Who wants to speak first? Endogenous debate design with career concerns. *

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Abstract

We model a committee made up of informed experts whose members have to reach an agreement on a particular decision. They reach an agreement in debating, following a protocol that determines the order of speech in the debate and the type of messages debaters are allowed to send. We consider that members of the committee have career concerns, namely that their payoff depends both on the decision made by the committee and the state of the world. We show that career concerns of the experts may imply that they disagree about the choice of the debate protocol. Yet we show that this disagreement can not be common knowledge. In other words, if expert's preferences about the debate protocol are common knowledge, then they have to be the same.

1 Introduction

When economic agents are interested in how their decisions are related to the state of the world, we say that they have career concerns. Implicitly, their payoffs depend on the evaluation made by an outside agent, who observes the state of the world after the decisions are made, and who has an opinion about what action should be made at each state of the world. Career concerns apply to experts whose market value depends on their reputation as giving good recommendations, to politicians who make their decisions according to electoral

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purposes, to firm managers whose promotion depends on the shareholders' opinion about their competence, and so on. There is a large literature concerning how individual decision makers behave when they are motivated by career concerns, which spans many types of decision makers, such as managers, financial advisers or politicians, in different environments (Holmström [1982], Crawford and Sobel [1982], Scharfstein and Stein [1990], Trueman [1994], Levy [2004], Dasgupta and Prat [2003], Morris [2001], Ottaviani and Sørensen [2002]). However, many economic and political decisions are taken by groups of decision makers, namely committees. Company boards, governments, monetary policy committees, consultants or juries are some examples.

The analysis of group decision making when the individuals members of the group have career concerns differs from the one of individual decision making. Any decision making in a committee implies a stage of debate, which is the process by which heterogenous decision makers produce a group decision. A debate protocol is characterized by the timing in the debate, which determines who gets to speak when in the debate, the communication rule between experts, which defines what are the words for communication, and the way the final decision is taken at the end of the debate. To the best of our knowledge, in the literature about decision making in committees, the debate protocol is always given by an outside designer. The timing can be sequential (Scharfstein and Stein [1990], Ottaviani and Sørensen [2001]) or simultaneous (Levy [2005], Sibert [2003], Visser and Swank [2005], Eliaz, Ray and Razin [2004]); experts can make a single, irreversible decision (Ottaviani and Sørensen [2000,2001],Levy [2005], Eliaz, Ray and Razin [2004]) or make a decision at each stage of a repeated game (Scharfstein and Stein (1990], Sibert [2003]); the final decision can be designed by a voting rule (Levy [2005], Sibert [2003], Visser and Swank [2005], Eliaz, Ray and Razin [2004]), or be the one supported by the common belief at the end of the debate (Ottaviani and Sørensen [2000,2001]). Finally, the communication rule is usually to say $m \in \{m_a, m_b\}$ which indicates that expert supports action a or b (Ottaviani and Sørensen [2001],Levy [2005], Sibert [2003], Visser and Swank [2005], Eliaz, Ray and Razin [2004]). The outside designer of the debate can optimize over the debate protocol (over the order of speech, the communication rule or the way to take the final decision) to improve the debate's efficiency. The optimal debate protocol depends then on the designer's preferences.

In this article, we address the issue of the design of a debate protocol by the members of the committee themselves. We consider what we call unanimous committees, namely committees whose members debate until they reach an agreement on the decision to make. The final decision will therefore be the one experts have reached an agreement about. Unanimous committee are treated in Feddersen and Pesendorfer [1998], or in Bognar and Smith [2004], but in both cases, unanimity is a rule imposed to the experts, who vote strategically in consequence. In our setting, experts learn information all along the debate, until they reach a consensus. In Levy [2005] and Feddersen and Pesendorfer [1998], experts make simultaneously a single, irreversible vote, and the decision made depends on the voting rule, which can be unanimity rule. Sibert [2003] model a repeated game where experts vote simultaneously at each period. In Ottaviani and Sørensen [2000], experts speak in sequence about the desirability of a public decision, according to an exogenous order of speech. They speak only once, and the decision made is the one supported by the common belief at the end of the debate.[1] Yet in decision committees as company boards, consulting teams, etc, even if members of the committee have career concerns, they often argue until they reach an agreement on a committee decision. Before debating, members of the committee may have different opinions about the decision to make. These divergences might come from different experiences, different private information or different interests. During the debate, decision makers try to convince other members that their opinion is the good one, and at the same time are influenced by others' arguments.

A way of tackling this debate issue could be the one of Glazer and Rubinstein [2003]: a speaker attempts to persuade a listener to accept a certain request. The conditions under which the listener should accept the request depend on the values of two aspects known only to the speaker. In order to persuade the listener, the speaker is able to send him a message. The listener can listen the speaker's claim is can check the value of at most one of the two aspects. Glazer and Rubinstein study the persuasion rules that minimize the probability of the listener making a mistake. Another possible approach is the one of the persuasion games literature. We choose to focus on the learning process that can occur during the debate rather than studying a rhetoric debate with argumentation and persuasion. In our setting, the debate is viewed as a learning process which achieves the consensus. Along the debate, decision makers learn information about the world from communicating with

[1] In their model, an informational cascade arises, that is from some date on , every experts send the same message. Note that if those who took their action before the cascade arises were allowed to revise their action, they would also take the cascade action and a consensus would obtain among all experts

others. In that extent, our model is related to the one of Scharfstein and Stein [1990]. As a consequence, it is essential that the communication process in the debate is informative. The actions made or the messages sent must, at least partially, depend on individuals' private information. In that sense, agents follow an action rule, which prescribes what to do as a function of any information situation they might be in. In Bala and Goyal [1998] or Gale and Kariv [2003], the action made by an individual is the one that maximizes his expected payoff given all the information at his disposal. In the classical herding model à la Bikhshandani, Hirshleifer and Welch [1992], Banerjee [1992][2] with two states of the world ω_0 and ω_1 and two actions a_0 and a_1, the action rule is exogenously given. An individual chooses action a_0 if the probability of state ω_0 is greater than one half, given his private information. In Ottaviani and Sørensen [2001,2002], experts send the message that maximizes their expected payoff, which is a reputational objective in their setting. We assume more generally that the message rule followed by decision makers belongs to the wide class of *union-consistency* message rule, which contains all action rules listed above. A way of interpreting the fact that debaters follow a message rule is to assume that they have to prove the arguments they use, and by consequence that they can not cheat in giving messages contrary to their private information. Remark that the messages sent during the debate may be votes for an action: «*We should hire the candidate x*», as well as more abstract messages. This is why we will use indifferently the words messages and actions, experts and decision makers, and speak and act. The essential assumption is the common knowledge of the fact that all experts follow the same message rule. The common knowledge of the message rules is necessary for learning to occur, and the fact that all message rules are the same is necessary for consensus to obtain.

During the debate, experts speak openly in front of other experts according to a given order of speech upon which they have agreed beforehand. The order of speech determines which agents are allowed to speak at each date. For learning to occur, we have to assume that the order of speech is *public*, namely that the messages sent during the debate are heard by all debaters, and that the order of speech is *fair*, namely that all experts have the possibility to speak infinitely many times. We consider any order of speech satisfying these two conditions. In particular we allow for several decision makers to speak at the same time. With the assumption that the order of speech is *fair*, we depart from Ottaviani and Sørensen [2001] and

[2]See Chamley [2004] for a detailed review of herding models.

from standard social learning models where experts make a single, irreversible decision. To this extent, the timing of agents' decisions is similar to the *social network models* of Bala and Goyal [1998] and Gale and Kariv [2003] and to the one in Parikh and Krasucki [1990], with the difference that they consider word of mouth communication whereas we make communication *public* in our setting.

The timing of the model is the following. Before debating, experts agree on a debate protocol, that is on a message rule and an order of speech. During the debate, decision makers learn from the statements made by others and update their private information. Those designed by the order of speech send messages again on the basis of their new information and, under the assumptions we made, the process goes on until it converges to the equality of all messages, a situation we will refer to as consensus. This first consensus result is a modified version of Parikh and Krasucki [1990] and Krasucki [1996].

The question we address in this article is how experts choose collectively a debate protocol? If all debate protocols had the same outcome, we could guess that experts may prefer the debate protocol that achieves consensus faster.[3] However, we give examples where different debate protocols lead to different final decisions. Furthermore, although experts agree on a consensus message at the end of the debate, they may still be in situation of asymmetric information. We also give examples where different protocols do not have the same efficiency in extracting information from the debate. By consequence, if experts have preferences over final decisions, or if experts discuss with others in order to be better informed[4], they may have preferences over debate protocols. In this article, we consider decision makers with career concerns. As a generale rule, a careerist decision maker is an agent whose payoff depend on the action he makes and the state of the world. Implicitly, careerist decision makers are evaluated by some outside agent who knows the state of the world and who has an opinion about what action should be made at each state of the world. As in our setting experts debate until they reach a consensus, we assume that they will be evaluated on the basis of the consensus decision.

We focus on careerist experts, namely decision makers whose utility function depends both on the decision made by the committee and the state of the world. This is the case in company

[3] In Eliaz, Ray and Razin [2004], Gul and Lundholm [1995], Bognar and Smith [?], experts bear a time cost that encourages them to produce a group decision the faster as possible.

[4] See Houy and Ménager [2005]

boards whose members will all benefit from a promotion if the shareholders observe that they made the appropriate decision for the firm. A particular case of careerist decision makers, called *biased* decision makers, have a utility function that depends only on the decision made by the committee. A typical example is the one of a recruitment commission whose members have preferences over the set of applicants for the job. Every decision maker wants his protégé to be hired for other purposes than the protégé's ability for the job.

We show that even if experts have the same payoff functions, they may prefer different debate protocols because of career concerns. It comes from the situation of asymmetric information that may stand before the debate, and that may still remain after. By consequence, how can careerist experts agree on a debate protocol? Our main result states if it may happen that some experts prefer different debate protocols at some state, it is not possible that this disagreement is common knowledge. In other words, experts can not agree to disagree on debate protocols. A corollary of this result is that if experts' preferences about debate protocols are common knowledge, then they have to be the same.

