_1^o(k_2^{o*}(\cdot; 1))$.

Multiple equilibria in the simultaneous action case make the comparison complicated. To avoid these difficulties, we consider only the case where there exists a unique equilibrium. The literature studying the condition of unique equilibrium in this kind of static coordination game and the example of such condition were presented in Section 3.1.

We showed in the proof of Proposition 3 that $\lim_{k_2^s \to \underline{x}} r_1^s(k_2^s) = \underline{k_1^s}$ and $\lim_{k_2^s \to \overline{x}} r_1^s(k_2^s) = \overline{k_1^s}$ while $r_2^s(\underline{k_1^s}) > \underline{x}$ and $r_2^s(\overline{k_1^s}) < \overline{x}$ for some $\underline{k_1^s}, \overline{k_1^s} \in X$. Therefore player 1's reaction function $r_1^s(k_1^p)$ should be steeper at the equilibrium than player 2's reaction function $r_2^s(k_2^p)$ for the unique equilibrium (Figure 3). Then since the increase of $v_i(\cdot)$ moves the reaction functions of player i inward, the following results are straightforward (Figure 4).

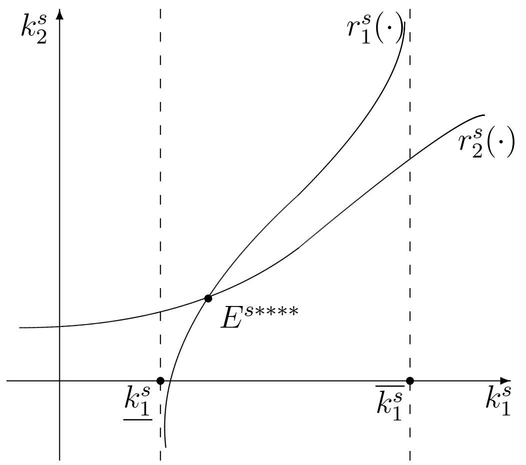

FIGURE 3. Here player 1's reaction curve is steeper than player 2's reaction curve at the unique equilibrium point E^{s****}. Note that in Figure 1, player 2's reaction curve is steeper than player 1's at one equilibrium point E^{s**} so that they produce (at least) two more intersections, that is, two more equilibria.

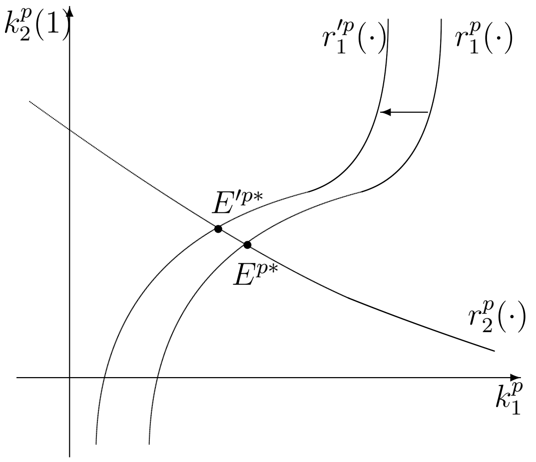

FIGURE 4. We illustrate the sequential action with private signal case when there is a positive shock to player 1. Player 1's reaction function shifts inward and equilibrium moves from E^{p*} to E'^{p*}

Proposition 9 *Suppose there exists a unique equilibrium in the simultaneous case.*

1. *If there is a positive shock to player 1, then in equilibrium,*
   - (a) *both players are more aggressive in the simultaneous action case.*
   - (b) *player 1 is more aggressive while player 2 do not change in sequential action with observed signal case.*
   - (c) *player 1 is more aggressive while player 2 is more conservative in sequential action with private signal case.*

2. *If there is a positive shock to player 2, then in equilibrium, both players are more aggressive in all cases.*

These results are driven by different strategic interactions which determine the slopes of reaction functions. The fact that the comparative statics analysis differs fundamentally depending on whether players' strategies are strategic complements or substitutes is well known since Bulow, Geanakoplos, and Klemperer (1985) and Fudenberg and Tirole (1984). We present Proposition 9 to underscore the importance of identifying information and timing structure in analyzing a given situation since they determine strategic interactions.

Consider again the example stated in Introduction. There are two firms, 1 and 2 in market A. They contemplate adopting a new technology which exhibits network externality. The payoff to the adoption of the technology depends on its unknown intrinsic value and the number of firms adopting it. Now, imagine firm 1 makes investment in market B where the technology can be also used. Hence this investment increases firm 1's overall payoff to adopting the new technology. Then, how does this firm 1's investment in market B change firms' decisions in market A? Proposition 9 implies that it depends on the information and timing structure in market A.

If firms are not able to commit to their adoption decisions and therefore make a decision simultaneously, the investment in market 1 promotes their adoptions of the technology. Now, suppose that firm 1 can commit to its decision in market A. If its assessment of the intrinsic value of the technology is exposed, for example, by its financial reports once it adopts the technology, then the investment induces firm 1's adoption while it does not change firm 2's decision. On the other hand, if firm 1's assessments of the value of the technology is not disclosed, the investment fosters firm 1's adoption while it impedes firm 2's. Finally, if firm 2 can commit to its decisions in market A, then the investment always promotes firm 1's adoption.

Proposition 9 also requires an empirical study of the economy to pay attention to the information and timing structure. For example, consider an economic boom where there are positive shocks to all economic agents in the economy but suppose that the sizes of the shocks are different. Proposition 9 implies that if the positive shock to player 1 is sufficiently bigger than that to player 2, then even the positive shocks to the economy makes player 2 conservative in the sequential action with private signal case (Figure 5). Therefore, if an empirical economist tries to statistically infer the state of the economy by obtaining data about the activities of the economic agents, he should control the variable representing the information and timing structure in his analysis.