In section 2 we present the formal setting, that consists of the information structure and the debate protocol, and the first consensus result, namely that given any debate protocol, the debate leads to equality of all messages. In section 3 we show that protocol matters in exhibiting examples where different protocols lead to different outcomes. In section 4 we give our main result, namely that experts can not agree to disagree on debate protocols. We discuss the results for different definitions of experts' preferences over protocols. In section 5, we discuss the implications of our results in a timing game. Section 6 concludes.

2 The Model

2.1 The information model

We cast our analysis in Aumann's setting. Let Ω be the set of states of the world. A state of the world $\omega \in \Omega$ describes all the relevant facts for the decision problem of the agents.

We consider a group of $N < \infty$ informed agents (experts or decision makers) indexed by i, whose information about the state of the world is given by a finite partition Π_i of Ω. We note $\Pi_i(\omega)$ the cell of the partition Π_i that contains the state ω. When the state $\omega \in \Omega$ occurs, agent i knows that the true state of the world belongs to $\Pi_i(\omega)$. Hence the partition

Π_i represents the ability of agent i to distinguish between the states of the world. The coarser her partition is, the less precise her information is, in the sense that she distinguishes among fewer states of the world. Furthermore, individual partitions are public and experts share a common prior P over Ω.

By contrast, most of the social learning models are based on a probabilistic information structure. A state of nature $s \in S$ is drawn and agents receive private random signals about s, that is to say signals whose probability distribution depends on the state of nature $s \in S$. In the standard model with two states of nature s_0, s_1, agents receive a signal $\sigma \in \{\sigma_0, \sigma_1\}$ such that $P(\sigma_1 \mid \omega_1) = P(\sigma_0 \mid \omega_0) = \mu$. For the sake of simplicity, suppose that there is only one agent, then there are four states of the world: $(\sigma_1, s_1), (\sigma_1, s_0), (\sigma_0, s_1)$ and (σ_0, s_0). The state of nature may be s_1 and the agent may have received either σ_1 or σ_0, or the state of nature may be s_0 and the agent may have received σ_1 or σ_0. An agent knows the value of the signal she received, but does not know the state of nature. Her private information can be rewritten with the following partition:

$$\Pi = \{(\sigma_1, s_1), (\sigma_1, s_0)\}\{(\sigma_0, s_1), (\sigma_0, s_0)\}$$

Hence a probabilistic information structure can always be expressed as a partitional information structure. A partitional information structure can be seen as a probabilistic information structure of the state of the world with a deterministic signal. Hence there is no loss of generality in considering a partitional instead of a probabilistic structure. The advantage of the partitional structure is that it allows to express in a simple way the concepts of knowledge and common knowledge of events.

An event is a subset of states of the world. In the former example, the event "The agent received signal σ_1" is $\{(\sigma_1, \omega_1,), (\sigma_1, \omega_0)\}$. We say that an agent i endowed with the partition Π_i knows the event $E \subseteq \Omega$ at the state ω if $\Pi_i(\omega) \subseteq E$.

In a group of agents, when everybody knows an event, everybody knows that everybody knows the event, everybody knows that everybody knows that everybody knows the event etc, the event is said to be *common knowledge*. The partitional structure allows to formalize this concept in a simple way. The finest common coarsening of the partitions $\Pi_1, \Pi_2, \ldots, \Pi_N$ is called the meet of these partitions and is defined as the finest partition M such that for all $\omega \in \Omega$, for all $i = 1, \ldots, N$, $\Pi_i(\omega) \subseteq M(\omega)$. Aumann [1976] showed that M is the partition of common knowledge, that is an event E is common knowledge at state ω if and only if $M(\omega) \subseteq E$.

The set of states of the world Ω, the set of experts and the information partitions $(\Pi_i)_i$ determine an *information model*.

2.2 The debate protocol

Before debating, agents have to agree on a debate protocol that will be applied throughout the debate. They have to agree on an order of speech, which determines who gets to speak when, and on a communication rule. It is actually the case in a lot of regulated debates. In the British Parliament or in the French Assemblée Nationale, the debate timing is decided before the debate. In trials, both timing and argumentation are regulated.

2.2.1 The order of speech

The order of speech determines which agents are allowed to speak at each date. We depart from Ottaviani and Sørensen [2001], Levy [2005] and from standard social learning models in allowing experts to revise their decisions rather than making a single, irreversible one. To this extent, the timing of agents' decisions is similar to the *social network models* of Bala and Goyal [1998] and Gale and Kariv [2003]. Our notion of order of speech is also quite similar to the one in Parikh and Krasucki [1990]. They consider word-of-mouth communication involving only two agents at each time. We generalize it by allowing for more than one agent to be senders at a given stage. The main difference with their setting and with Bala and Goyal [1998] and Gale and Kariv [2003] is that we make the assumption that communication is public, namely that all agents are recipient of the messages sent at any time. The reason for this restriction is that we want to be able to consider any kind of message space and any communication rule, whereas Parikh and Krasucki [1990]'s setting impose that the message space is the real line.[5]

Definition 1 *An order of speech is a function $\alpha : \mathbb{N}^* \rightarrow 2^{\{1,\ldots,N\}}$. If $\alpha(t) = S$, then we interpret S as the set of senders of the communication which takes place at time t.*
We note Γ the set of possible orders of speech and we assume that Γ is finite.

[5] Parikh and Krasucki [1990] show that if communication is not public, the message space has to be an interval of the real line for consensus to obtain.

To make sure the debate leads to a consensus, we have to assume that the order of speech is $fair$. We adapt Parikh and Krasucki's definition of a $fair$ order in our setting, but the meaning remains the same: an order is $fair$ if and only if every expert is the sender of the communication infinitely many times.

Definition 2 *An order* α *is fair if and only if* $\forall\ i \in \{1, \dots, N\}$, $\forall\ t \in \mathbb{N}$, $\exists\ t' > t$ *and* $S \subseteq \{1, \dots, N\}$ *such that* $\alpha(t') = S$ *and* $i \in S$.

This assumption is equivalent to the one of connectedness in Bala and Goyal [1998] and Gale and Kariv [2003]: consider the graph where vertices are agents $\{1, \dots, N\}$ and such that there is an edge from i to every $j \in \{1, \dots, N\}$ iff there are infinitely many t such that $i \in \alpha(t)$. The protocol is fair if and only if the graph defined above is connected.

These $fairness$ or *connectedness* assumptions are necessary for uniformity to arise. If some agent is excluded from communication, no learning can occur for this agent.

2.2.2 The communication rule

Along the debate, experts communicate by sending messages, which we assume to be delivered instantaneously, that is at time t, messages are simultaneously sent by every $i \in s(t)$ and heard by every $j \in N$. The messages are the words for communication. In social learning models, where individuals learn from observing others' actions, individual actions play the role of words, in the sense that they are the means of communication between agents. Hence a message $m \in \mathcal{M}$ can be an action, an advice, a belief and so on.

For learning to arise, individual messages have to be informative, namely messages have to reflect, at least partially, the private information of each acting individual. Two assumptions are necessary for that. First, the messages sent by individuals must depend on their private information. Second, the communication rule followed by individuals must be perfectly known by others.

We define the communication rule as follows. The messages sent by the experts are the values of some function $f : 2^\Omega \to \mathcal{M}$, from the set of subsets of Ω into some set of messages $\mathcal{M}$. The messages sent depend on private information in the following way. When an agent knows that the true state of the world lies in $X \subseteq \Omega$ and knows that it does not lie in $\Omega \setminus X$, she sends the message $f(X)$. In other words, $f(X)$ is the message sent by an agent with private information X.

Consider a set of states of the world $\Omega = \{1, 2, 3, 4, 5\}$ and an agent endowed with the following partition: $\Pi = \{1, 2\}\{3, 4\}\{5\}$. Suppose that the set of messages is $\mathcal{M} = \{m_0, m_1\}$ and that the communication rule is to say m_0 if the states 2 or 3 are possible, and to say m_1 otherwise. Formally, the communication rule is f such that:

$$f(X) = \begin{cases} m_0 & \text{if} X \cap \{2,3\} \neq \emptyset \\ m_1 & \text{otherwise} \end{cases}$$

If state 1 or 2 occurs, the agent knows that the state of the world belongs to $\{1, 2\}$. As $\{2, 3\} \cap \{1, 2\} = \{2\}$, he sends the message $f(\{1, 2\}) = m_0$. He also sends the message m_0 at states 3 or 4, but sends m_1 if state 5 occurs.

In most of social learning models[6], agents have a utility function $U : \mathcal{M} \times \Omega \to \mathbb{R}$ that depends both of action chosen and the state of the world. In reputational cheap talk models of Ottaviani and Sørensen [2000,2001] or Levy [2005], U is a reputational objective. Then the decision rule followed by individuals is to choose the action that maximizes their expected utility with respect to their private information. In our setting, this decision rule would be written:

$$f(X) \in \text{argmax}_{m \in \mathcal{M}} E[U(m, .) \mid X] \tag{1}$$

The fact that agents follow a communication rule f implies that they cannot cheat by making an action that is contrary to their private information. A message reveals partially but truthfully someone's private information. This communication setting can be seen as a debate where agents have to prove any of their arguments.

To make sure that experts reach a consensus at the end of the debate, we also have to assume that the communication rule f satisfies a consistency property.[7]

Definition 3 *f is union-consistent if for all X, Y such that $X \cap Y = \emptyset$, $f(X) = f(Y) \Rightarrow f(X \cup Y) = f(X) = f(Y)$. We note Δ the set of communication rules.*

Most of message rules used in social learning literature are union-consistent. This is for instance the case when agents communicate their posterior beliefs for a particular event E ($f(X) = P(E \mid X)$). This is also the case when the communication rule is defined as in (1).

[6]BHW [1992,1998], Banerjee [1992], Bala and Goyal [1998], Gale and Kariv [2003]

[7]Due to Cave [1983]

An order of speech α and a message rule f define a debate protocol (α, f). The set of debate protocols is $\Gamma \times \Delta$. In the sequel, we will consider only fair orders of speech and union-consistent communication rules.

2.3 A first consensus result

We now describe how information is aggregated during the debate. At a given date t, agents selected by the order α send a message heard by all other agents. Then everybody infers the set of states of the world that are compatible with the messages sent and updates her private information accordingly. Let us describe the way agents revise their private information with the following example.

Example 1 *The set of states of the world is* $\Omega = \{1, 2, 3, 4, 5\}$. *There are two agents endowed with the partitions* $\Pi_1 = \{1, 2\}\{3, 4\}\{5\}$ *and* $\Pi_2 = \{1, 2, 5\}\{3, 4\}$. *The communication rule is* f *such that:*

$$f(X) = \begin{cases} m_0 & \text{if} X \cap \{2, 3\} \neq \emptyset \\ m_1 & \text{otherwise} \end{cases}$$

We saw that at states 1,2,3 and 4, agent 1 says m_0*, and says* m_1 *at state 5. Suppose that agent 1 is the first to speak. As agent 2 knows the partition of agent 1, he can infer from the message* m_0 *that agent 1 thinks the state of the world is in* $\{1, 2\}$ *or in* $\{3, 4\}$*, but is not 5, and he can infer from message* m_1 *that agent 1 knows the state of the world is 5. Therefore, agent 1's message allows agent 2 to distinguish between states 1,2,3,4 on one hand and state 5 on another hand. With his initial private information* Π_2*, agent 2 could already distinguish between 1,2,5 and 3,4. After hearing agent 1's message, his partition becomes* $\Pi_2' = \{1, 2\}\{5\}\{3, 4\}$*. He is now able to distinguish state 5 from states 1 and 2.*

The formal description of the way information is aggregated during the debate is the following. Given an information model $\langle \Omega, (\Pi_i)_i \rangle$, a debate protocol (α, f), and a state of the world ω, we define the process $t \to H_t^{\alpha,f}(\omega) \subseteq 2^\Omega$, where $H_t^{\alpha,f}(\omega)$ is the set of states of the world that all agents are able to infer from hearing the messages sent at date $t-1$. Each agent i combines the public information $H_t^{\alpha,f}(\omega)$ with his private information $\Pi_i(\omega)$. We note $\Pi_{i,t}^{\alpha,f}(\omega)$ i's updated private information at date t. The dynamic of the process is the following.

- $H_0^{\alpha,f}(\omega) = M(\omega)$, and $\forall\, i,\ \Pi_{i,0}^{\alpha,f}(\omega) = \Pi_i(\omega) \cap M(\omega)$ and $\forall\, t \geq 0,\ \forall\, i$

- $H_{t+1}^{\alpha,f}(\omega) = H_t^{\alpha,f}(\omega) \cap \{\omega' \in \Omega \mid \forall\, j \in \alpha(t),\ f(\Pi_{j,t}^{\alpha,f}(\omega')) = f(\Pi_{j,t}^{\alpha,f}(\omega))\}$
- $\Pi_{i,t+1}^{\alpha,f}(\omega) = \Pi_{i,t}^{\alpha,f}(\omega) \cap H_{t+1}^{\alpha,f}(\omega)$

For all t, $H_t^{\alpha,f}$ defines a partition of Ω. The next result states it is the partition of common knowledge at date t if the debate protocol is (α, f).

Proposition 1 *For all t, for all (α, f), $H_t^{\alpha,f}$ is the partition of common knowledge at date t.*

The next theorem is a different version of Parikh and Krasucki [1990] and Parikh [1996]. We show that the process defined above converges to an equilibrium characterized by equality of all messages, in a case where the message rule is union-consistent and the communication is public and fair.

Theorem 1 *There exists T such that for all $t \geq T$, $H_{t+1}^{\alpha,f}(\omega) = H_t^{\alpha,f}(\omega)$ for all ω. Furthermore, if α is fair, and if f is union-consistent, then for all i, for all ω, $f(\Pi_i^{\alpha,f}(\omega, T)) = f(H_T^{\alpha,f}(\omega))$.*

The sketch of the proof is the following. We first show that the process $H_t^{\alpha,f}(\omega)$ converges to a steady equilibrium in a finite number of steps. At equilibrium of the process, agents do not infer information from other's statement anymore. By definition, we have at equilibrium $H_t^{\alpha,f}(\omega) \subseteq \{\omega' \mid f(\Pi_{i,t}^{\alpha,f}(\omega')) = f(\Pi_{i,t}^{\alpha,f}(\omega))\ \forall ß\}$. By proposition 1, it means that individual messages have become common belief among the agents. By Cave [1983], it implies that individual messages are the same.

In the sequel, $\Pi_i^{\alpha,f}(\omega)$ will simply denote the set of possible states for agent i at equilibrium of the process if the true state is ω and the debate protocol is (α, f), and $\Pi_i^{\alpha,f}$ i's information partition at equilibrium. Furthermore, $H^{\alpha,f}$ denotes the partition of common knowledge at equilibrium if the protocol is (α, f). We will call $f(H^{\alpha,f}(\omega))$ the *consensus message* at state ω if the debate protocol is (α, f).

3 Protocol matters

The previous result states that for any protocol chosen by the experts, the debate leads to a consensus on a particular message (decision, recommendation..). We now show that the outcome of the debate depends on the chosen protocol, in terms of consensus message as well as of equilibrium partitions.

3.1 Order matters

Proposition 2 *There exist an information model* $\langle \Omega, (\Pi_i)_i, \rangle$*, a message rule* f *and two protocols* α, β *for which there is* ω *such that* $f(H^{\alpha,f}(\omega)) \neq f(H^{\beta,f}(\omega))$*, and* $\Pi_i^{\alpha,f} \neq \Pi_i^{\beta,f}$ *for some* i*.*

Example 2 *Consider* $\Omega = \{1, 2, 3, 4, 5\}$ *and three experts with career concerns endowed with a uniform prior* P *on* Ω *and with the following partitions:*

$$\Pi_1 = \{1, 2\}\{3, 4\}\{5\}$$
$$\Pi_2 = \{1, 2, 3\}\{4, 5\}$$
$$\Pi_3 = \{1, 3, 4\}\{2, 5\}$$

An uninformed decision maker has to take an action $m \in \{m_0, m_1\}$ *and asks the three experts for advice. According to the decision maker's payoff, she should take action* m_1 *at states 2 or 3, and action* m_0 *otherwise. As experts are careerist, they want the decision maker to think they possess good information, so their optimal strategy is to recommend* m_1 *if the probability of event* $\{2, 3\}$ *is greater than one half, and to recommend* m_0 *otherwise. Hence experts follow a message rule that is biased toward* m_1 *and that is defined by:*

$$f(E) = \begin{cases} m_1 \Leftrightarrow P(\{2, 3\} \mid E) \geq 1/2 \\ m_0 \text{ otherwise} \end{cases}$$

Experts speak in turn, following a round-robin protocol. That is to say expert 1 speaks after expert 3, who speaks after expert 2, who speaks after expert 1. Before debating, the three experts must only decide who of them is going to speak first. In the sequel, we put in subscript of each cell of individual partitions the associated recommendation.

Suppose that expert 1 speaks first (which corresponds to the order α*).*

- $t = 1$*: At the first stage, expert 1 announces what action she recommends. If she supports* m_1*, the other experts infer that expert 1 thinks the state of the world belongs to* $\{1, 2, 3, 4\}$*. If she recommends* m_0*, everybody infers that the state of the world is 5. Then after hearing expert 1's recommendation, updated individual partitions are:*

$$\begin{cases} \Pi^{\alpha}_{1,1} = \{1, 2\}_{m_1}\{3, 4\}_{m_1}\{5\}_{m_0} \\ \Pi^{\alpha}_{2,1} = \{1, 2, 3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\alpha}_{3,1} = \{1, 3, 4\}_{m_0}\{2\}_{m_1}\{5\}_{m_0} \end{cases}$$

- $t = 2$*: As initial partitions are public, updated partitions are public too. Then if expert 2 support* m_1*, everybody infers that the state of the world belongs to* $\{1, 2, 3\}$*, and belongs to* $\{4, 5\}$ *if she supports* m_0*. Hence after expert 2's recommendation, individual partitions become:*

$$\begin{cases} \Pi^{\alpha}_{1,2} = \{1, 2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\alpha}_{2,2} = \{1, 2, 3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\alpha}_{3,2} = \{1, 3\}_{m_1}\{4\}_{m_0}\{2\}_{m_1}\{5\}_{m_0} \end{cases}$$

- $t \geq 3$*: Expert 3's recommendation does not tell anything new to other experts. It allows to distinguish between states 1,2,3 and 4,5, something all experts were already able to do after stage 2. Then the consensus takes two steps to obtain, and the equilibrium partitions with order* α *are:*

$$\begin{cases} \Pi^{\alpha}_{1,3} = \{1, 2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\alpha}_{2,3} = \{1, 2, 3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\alpha}_{3,3} = \{1, 3\}_{m_1}\{4\}_{m_0}\{2\}_{m_1}\{5\}_{m_0} \end{cases}$$

Suppose now that expert 3 speaks first (which corresponds to order β*).*

- $t = 1$*: After hearing expert 3's recommendation, everybody can distinguish between states 1,2,3 and states 4,5. Then individual partitions become:*

$$\begin{cases} \Pi^{\beta}_{1,1} = \{1\}_{m_0}\{2\}_{m_1}\{3, 4\}_{m_1}\{5\}_{m_0} \\ \Pi^{\beta}_{2,1} = \{1, 2, 3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\beta}_{3,1} = \{1, 3\}_{m_1}\{4\}_{m_0}\{2, 5\}_{m_1} \end{cases}$$

- $t = 2$*: According to her updated partition, expert 1's recommendation allows to distinguish between states 1,5 and states 2,3,4. Then after hearing 1's recommendation, updated partitions are:*

$$\begin{cases} \Pi^{\beta}_{1,2} = \{1\}_{m_0}\{2\}_{m_1}\{3, 4\}_{m_1}\{5\}_{m_0} \\ \Pi^{\beta}_{2,2} = \{1\}_{m_0}\{3\}_{m_1}\{2\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\beta}_{3,2} = \{1\}_{m_0}\{3, 4\}_{m_1}\{2\}_{m_1}\{5\}_{m_0} \end{cases}$$