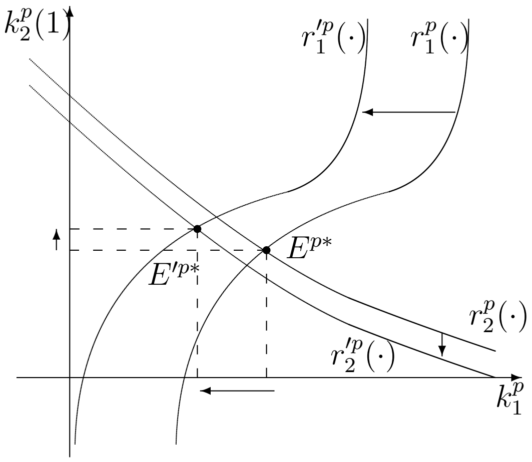

FIGURE 5. Even the overall positive shocks can make player 2 conservative in the sequential action with private signal case if the shift of player 1's reaction function is sufficiently bigger than that of player 2's reaction function.

5 Concluding Remarks

In this paper, we have investigated the role of dynamics and learning in a sequence of coordination games under incomplete information. We have shown that dynamics and learning make a fundamental difference in strategic interaction between players.

We construct the model under a general condition regarding the distribution of state variable and signal. It makes our main result robust to various assumptions on distribution. Additionally, though we restrict our analysis to two-period, two-player case in this paper, it is clear from the arguments that the main result still hold in N-period, N-player case: in the static coordination game, players' strategies are strategic complements to each other; in the dynamic coordination game without learning, there exists only one-way strategic interaction, that is, only player 2's strategy is a strategic complement to player 1's; in the dynamic coordination game with learning, strategic asymmetry arises where player 1's strategy is a strategic substitute for player 2's while the latter is a strategic complement to the former.

However, we have assumed the order of player's actions is exogenously given in the dynamic cases which is the main limitation of this paper. We conjecture it is possible to extend the model so that the order is endogenously determined by the parameter representing heterogeneity among players' utilities as Farrell and Saloner (1985) does under the different assumption of the complete information about the state.

Finally, by focusing on two-period, two-player case, we need not consider the possibility of herd behavior: two players are too small to make a herd. In fact, we assume that the signal is always important (Assumption 2) so that we remove the possibility of player's ignoring his own signal. However, allowing the herd behavior will enrich the model especially when we generalize the model to N-period, N-player case.

We are studying these and related issues and we believe there are many other interesting questions in the dynamic coordination problems.

Appendix

The proofs of Lemma 1 and Lemma 2 are provided here.

Proof of Lemma 1 1. Since $G(\theta|x) = \frac{\int_{\theta' \leq \theta} f(x|\theta')dG(\theta')}{\int_{\theta' \in \Theta} f(x|\theta')dG(\theta')}$, the continuity of $f(x|\theta')$ in x implies the result. The other continuity results in Lemma 1 are obtained by the continuity of $G(\theta|x)$ or the integration by the corresponding factors. As already stated, Assumption 1 implies $G(\theta|x)$ is decreasing in x.

2. Note that $G(\theta|x < k) = \frac{\int_{\underline{x}}^{k} G(\theta|x)f(x)dx}{\int_{\underline{x}}^{k} f(x)dx}$ where $f(x) = \int_{\theta' \in \Theta} f(x|\theta')dG(\theta')$ is the unconditional density of x. Since $G(\theta|x < k)$ is a weighted average of $G(\theta|x)$ with the weight $\frac{f(x)}{\int_{\underline{x}}^{k} f(x)dx} > 0$ and $G(\theta|x)$ is decreasing in x, $G(\theta|x < k)$ is decreasing in k. Likewise, it can be shown that $G(\theta|x \geq k)$ is decreasing in k.

3. and 4. Since the signals x and x' are conditionally independent, it holds that $f(x, x'|\theta) = f(x|\theta)f(x'|\theta)$ where $f(x, x'|\theta)$ is the joint conditional density of x and x'. Therefore, $f(x, x'|\theta)$ is continuous in x, x' and satisfies the strict MLRP. Then the results follows in the same way as 1. and 2.

5. Note that $\mathbb{P}(x' \leq l|x) = \int_{\theta \in \Theta} F(l|\theta)dG(\theta|x)$ and $F(l|\theta)$ is decreasing in θ by the strict MLRP. Then, the fact that $G(\theta|x)$ is decreasing in x implies the result.

6. The relation between $\mathbb{P}(x' \leq l|x)$ and $\mathbb{P}(x' \leq l|x < k)$ corresponds to that between $G(\theta|x)$ and $G(\theta|x < k)$. Therefore, the result can be shown in the same way as 2.

7. to 9. These results follow by the strict first-order stochastic dominance proved in 2. to 5. ■

Proof of Lemma 2 Since $\mathbb{E}(\theta|x \geq k, x') = \frac{\int_k^{\overline{x}} \mathbb{E}(\theta|x,x')f(x,x')dx}{\int_k^{\overline{x}} f(x,x')dx}$ and $\mathbb{E}(\theta|x < k, x') = \frac{\int_{\underline{x}}^k \mathbb{E}(\theta|x,x')f(x,x')dx}{\int_{\underline{x}}^k f(x,x')dx}$ where $f(x, x') = \int_{\theta' \in \Theta} f(x, x'|\theta')dG(\theta')$ is the unconditional joint density of x and x', (2) follows by (1). (3) is obtained by $k \to \underline{x}$ and $k \to \overline{x}$ in (2). ■

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