- $t = 3$*: At stage 3, expert 2's recommendation allows to distinguish between states 1,4,5* *and states 2,3. Then experts 1 and 3 update their partition which become:*

$$\begin{cases} \Pi^{\beta}_{1,3} = \{1\}_{m_0}\{2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\beta}_{2,3} = \{1\}_{m_0}\{2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\beta}_{3,3} = \{1\}_{m_0}\{2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \end{cases}$$

- $t \geq 4$*: From stage 4 on, no expert can infer more information from communication. Then the consensus takes three steps to obtain, and equilibrium partitions are:*

$$\begin{cases} \Pi^{\beta}_{1,4} = \{1\}_{m_0}\{2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\beta}_{2,4} = \{1\}_{m_0}\{2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \\ \Pi^{\beta}_{3,4} = \{1\}_{m_0}\{2\}_{m_1}\{3\}_{m_1}\{4\}_{m_0}\{5\}_{m_0} \end{cases}$$

This example exhibits several features that stress the relevancy of the ordering issue in communication. First, it shows that different orders of speech lead to different consensus messages. At state 1, the committee will recommend the action m_1 if expert 1 speaks first, and the action m_0 if expert 3 speaks first. Yet from the point of view of the decision maker, action m_0 is the right action to make at state 1. In this example, the experts and the decision maker have the same preferences, so both prefer expert 3 to speak first, so that no mistake is made at state 1. Second, individual equilibrium partitions are finer when expert 3 speaks first, which means that the debate extracts more information when expert 3 speaks first than when expert 1 does. Ottaviani and Sørensen [2001]'s model exhibits the same feature, namely that different orders of speech may not have the same informational efficiency. They show that modifying the order of speech may decrease the incidence of herding.

3.2 Message rule matters

Suppose that experts communicate simultaneously. The next example shows that there exist an information model $\langle \Omega, (\Pi_i)_i \rangle$ and two message rules f, g for which there is an ω such that $f(H^{\alpha,f}(\omega)) \neq g(H^{\alpha,g}(\omega))$, and $\Pi_i^{\alpha,f} \neq \Pi_i^{\alpha,g}$ for some i.

Example 3 *Consider* $\Omega = \{1, 2, 3, 4\}$*, and two experts endowed with the following partitions:*

$$\Pi_1 = \{1, 3\}\{2, 4\}$$
$$\Pi_2 = \{1, 2\}\{3, 4\}$$

- *Consider the message rule* $f : 2^\Omega \to \{a, b\}$ *defined by:*

$$f(1) = f(2) = f(1,2) = a$$
$$f(E) = b \; \forall \; E \subseteq \Omega, \; E \neq \{1\}, \{2\}, \{1,2\}$$

We rewrite the initial partitions with associated messages in subscript.

$$\Pi_1 = \{1,3\}_b \{2,4\}_b$$
$$\Pi_2 = \{1,2\}_a \{3,4\}_b$$

Simultaneous communication leads to the equilibrium partitions:

$$\Pi_1^f = \{1\}_a \{3\}_b \{2\}_a \{4\}_b$$
$$\Pi_2^f = \{1,2\}_a \{3,4\}_b$$

- *Consider now the message rule* $g : 2^\Omega \to \{a, b\}$ *defined by:*

$$g(1) = g(3) = g(1,3) = a$$
$$g(E) = b \; \forall \; E \subseteq \Omega, \; E \neq \{1\}, \{3\}, \{1,3\}$$

We rewrite the initial partitions with associated messages in subscript.

$$\Pi_1 = \{1,3\}_a \{2,4\}_b$$
$$\Pi_2 = \{1,2\}_b \{3,4\}_b$$

Simultaneous communication leads to the equilibrium partitions:

$$\Pi_1^g = \{1,3\}_a \{2,4\}_b$$
$$\Pi_2^g = \{1\}_a \{2\}_b \{3\}_a \{4\}_b$$

This example first shows that at state 2, communication leads to consensus on message a with rule f, and on message b with rule g. The fact that the consensus message may change with the communication rule is not surprising, as the communication rule is what associates messages to private information. What is more striking is that different communication rules may have different informational efficiency, in the sense that debating with two different rules may lead to different equilibrium partitions. In this example, Π_1^f is finer than Π_1^g and Π_2^g is finer than Π_2^f. If experts communicated in order to be better informed, as in Houy and Ménager [2005], expert 1 would prefer the communication rule f whereas expert 2 would prefer g.

4 Agreeing to disagree on debate protocols

In the previous section, we have shown that different debate rules result in different outcomes, in terms of consensus messages as well as of information revealed at equilibrium. Given a group of experts, it matters in which order they speak and with which communication rule. It is therefore relevant to ask the question of wether career concerns of experts can be a source of disagreement about the debate protocol, and if so, to what extent. In other words, we look at wether a disagreement on the debate protocol can be common knowledge among experts, *i.e.* wether experts can agree to disagree on debate protocols.

We consider careerist decision makers, that we define as agents preoccupied by how their decisions are related to the state of the world. They can be professional experts who are concerned with the public perception of the quality of their information, CEOs motivated by their reputation as good leaders, judges who want to be perceived as making the right sentences etc. The underlying assumption of this concept of career concerns is that there is an outside evaluator who knows the state of the world, and who has an opinion about what action should be made at this state of the world. We could assume that the evaluator is *imperfectly* informed about the state of the world. In this case, experts would take into account the beliefs of the evaluator in their expected payoff, and our results would be the same. As we consider unanimous committee where experts debate until they reach an agreement on a particular action, we assume that each member of the committee is evaluated on the basis of the consensus action. Therefore, careerist decision makers have a utility function $U : \mathcal{M} \times \Omega \to \mathbb{R}$ which depends both on the message sent at the end of the debate and on the state of the world. A particular case of careerist decision makers are *biased* decision makers. They are interested only in the final decision, and not in how it is related to the state of the world. It can be the case in some recruitment committees, whose members have to choose an applicant to hire. Some members of the committee may have a protégé among the applicants that they want to hire independently of the protégé's ability for the job. In this case, the underlying assumption is that experts are judged by an outside evaluator who is also only concerned by the decision made. Therefore, biased decision makers have a utility function on $U : \mathcal{M} \to \mathbb{R}$ which depends only on the final decision taken by the committee. Biased decision makers are particular careerist decision makers with utility function $U(m, \omega) = U(m)\ \forall\, m,\ \forall\, \omega$.

The difference between biased and opportunist experts can be illustrated as follows. Consider for instance a person on trial who is either innocent (state ω_0) or guilty (state ω_1). This person can be either acquitted (action a_0) or convicted (action a_1). An opportunist prosecutor who wants to have the reputation of being a good instructor wants to convict the defendant if she is guilty and relax her if she is innocent. In this case, his utility function is such that $U(a_1, \omega_1) = U(a_0, \omega_0) > U(a_0, \omega_1) = U(a_1, \omega_0)$. On the contrary, a biased prosecutor may need a conviction for promotion purpose. In this case, he may want to convict the defendant regardless of wether she is guilty or not, *i.e* his utility function is defined on $\{a_0, a_1\}$ and is such that $U(a_1) > U(a_0)$. The reason we present the particular case of biased decision makers is that some results are slightly different for both cases according to the definition of preferences over debate protocols we use.

We first consider the case of homogenous experts, namely experts endowed with the same preferences. We show that it can not be common knowledge that two experts have different preferences about two debate protocols. This result has a corollary that state that if individual preferences over the debate protocols (even if incomplete) are common knowledge, then they have to be the same. In other words, experts can not agree to disagree on debate protocols. We show that these results do not hold in the case of heterogenous experts, except in the very particular situation where experts are biased and the set of possible actions contains two elements.

4.1 Defining preferences over debate protocols

Let us consider a careerist decision maker i endowed with a utility function $U_i : \mathcal{M} \times \Omega \to \mathbb{R}$. For the particular case where i is a biased decision maker, we will have $U_i(m, \omega) = U_i(m)$ for all ω. We assume that experts share a common prior P over Ω, and expert i's private information is represented by Π_i.

As individual partitions $(\Pi_i)_i$ are public, given any debate protocol (α, f), equilibrium partitions $(\Pi_i^{\alpha,f})_i$ are public too. And so is the function which associates the consensus message to each state of the world $\omega \mapsto f(H^{\alpha,f}(\omega))$. Therefore, everyone knows that at the end of the day, agent i's expected utility at state ω given the debate protocol (α, f) is:

$$E(U_i(f(H^{\alpha,f}(\omega)), .) \mid \Pi_i^{\alpha,f}(\omega)) := \frac{1}{P(\Pi_i^{\alpha,f}(\omega))} \sum_{\omega' \in \Pi_i^{\alpha,f}(\omega)} P(\omega') U_i(f(H^{\alpha,f}(\omega)), \omega')$$

However, before the debate takes place, i only knows that the state of the world belongs to

$\Pi_i(\omega)$ at state ω, so she does not know that her expected utility given the order α will be $E(U_i(f(H^{\alpha,f}(\omega)),.) \mid \Pi_i^\alpha(\omega))$. The following definition represent a usual way of coping with the *ex ante* uncertainty about i's future expected utility: i computes her expectation at state ω of her expected utility at equilibrium, and compares it according to the different debate protocols. We have:

$$E[E(U(f(H^{\alpha,f}(\textbf{.})),.) \mid \Pi_i^{\alpha,f}(\textbf{.})) \mid \Pi_i(\omega)] := \frac{1}{P(\Pi_i(\omega))} \sum_{\omega' \in \Pi_i(\omega)} P(\omega')E(U(f(H^{\alpha,f}(\omega')),.) \mid \Pi_i^\alpha(\omega')) \tag{2}$$

Definition 4 *Consider an information model $\langle \Omega, (\Pi_i)_i \rangle$, a prior belief P over Ω, and (α, f) and (β, g) two debate protocols. Agents compute their expected utility with respect to the prior P.*

(i) We say that i prefers (α, f) to (β, g) at state ω, which is denoted $(\alpha, f) \succ_i^\omega (\beta, g)$, iff:

$$E[E(U_i(f(H^{\alpha,f}(\textbf{.})),.) \mid \Pi_i^{\alpha,f}(\textbf{.})) \mid \Pi_i(\omega)] > E[E(U_i(g(H^{\beta,g}(\textbf{.})),.) \mid \Pi_i^{\beta,g}(\textbf{.})) \mid \Pi_i(\omega)]$$

(ii) We say that i is indifferent between (α, f) and (β, g) at state ω, which is denoted $(\alpha, f) \sim_i^\omega (\beta, g)$, iff:

$$E[E(U_i(f(H^{\alpha,f}(\textbf{.})),.) \mid \Pi_i^{\alpha,f}(\textbf{.})) \mid \Pi_i(\omega)] = E[E(U_i(g(H^{\beta,g}(\textbf{.})),.) \mid \Pi_i^{\beta,g}(\textbf{.})) \mid \Pi_i(\omega)]$$

According to this definition, $\succeq_i^\omega$ is a complete, state dependent preference. Given any state ω, an expert is always able to compare two debate protocols.

4.2 Homogenous experts

We first consider the case where members of the committee have all the same payoff function. Careerist decision makers are all motivated by how the consensus message is related to the state of the world in the same way, or biased decision makers have all the same preferences on $\mathcal{M}$. Implicitly, this setting implies that decision makers are all evaluated by the same outside agent. This is the case in Scharfstein and Stein [1990], Ottaviani and Sørensen [2001], in Levy [2005] etc. As decision makers have all the same objective, the first intuition would be that they can not disagree on the protocol. Imagine a recruitment commission whose members have all the same protégé. They all want the protocol to lead them to hire their protégé. However, even in that case, it may happen that they disagree on the protocol to choose. This surprising disagreement comes from the asymmetric information experts bear before the debate, and that possibly persists after the debate.

The following example shows homogenous biased decision makers who happen to disagree on the order of speech. This example is even stronger in the particular case of homogenous biased decision makers as their ex post expected utility at a given state are all the same.

Example 4 *Let the set of states of the world be* $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ *and two recruiters endowed with a uniform prior on* Ω *and with the partitions:*

$$\Pi_1 = \{1, 2, 3\}\{4, 5, 6\}\{7, 8, 9\}$$
$$\Pi_2 = \{1, 2, 5, 8, 9\}\{3, 4\}\{6, 7\}$$

They have to hire applicant a *or applicant* b*. They communicate with the message rule* $f : 2^\Omega \to \{a, b\}$ *such that:*[8]

$f(\{1, 2\}) = f(\{4\}) = f(\{5\}) = f(\{6\}) = f(\{1, 2, 3\}) = f(\{4, 5, 6\}) = f(\{1, 2, 5, 8, 9\}) = b$

$f(\{3\}) = f(\{4, 6\}) = f(\{7\}) = f(\{8, 9\}) = f(\{3, 4\}) = f(\{6, 7\}) = f(\{1, 2, 3\}) = f(\{7, 8, 9\}) = f(\{1, 2, 8, 9\}) = a$

If 1 speaks first, the equilibrium partitions will be:

$$\Pi_1 = \{1, 2, 3\}_a\{4, 5, 6\}_b\{7, 8, 9\}_a$$
$$\Pi_2 = \{1, 2, 8, 9\}_a\{3\}_a\{4\}_b\{5\}_b\{6\}_b\{7\}_a$$

If 2 speaks first, the equilibrium partitions will be:

$$\Pi_1 = \{1, 2\}_b\{3\}_a\{4, 6\}_a\{5\}_b\{7\}_a\{8, 9\}_a$$
$$\Pi_2 = \{1, 2, 5\}_b\{8, 9\}_a\{3, 4\}_a\{6, 7\}_a$$

Suppose that the two recruiters are biased in favor of a, say their utility function is such that $U(a) = 1$ and $U(b) = 0$.

- At state 3, recruiter 1 knows that if he speaks first, he will end up with private information $\{1, 2, 3\}$ and will hire candidate a. Hence at state 3 his ex ante expected utility if he speaks first is $E[E(U(a) \mid \{1, 2, 3\}) \mid \{1, 2, 3\}] = 1$. If he speaks second, he knows that at the end of the day, his utility will be 0 at state 1 and 2, and 1 at state 3. Hence his expected utility if he speaks second is $\dfrac{P(\{1, 2\}) * 0 + P(\{3\}) * 1}{P(\{1, 2, 3\})} = 1/3$. Then recruiter 1 prefers to speak first at state 1,2 and 3.

[8] The rest of the message rule is derived by union-consistency or is irrelevant.

- At state 3, recruiter 2 knows that if he speaks first, he will end up with private information $\{3,4\}$ and will hire the applicant a. Hence his ex ante expected utility is $E[E(U(a) \mid \{3,4\}) \mid \{3,4\}] = 1$. If he speaks second, he knows that at the end of the day, his utility will be 1 at state 3 and 0 at state 4. Hence his expected utility if he speaks second is $\dfrac{P(\{3\}) * 1 + P(\{4\}) * 0}{P(\{3,4\})} = 1/2$. Then recruiter 2 prefers to speak first at state 3 and 4.

Despite both recruiters prefer to hire a than b, and despite they have the same utility ex post, they both prefer to speak first because at state 3 of the ex ante situation of asymmetric information. Similarly, they both want to speak second at state 5.

This example shows that even homogenous decision makers might prefer different debate protocols. The two recruiters of the previous example, who both want to speak in second at state 5, may stand by, one in front of the other, waiting for the other to speak first. However, the recruiters are both able to make inferences from observing that the other does not speak. When 1 observes that 2 does not speak, he knows that 2 does not want to speak first, and then he understand that 2 thinks the state of the world is in $\{1,2,5,8,9\}$, otherwise he would have spoken first. As 1 does not speak either, 2 understands that 1 thinks the state of the world is in $\{4,5,6\}$, otherwise he would have spoken first. They update their private information and both conclude that the state of the world is 5. From that time on, experts 1 and 2 become indifferent between the two orders of speech as they both bring them the same expected utility.

If recruiters 1 and 2 do not speak, they both understand that no one wants to be the first to speak. The fact that they both want to speak second is *mutual knowledge*. As both of them remain silent, they both understand that the other know that they know that none of them wants to speak first, etc...As the time goes by with no one speaking, the fact that they both want to speak second becomes common knowledge. The example described above gives the intuition that common knowledge of a disagreement about the protocol makes people indifferent between protocols.

The next result states that it can not be common knowledge that two careerist decision makers have opposite preferences about two debate protocols. In other words, given two protocols (α, f) and (β, g), it can not be common knowledge that some agent strictly prefers (α, f) to (β, g) and another one prefers (β, g) to (α, f).

Theorem 2 *Consider two homogenous careerist experts i, j and two debate protocols $(\alpha, f) \neq (\beta, g)$. If it is common knowledge at some state ω that $(\alpha, f) \succeq_i^\omega (\beta, g)$ and $(\beta, g) \succeq_j^\omega (\alpha, f)$, then it is common knowledge at ω that $(\alpha, f) \sim_i^\omega (\beta, g)$ and $(\alpha, f) \sim_j^\omega (\beta, g)$.*

According to the definition we use, experts' preferences over protocols are complete. Therefore, given any state ω, every expert is able to rank the set of debate protocols. A corollary of the above theorem is that if agents' ranking over protocols are common knowledge, then they have to be the same. In other words, it can not be common knowledge that two opportunist decision makers have different preferences on the set of debate protocols.

Corollary 1 *Consider an information model $\langle \Omega, (\Pi_i)_i \rangle$ with opportunist experts. For all state ω, if $\succeq_i^\omega$ is common knowledge at ω for all i, then $\succeq_i^\omega = \succeq_j^\omega$ for all i, j.*

4.3 Heterogenous experts

We now turn to the case of heterogenous careerist experts, who do not have the same objective functions. Implicitly, they are judged by different evaluators, who do not have the same opinion about what action should be taken at each state of the world, or, for biased decision makers, by evaluators who do not have the same preferences over final decisions. It is for instance the case when middlemen are put in charge of negotiating something in somebody's place, or when lawyers in trial want both that their client wins the affair.

The following example shows that, even with $\mid \mathcal{M} \mid = 2$, it can be common knowledge that heterogenous biased decision makers disagree about the debate protocol (and by consequence careerist decision makers). In this example we assume that the message rule is the same in the two protocols, but we can easily find examples where it is common knowledge that heterogenous decision makers prefer different message rules.

Example 5 *Consider $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and two agents endowed with the following partitions:*

$$\Pi_1 = \{1, 2, 3\}\{4, 5, 6\}\{7, 8, 9\}$$

$$\Pi_2 = \{1, 3, 4\}\{2, 5, 8\}\{6, 7, 9\}$$

Consider the decision rule $f : 2^\Omega \to \{a, b\}$ defined by:[9]

[9] For other $E \subseteq \Omega$, $f(E)$ is given by union-consistency or is irrelevant.

$f(\{2\}) = f(\{4\}) = f(\{6\}) = f(\{8\}) = f(\{2,5\}) = f(\{1,3,4\}) = f(\{1,2,3\}) = f(\{4,5,6\}) = a$

$f(\{1,3\}) = f(\{5,6\}) = f(\{5,8\}) = f(\{7,9\}) = f(\{7,8,9\}) = f(\{6,7,9\}) = f(\{2,5,8\}) = b$

If expert 1 speaks first (order α), equilibrium partitions are:

$$\Pi_1 = \{1,2,3\}_a\{4,5,6\}_a\{8\}_a\{7,9\}_b$$
$$\Pi_2 = \{1,3,4\}_a\{2,5\}_a\{8\}_a\{6\}_a\{7,9\}_b$$

If expert 2 speaks first (order β), equilibrium partitions are:

$$\Pi_1 = \{1,3\}_b\{2\}_a\{4\}_a\{5,6\}_b\{7,8,9\}_b$$
$$\Pi_2 = \{1,3\}_b\{4\}_a\{2\}_a\{5,8\}_b\{6,7,9\}_b$$

Suppose that expert 1's utility function is such that $U_1(a) = 1$, $U_1(b) = 0$ and expert 2's utility function is $U_2(a) = 0$ and $U_2(b) = 1$.

- At state 1, 2, 3, 4, 5 and 6, expert 1's expected utility if he speaks first is 1 and is 1/3 if he speaks second. At state 7,8 and 9, his expected utility if he speaks first is 1/3 and is 0 otherwise. Then expert 1 prefers to speak first at every state of the world.

- At states 1,2,3,4,5 and 8, expert 2's expected utility is 2/3 if he speaks first, and is 0 if he speaks second. At states 6, 7 and 9, his expected utility is 1 if he speaks first and is 2/3 otherwise. Hence expert 2 prefers to speak first at every state of the world.

4.4 A special case

In this section, we show that the results of the previous subsection are slightly different in the particular case of biased expert if we define preferences over protocols in an alternative way. Let us call the former definition of preferences definition A, and the present one definition B. Consider now that an expert prefers a debate protocol to another one at some state ω if her expected utility with this protocol is strictly greater than with the other one at every state that she judges possible *ex ante* at ω. This definition of preferences is "stronger" than the one we use before, in the sense that if an expert prefers some protocol to some other one in the sense of definition B, then he also does in the sense of definition A.

Definition 5 *Consider an information model $\langle \Omega, (\Pi_i)_i \rangle$, and (α, f) and (β, g) two debate protocols.*

(i) We say that i prefers (α, f) to (β, g) at state ω, which is denoted $(\alpha, f) \succ_i^{\omega} (\beta, g)$, iff:

$$\forall\ \omega' \in \Pi_i(\omega),\ E[U_i(f(H^{\alpha,f}(\omega')), .) \mid \Pi_i^{\alpha,f}(\omega')] > E[U_i(g(H^{\beta,g}(\omega')), .) \mid \Pi_i^{\beta,g}(\omega')]$$

(ii) We say that i is indifferent between (α, f) and (β, g) at state ω, which is denoted $(\alpha, f) \sim_i^{\omega} (\beta, g)$, iff:

$$\forall\ \omega' \in \Pi_i(\omega),\ U_i(f(H^{\alpha,f}(\omega'))) = U_i(g(H^{\beta,g}(\omega')))$$

We can notice that these preferences are not complete. It may happen that an expert is not able to compare two protocols.

The first difference with previous results is that in the case of homogenous, biased experts, the ex ante asymmetry of information is not sufficient to induce a disagreement about the debate protocol, whereas it would have been the case with definition A.

Theorem 3 *Let $\langle \Omega, (\Pi_i)_i \rangle$ be an information model and suppose that preferences are defined as in definition B. For all debate protocols (α, f), (β, g), for all i, j biased homogenous decision makers, there is no $\omega \in \Omega$ such that*

(i) $(\alpha, f) \succ_i^{\omega} (\beta, g)$ and $(\beta, g) \succeq_j^{\omega} (\alpha, f)$, or

(ii) $(\alpha, f) \succ_i^{\omega} (\beta, g)$ and $(\beta, g) \sim_j^{\omega} (\alpha, f)$, or

(iii) $(\alpha, f) \succ_i^{\omega} (\beta, g)$ and j can not compare (β, g) and (α, f) at ω.

This result implies clearly that it can not be common knowledge that two biased experts have different preferences, even incomplete, over debate protocols.

The second difference with previous results concerns the case of heterogenous experts. Suppose that the message rule is given, and that the set of messages contains only two elements. To the best of our knowledge, these two assumptions are made in most of papers on decision making in committees. Consider a recruitment commission whose members have different protégés among a set of applicant $\{a, b\}$. The debate inside the commission is regulated, and the only free parameter which has an effect on the final decision is the order of speech in the commission. We showed in the previous section that with definition A, heterogenous biased decision makers may well agree to disagree on debate protocols. We show that with definition B, it can not even be common knowledge that some biased expert prefers an order of speech to another one.

Theorem 4 *Consider two debate protocols (α, f) and (β, f), and suppose that $\mid \mathcal{M} \mid= 2$. Suppose that preferences over protocols are defined as in B. If it is common knowledge that for some biased expert i, $(\alpha, f) \succeq_i (\beta, f)$, then $(\alpha, f) \sim_i (\beta, f)$.*

Imagine that a member a the commission is a woman whose husband is one of the two applicants. She obviously wants him to be hired and every other members of the commission knows it. By consequence, other members may know that she prefers an order of speech to another one at some state ω. However, this cannot be common knowledge among them, that is this cannot be the case for every state in $M(\omega)$.

The intuition of the result is the following. As there are only two final decisions (say *husband* and *unknown applicant*), common knowledge of the fact that she prefers an order α to an order β implies that it is common knowledge that the final decision according to order α is «hiring the husband» and is «hiring the unknown guy» according to the order β. This implies that the final decision which should be taken knowing the set of states that are common knowledge is at the same time «hiring the husband» and «hiring the unknown applicant». However, this result does not hold for $\mid \mathcal{M} \mid> 2$.

Example 6 *Consider $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and two experts endowed with the following information partitions.*

$$\Pi_1 = \{1, 2\}\{3, 4, 5, 6\}\{7, 8\}$$
$$\Pi_2 = \{1, 4\}\{2, 3\}\{5, 8\}\{6, 7\}$$

Let consider the message rule $f : 2^\Omega \to \{a, b, c, d, e, f, g, h, i, j, k\}$ defined by:
$f(\{1\}) = a, f(\{2\}) = b, f(\{3\}) = c, f(\{4\}) = d, f(\{5\}) = e, f(\{6\}) = f, f(\{7\}) = g, f(\{8\}) = h$
$f(\{1, 2\}) = f(\{3, 4\}) = f(\{1, 4\}) = f(\{2, 3\}) = i$
$f(\{5, 6\}) = f(\{7, 8\}) = f(\{5, 8\}) = f(\{6, 7\}) = j$

$f(\{3,4,5,6\}) = k$

If expert 1 speaks first, equilibrium partitions are

$$\Pi_1 = \{1\}_a\{2\}_b\{3\}_c\{4\}_d\{5\}_e\{6\}_f\{7\}_g\{8\}_h$$

$$\Pi_2 = \{1\}_a\{2\}_b\{3\}_c\{4\}_d\{5\}_e\{6\}_f\{7\}_g\{8\}_h$$

If expert 2 speaks first, equilibrium partitions are

$$\Pi_1 = \{1,2\}_i\{3,4\}_i\{5,6\}_j\{7,8\}_j$$

$$\Pi_2 = \{1,4\}_i\{2,3\}_i\{5,8\}_j\{6,7\}_j$$

Suppose now that agents have the following utility functions on $\mathcal{M}$:

$U_1(a) = U_1(b) = U_1(c) = U_1(d) = U_1(e) = U_1(f) = U_1(g) = U_1(h) = 1,\ U_1(i) = U_1(j) = 0$ and $U_2(a) = U_2(b) = U_2(c) = U_2(d) = U_2(e) = U_2(f) = U_2(g) = U_2(h) = 0,\ U_2(i) = U_2(j) = 1$. Then it is common knowledge at every $\omega \in \Omega$ that both experts prefer to speak first in the sense defined in 5.

5 A timing game

In our setting, agents have to reach an agreement on a particular debate protocol before debating, that is on the whole sequence of decision making. This setting fit some situations of regulated debates, but not necessarily all debate situations. In particular, in daily life conversations, people may wait to see what others will say, especially agents with career concerns. This kind of "wait-and-see" behaviors are treated in Chamley and Gale [1994], Gul and Lundholm [1995],...In this kind of models, an agent has to choose an action and a place in the decision-making queue. What are the implications of our main result, namely that it can not be common knowledge that two experts prefer different orders of speech, in a timing game?

We consider that experts must coordinate on the same order of speech for the debate to take place. In the case of two experts, it is easy to see that if nobody wants to speak first, that is if player i prefers order $j, i, j, i...$ and player j prefers order $i, j, i, j...$, the debate will not begin. In the case of three experts, if for instance nobody wants to speak after expert 1, the debate will be stuck from a certain stage on. Hence the fact that experts coordinate on a debate protocol guarantees that at each date, the individual action of speaking or not will be optimal for each player. In Chamley and Gale [1994]'s timing game, at date t player i chooses an action $x_{it} \in \{0, 1\}$, where $x_{it} = 1$ if player i invests at date t $x_{it} = 0$ if he does not. In our setting, player i's action at date t is twofold: i chooses to speak ($x_{it} = 1$) or not ($x_{it} = 0$), and if he speaks, he chooses a message m_{it} which depends on the history of the game at date t that determines his private information at date t. The difference with Chamley and Gale [1994] is that experts are allowed to speak more than once. In their game, a strategy is a function λ from the set of histories into $[0, 1]$, which associates to an history h the probability $\lambda(h)$ of choosing action 1 after observing the history h. In that case, the choice of speaking or not at date t is optimal given the history of the game at date t. In our setting, individual payoffs depend on the whole sequence of actions and messages, then what is optimal for i is not to speak at some date t given the history at t, but is that some agent speaks at date $t-1$, some other at date $t+1$, and so on. By consequence, the relevant strategic setting for the issue we consider is a game where the set of actions is the set of debate protocols.

Let us consider the following coordination game. Players are experts endowed with information partitions of Ω. They have to choose simultaneously an action in $\mathcal{A}$, where the set of actions is the set of debate protocols Γ. We assume that experts are homogenous and opportunist, so that each expert has a complete preference order on protocols at each state. We note $a_i(\omega)$ the protocol chosen by expert i at state ω. We assume that if experts fail in reaching an agreement on a protocol, the debate can not run and every expert receives an infinite penalty. On the contrary, if experts coordinate on some protocol (α, f), expert i receives the payoff $U_i^{\omega}(\alpha, f) := E[E(U(f(H^{\alpha,f}(\cdot), .) \mid \Pi_i^{\alpha,f}(\cdot) \mid \Pi_i(\omega)]$ as defined in definition 4. The payoffs, and by consequence the game, depend on the state of the world. Formally, at state ω, payoffs are the following:

$$U_i^{\omega}(a_i, a_{-i}) = \begin{cases} -\infty \; if \; \exists \, j, j' \; s.t. \; a_j \neq a_{j'} \\ U_i^{\omega}(a) \; if \; a_j = a \; \forall \, j \end{cases}$$

For instance, consider two players and suppose that there are only two possible protocols for the sake of simplification. Both experts speak in turn, in protocol α, player 1 speaks first, and in protocol β, player 2 speaks first.[10]

At state ω, the payoff matrix is :

		Expert 1	
		α	β
Expert 2	α	$U_1^\omega(\alpha), U_2^\omega(\alpha)$	$-\infty, -\infty$
	β	$-\infty, -\infty$	$U_1^\omega(\beta), U_2^\omega(\beta)$

It is usual in game theory to assume that players know the game they are playing. In particular, it is usual to assume that the payoff structure is common knowledge among the players. Yet common knowledge of the matrix payoff at some state ω implies the common knowledge of $U_1^\omega(.)$ and $U_2^\omega(.)$, that is common knowledge of individual preferences $\succeq_i^\omega$ on Γ. Whatever the type of experts and preferences we consider, Corollaries 1, 2 and 3 imply equality of individual preferences, namely $U_1(.) = U_2(.)$ in this example. By consequence, there exists an equilibrium outcome that Pareto-dominates the other outcomes.

Under certain conditions (backward induction, imitation...), this equilibrium PD is going to be selected.

6 Discussion and concluding remarks

6.1 The outside evaluator

Careerist decision makers have necessarily the same preferences as the agent who evaluates them. Then if experts are homogenous, the evaluator is indifferent between choosing the debate protocol and letting experts find an agreement on their own.

On contrary, heterogenous experts may "agree to disagree" on the debate protocols, which prevents the debate to take place. In that case, evaluators can not design a debate protocol as they have the same preferences that experts. They would also agree to disagree.

6.2 Related literature

Our result can be viewed as belonging to the agreement literature following Aumann [1976], Geanakoplos and Polemarchakis [1982], Parikh and Krasucki [1990], Krasucki [1996]

[10] Much orders are possible if we allow experts to speak together at some dates.

about common knowledge and consensus. These results study the conditions under which common knowledge achieves consensus. If experts are careerist, common knowledge of individual preferences over the set of debate protocols implies equality of these preferences.

The most closely related work is Ottaviani and Sørensen [2001]. They model a sequential transparent process where experts speak openly in front of other members of the committee about the desirability of a public decision. Each expert speaks only once, according to an order of speech designed by an outside decision maker. They discuss the optimality of the order of speech according to the outside decision maker who wants to learn the more information as possible from the debate. They cast their analysis in a probabilistic information structure. There is no conceptual difference with our partitional structure, however their setting allows to consider experts of different abilities. In simplifying, an expert's ability is the probability of receiving the good signal. It allows them to have the result that with two experts of different abilities, say a junior and a senior, it is better for the extraction of information that the junior speaks first. In our setting, if we consider that an expert is more able than one other if he has a finer partition, that is if expert A is more able than expert B if and only if $\forall\ \omega \in \Omega$, $\Pi_A(\omega) \subseteq \Pi_B(\omega)$, we do not have the same result. The senior with the finer partition will never learn anything from the junior. Whatever the order of speech is, the debate can not make better than the join partition, which will be the senior partition in this case. If we consider abilities of experts state by state, that is if we say that expert A is more able than expert B at ω if and only if $\Pi_A(\omega) \subseteq \Pi_B(\omega)$, then we can not have systematic results they have.

Levy [2005] considers that two types of committees should be distinguished in order to study the impact of career concerns on group decision making: the secretive and the transparent. In transparent committees, individual recommendations are public. This is the case of deliberations of the US Supreme Court Justice, of French Assemblée Nationale debates. But those of European Central Bank or European Union Court of Justice are hidden from the public eye. Only the final decisions of these institutions are publicly observed, but not the views of individual members. If careerist committee members use their decision (vote, recommendation, etc) to impress some evaluator, then it is important whether committees' meetings are public or not. She addresses the question of wether experts support an action more or less often when their recommendations are public compared to when they remain secret. She shows that when the committee is secretive, a group reputation effect arises which encourages experts to be more conservative, that is to accept more often the action towards which the voting rule is biased, or to accept more often the action that goes in the sense of the initial prior belief. Our setting is closest to a secretive committee as experts debate until they reach a consensus.

Sibert [2003] and Stasavage [2004] analyse behavior in monetary policy committees and the effect of transparency. Both assume that agents care about acquiring a reputation for having some particular preferences, and not for being able experts.

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Proofs

Proof: [Proposition 1]

• Let us show by induction that H_t^α defines a partition of Ω for all t.

$H_0^\alpha = M$ by construction.

Suppose that H_t^α is a partition for a given t, and show that H_{t+1}^α is a partition. First, $H_t^\alpha(\omega) \subseteq \Omega \; \forall \, \omega$ so $H_{t+1}^\alpha(\omega) \subseteq \Omega \; \forall \, \omega$. Let $\omega' \in H_{t+1}^\alpha(\omega)$, with ω given in Ω. Then $\omega' \in H_t^\alpha(\omega)$ and $\forall \, j \in \alpha(t), \; m(\Pi_t^{j,\alpha}(\omega')) = m(\Pi_t^{j,\alpha}(\omega))$.

If $z \in H_{t+1}^\alpha(\omega')$, then $z \in H_t^\alpha(\omega')$ and $\forall \, j \in \alpha(t), \; m(\Pi_t^{j,\alpha}(z)) = m(\Pi_t^{j,\alpha}(\omega'))$. Hence $z \in H_t^\alpha(\omega)$ and $\forall \, j \in \alpha(t), \; m(\Pi_t^{j,\alpha}(z)) = m(\Pi_t^{j,\alpha}(\omega)) \Rightarrow z \in H_{t+1}^\alpha(\omega)$. So $H_{t+1}^\alpha(\omega') \subseteq H_{t+1}^\alpha(\omega)$.
If $\exists z \in H_{t+1}^\alpha(\omega)$ such that $z \notin H_{t+1}^\alpha(\omega')$, then $\exists z$ such that $\exists \, j \in \alpha(t)$ such that $m(\Pi_t^{j,\alpha}(z)) = m(\Pi_t^{j,\alpha}(\omega))$ and $m(\Pi_t^{j,\alpha}(z)) \neq m(\Pi_t^{j,\alpha}(\omega'))$, that is a contradiction.
Hence $H_{t+1}^\alpha(\omega) = H_{t+1}^\alpha(\omega')$.

• Let us show by induction that H_t^α is the partition of common knowledge at date t if the protocol is α.

$H_0^\alpha = M$

Suppose that H_t^α is the partition of common knowledge at date t. It means that $\forall \, \omega$, $H_t^\alpha(\omega)$

is the smallest set such that $\Pi_t^{i,\alpha}(\omega) \subseteq H_t^\alpha(\omega) \forall\, i$.

As α is public, $\Pi_{t+1}^{i,\alpha}(\omega) \subseteq H_{t+1}^\alpha(\omega)\ \forall\ i,\ \omega$ by construction. If $\exists\ z \in H_{t+1}^\alpha(\omega)$ such that $z \notin \Pi_{t+1}^{i,\alpha}(\omega)\ \forall\ i$, then $\exists\ z \in H_t^\alpha(\omega)$ such that $z \notin \Pi_t^{i,\alpha}(\omega) \forall\, i$, that is a contradiction. $\square$

Proof: [Theorem 1]

• H_t^α converges in finite time because of the finiteness of individual partitions. Hence there exists T such that $\forall\ t \geq T,\ H_{t+1}^\alpha(\omega) = H_t^\alpha(\omega)$ for all ω.

• As $H_{t+1}^\alpha(\omega) = H_t^\alpha(\omega)$ for all $t \geq T$, we have $H_t^\alpha(\omega) \subseteq \{\omega' \in \Omega \mid\ \forall\ j \in \alpha(t),\ f(\Pi_j^\alpha(\omega', t)) = f(\Pi_j^\alpha(\omega, t))\}$. As H_t^α is the partition of common knowledge at date t, $f(\Pi_j^\alpha(\omega))$ is common knowledge at ω for all $j \in \alpha(t)$. Hence $f(H_t^\alpha(\omega)) = f(\Pi_j^\alpha(\omega))$ for all $j \in \alpha(t)$. As α is fair, for all i there is $t' \geq T$ such that $i \in \alpha(t')$, so $f(H_t^\alpha(\omega)) = f(\Pi_i^\alpha(\omega))$ for all i. $\square$

Proof: [Theorem 2]

• Let us first prove that if it is common knowledge at ω that i prefers (α, f) to (β, g) and j prefers (β, g) to (α, f), then:

$E[E(U(f(H^{\alpha,f}(\centerdot)), .) \mid \Pi_i^{\alpha,f}(\centerdot)) \mid M(\omega)] = E[E(U(g(H^{\beta,g}(\centerdot)), .) \mid \Pi_i^{\beta,g}(\centerdot)) \mid M(\omega)]$

and

$E[E(U(f(H^{\alpha,f}(\centerdot)), .) \mid \Pi_j^{\alpha,f}(\centerdot)) \mid M(\omega)] = E[E(U(g(H^{\beta,g}(\centerdot)), .) \mid \Pi_j^{\beta,g}(\centerdot)) \mid M(\omega)]$

If it is common knowledge at ω that i prefers (α, f) than (β, g) and j prefers (β, g) than (α, f), then

$$M(\omega) \subseteq \{\omega' \in \Omega \mid (\alpha, f) \succeq_i^{\omega'} (\beta, g)\ and\ (\beta, g) \succeq_j^{\omega'} (\alpha, f)\}$$

that is

$$M(\omega) \subseteq \left\{ \omega' \in \Omega \mid \begin{array}{c} E[E(U(f(H^{\alpha,f}(\centerdot)), .) \mid \Pi_i^{\alpha,f}(\centerdot)) \mid \Pi_i(\omega')] \geq E[U(g(H^{\beta,g}(\centerdot)), .) \mid \Pi_i^{\beta,g}(\centerdot)) \mid \Pi_i(\omega')] \\ E[E(U(f(H^{\alpha,f}(\centerdot)), .) \mid \Pi_j^{\alpha,f}(\centerdot)) \mid \Pi_j(\omega')] \leq E[U(g(H^{\beta,g}(\centerdot)), .) \mid \Pi_j^{\beta,g}(\centerdot)) \mid \Pi_j(\omega')] \end{array} \right\}$$

As $M(\omega)$ is a disjoint union of cells of Π_i and of cells of Π_j, we have

$$E[E(U(f(H^{\alpha,f}(\centerdot)), .) \mid \Pi_i^{\alpha,f}(\centerdot)) \mid M(\omega)] \geq E[E(U(g(H^{\beta,g}(\centerdot)), .) \mid \Pi_i^{\beta}(\centerdot)) \mid M(\omega))] \quad (3)$$

and

$$E[E(U(g(H^{\beta,g}(\centerdot)), .) \mid \Pi_j^{\beta,g}(\centerdot)) \mid M(\omega)] \geq E[E(U(f(H^{\alpha,f}(\centerdot)), .) \mid \Pi_j^{\alpha,f}(\centerdot)) \mid M(\omega))] \quad (4)$$

Recall that for all ω and for all debate protocol (α, f), $M(\omega)$ is a disjoint union of cells of the partition of common knowledge at equilibrium $H^{\alpha,f}$. $\sum_{H^{\alpha,f}(k)\subseteq M(\omega)}$ will denote the sum on all cells of $H^{\alpha,f}$ composing $M(\omega)$. We have:

$$\begin{aligned}
E[E(U(f(H^{\alpha,f}(\boldsymbol{.})),.) \mid \Pi_i^{\alpha,f}(\boldsymbol{.})) \mid M(\omega)] &= \sum_{z\in M(\omega)} P(z)E(U(f(H^{\alpha,f}(z)),.) \mid \Pi_i^{\alpha}(z)) \\
&= \sum_{H^{\alpha,f}(k)\subseteq M(\omega)} \sum_{z\in H^{\alpha,f}(k)} P(z)E(U(f(H^{\alpha,f}(z)),.) \mid \Pi_i^{\alpha,f}(z)) \\
&= \sum_{H^{\alpha,f}(k)\subseteq M(\omega)} \sum_{z\in H^{\alpha,f}(k)} P(z)E(U(f(H^{\alpha,f}(k)),.) \mid \Pi_i^{\alpha,f}(z)) \\
&= \sum_{H^{\alpha,f}(k)\subseteq M(\omega)} \sum_{\Pi_i^{\alpha,f}(k')\subseteq H^{\alpha,f}(k)} P(\Pi_i^{\alpha,f}(k'))E(U(f(H^{\alpha,f}(k)),.) \mid \Pi_i^{\alpha,f}(k')) \\
&= \sum_{H^{\alpha,f}(k)\subseteq M(\omega)} P(H^{\alpha,f}(k))E(U(f(H^{\alpha,f}(k)),.) \mid H^{\alpha,f}(k))
\end{aligned}$$

By the same computation, we have:

$$E[E(U(g(H^{\beta,g}(\boldsymbol{.})),.) \mid \Pi_i^{\beta,g}(\boldsymbol{.})) \mid M(\omega)] = \sum_{H^{\beta,g}(k)\subseteq M(\omega)} P(H^{\beta,g}(k))E(U(g(H^{\beta,g}(k)),.) \mid H^{\beta,g}(k))$$

Then expressions (1) and (2) imply that

$$\sum_{H^{\alpha,f}(k)\subseteq M(\omega)} P(H^{\alpha,f}(k))E(U(f(H^{\alpha,f}(k)),.) \mid H^{\alpha,f}(k)) = \sum_{H^{\beta,g}(k)\subseteq M(\omega)} P(H^{\beta,g}(k))E(U(g(H^{\beta,g}(k)),.) \mid H^{\beta,g}(k))$$

By consequence,

$$E[E(U(f(H^{\alpha,f}(\boldsymbol{.})),.) \mid \Pi_i^{\alpha,f}(\boldsymbol{.})) \mid M(\omega)] = E[E(U(g(H^{\beta,g}(\boldsymbol{.})),.) \mid \Pi_i^{\beta,g}(\boldsymbol{.})) \mid M(\omega)]$$

and

$$E[E(U(f(H^{\alpha,f}(\boldsymbol{.})),.) \mid \Pi_j^{\alpha,f}(\boldsymbol{.})) \mid M(\omega)] = E[E(U(g(H^{\beta,g}(\boldsymbol{.})),.) \mid \Pi_j^{\beta,g}(\boldsymbol{.})) \mid M(\omega)]$$

- Let us now show that $(\alpha, f) \sim_i^{\omega} (\beta, g)$ and $(\alpha, f) \sim_j^{\omega} (\beta, g)$. For all $\omega' \in M(\omega)$,

$$E[E(U(f(H^{\alpha,f}(\boldsymbol{.})),.) \mid \Pi_i^{\alpha,f}(\boldsymbol{.})) \mid \Pi_i(\omega')] \geq E[E(U(g(H^{\beta,g}(\boldsymbol{.})),.) \mid \Pi_i^{\beta,g}(\boldsymbol{.})) \mid \Pi_i(\omega')] \qquad (5)$$

If there exists $\omega' \in M(\omega)$ such that the inequality is strict in (5), then

$$\sum_{\omega'\in M(\omega)} P(\omega')E[E(U(f(H^{\alpha,f}(\boldsymbol{.})),.) \mid \Pi_i^{\alpha,f}(\boldsymbol{.})) \mid \Pi_i(\omega')] > \sum_{\omega'\in M(\omega)} P(\omega')E[E(U(g(H^{\beta,g}(\boldsymbol{.})),.) \mid \Pi_i^{\beta,g}(\boldsymbol{.})) \mid \Pi_i(\omega')]$$

$$\Longrightarrow P(M(\omega))E[E(U(f(H^{\alpha,f}(\boldsymbol{.})),.) \mid \Pi_i^{\alpha,f}(\boldsymbol{.})) \mid M(\omega)] > P(M(\omega))E[E(U(g(H^{\beta,g}(\boldsymbol{.})),.) \mid \Pi_i^{\beta,g}(\boldsymbol{.})) \mid M(\omega)]$$

which contradicts the former result.

Hence $E[E(U(f(H^{\alpha,f}(\centerdot)),.) \mid \Pi_i^{\alpha,f}(\centerdot)) \mid \Pi_i(\omega')] = E[E(U(g(H^{\beta,g}(\centerdot)),.) \mid \Pi_i^{\beta,g}(\centerdot)) \mid \Pi_i(\omega')]$ for all $\omega' \in M(\omega)$. Applying the same reasoning for j, we get $(\alpha, f) \sim_i^{\omega'} (\beta, g)$ and $(\alpha, f) \sim_j^{\omega'} (\beta, g)$ for all $\omega' \in \Omega$. $\square$

Proof: [Corollary 1]

Let us prove Corollary 1 *ad absurdum.* If $\succeq_i^{\omega} \neq \succeq_j^{\omega}$, then $\exists\ (\alpha, f) \neq (\beta, g) \in \Gamma \times \Delta$ such that $(\alpha, f) \succ_i^{\omega} (\beta, g)$ and $(\alpha, f) \preceq_j^{\omega} (\beta, g)$. By Theorem 2, it implies that it is not common knowledge at ω that $(\alpha, f) \succeq_i (\beta, g)$ and $(\beta, g) \succeq_j (\alpha, f)$. Then $\exists\ \omega' \in M(\omega)$ such that $(\alpha, f) \prec_i^{\omega'} (\beta, g)$ or $(\beta, g) \prec_j^{\omega'} (\alpha, f)$. Yet if $\succeq_i$ and $\succeq_j$ were common knowledge at ω, we would have $M(\omega) \subseteq \{\omega' \in \Omega \mid \succeq_i^{\omega'} = \succeq_i^{\omega}\ \ \textit{and}\ \ \succeq_j^{\omega'} = \succeq_j^{\omega}\}$, that is for all $\omega' \in M(\omega)$, we would have $(\alpha, f) \succ_i^{\omega'} (\beta, g)$ and $(\alpha, f) \preceq_j^{\omega'} (\beta, g)$. Then $\succeq_i$ and $\succeq_j$ are not common knowledge at ω. $\square$

Proof: [Theorem 3]

Ad absurdum. Suppose that there exists some state ω at which $(\alpha, f) \succ_i^{\omega} (\beta, g)$ and $(\beta, g) \succeq_i^{\omega} (\alpha, f)$. It implies that for all $\omega' \in \Pi_i(\omega)$, $f(H^{\alpha,f}(\omega')) > g(H^{\beta,g}(\omega'))$ and for all $\omega' \in \Pi_j(\omega)$, $g(H^{\beta,g}(\omega')) \gtrsim f(H^{\alpha,f}(\omega'))$. Yet $\omega \in \Pi_i(\omega) \cap \Pi_j(\omega)$, so it implies that $g(H^{\beta,g}(\omega)) \gtrsim f(H^{\alpha,f}(\omega))$ and $f(H^{\alpha,f}(\omega)) > g(H^{\beta,g}(\omega))$ which is a contradiction. $\square$

Proof: [Theorem 4]

If it is common knowledge at ω that $(\alpha, f) \succeq_i (\beta, f)$, then

$$M(\omega) \subseteq \{\omega' \in \Omega \mid f(H^{\alpha,f}(\omega')) \succeq_i f(H^{\beta,f}(\omega'))\} \tag{6}$$

Let $\mathcal{M}$ be $\{a, b\}$, $a \neq b$, and assume without loss of generality that $a \succ_i b$. Then 6 implies that

$$M(\omega) \subseteq \{\omega' \in \Omega \mid f(H^{\alpha,f}(\omega')) = a \textit{ and } f(H^{\beta,f}(\omega')) = b\}$$

Yet $M(\omega)$ is a disjoint union of cells of $H^{\alpha,f}(.)$. Hence $f(M(\omega)) = a$. As $M(\omega)$ is also a disjoint union of cells of $H^{\beta,f}(.)$, we have $f(M(\omega)) = b$. Then $a = b$ which is a contradiction. $\square$