Parimutuel versus Fixed-Odds Markets

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Abstract

This paper compares the outcomes of parimutuel and competitive fixed-odds betting markets. In the model, there is a fraction of privately informed bettors that maximize expected monetary payoffs. For each market structure, the symmetric equilibria are characterized. In parimutuel betting, the return on longshots is driven to zero as the number of insiders grows large. In fixed odds betting instead, this return is bounded below. Conversely, the expected return on longshots is increasing in the number of insiders in the parimutuel market, but decreasing in the fixed-odds market. The extent of the favorite-longshot bias is shown to depend on the market rules.

Keywords: Parimutuel, fixed odds, betting, favorite-longshot bias, private information.

JEL Classification: D82 (Asymmetric and Private Information), D84 (Expectations; Speculations), G13 (Contingent Pricing; Futures Pricing).

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1. Introduction

This paper compares the performance of parimutuel with fixed-odds markets, the two most common market structures used for betting. In *parimutuel* markets, a winning bet pays off a proportional share of the total money bet on all outcomes, so that the odds offered to a bet are determined only after all the bets are placed. In *fixed-odds* markets instead, bookmakers compete to set odds at which they accept bets from the public.

Differently from regular financial markets, in betting markets the uncertainty is resolved unambiguously and the fundamental values are publicly observed. In addition, these values are exogenous with respect to the market prices. Because of these features, betting markets provide an ideal testbed for evaluating theories of price formation (see Thaler and Ziemba 1988, Hausch and Ziemba 1995, Sauer 1998, and Jullien and Salanié 2002 for surveys).

A commonly observed empirical pattern is the *favorite-longshot bias*, according to which horses with short market odds (favorites) offer higher average payoff than horses with long market odds (longshots). In this paper, we show that this bias can result from the presence of privately informed bettors in both the parimutuel and fixed-odds market structures. We further argue that these two structures induce different systematic relations between empirical and market odds, depending on the amount of information present in the market.

In the model, there are two classes of bettors, outsiders and insiders. While outsider are uniformed and place an exogenous amount of bets, insiders are privately informed and maximize their expected monetary payoff. We characterize how the symmetric equilibrium depends on the market structure.

First, consider the outcome of a competitive *fixed-odds* market. Relative to favorites, longshots attract a relatively higher proportion of insiders and pay out more conditional on winning. To counteract this more severe adverse selection problem, competitive bookmarkers quote relatively shorter odds on longshots. This is the equivalent of a larger bid-ask spread in the presence of more insider trading in a standard financial market making mechanism (Glosten and Milgrom 1985). Since the adverse selection problem is greater on the longshot than on the favorite, a favorite-longshot bias arises.

Consider next the case of *parimutuel markets*. The insiders simultaneously use their private information to decide where to place their bets. As a result of insider cash constraints, the market odds do not move sufficiently far to exhaust the gains revealed by the informed bettors, and the favorite-longshot bias arises. Many (few) informed bets on a horse indicates that this favorite (longshot) is less (more) likely to win than indicated by the realized bet distribution.

Our analysis of fixed-odds markets is closely related to Shin (1991 and 1992). Shin argues that a monopolistic bookmaker sets odds with a favorite-longshot bias in order to limit the subsequent losses to the better informed insiders. We derive a similar bias in a competitive bookmaking market. Our informational assumptions are similar to those made by Shin, but we consider the case of ex-post rather than ex-ante competition among bookmakers. Our results on the favorite-longshot bias in fixed-odds betting are therefore new. Our analysis of parimutuel markets builds extensively on the results obtained by Ottaviani and Sørensen (2004).

To the best of our knowledge, this is the first paper that compares the performance of trading structures used in betting markets. We study these two structures in isolation. When both markets coexist, bettors have the additional choice of participating in either market, possibly depending on their information. The investigation of this selection issue is left to future research.[1]

A number of alternative theories have been formulated to explain the favorite-longshot bias. First, Griffith (1949) suggested that the bias might be due to the tendency of individual decision makers to overestimate small probabilities events. Second, Weitzman (1965) and Ali (1977) hypothesized that individual bettors are risk loving, and so are willing to accept a lower expected payoff when betting on the riskier longshots. Third, Isaacs (1953) noted that an informed monopolist bettor would not bet until the marginal bet has zero value, if the marginal bet reduces the return to the inframarginal bets. Fourth, Hurley and McDonough (1995) noted that the presence of the track take limits the amount of arbitrage by the informed bettors, who are prevented from placing negative bets and so cannot take advantage of negative returns on longshots.

While Isaacs' (1953) market power explanation and Hurley and McDonough's (1995) limited arbitrage explanation are specific to parimutuel markets, the informational explanation of the favorite-longshot bias proposed here applies both to parimutuel and fixed-

[1] For example, parimutuel and fixed odds markets coexist in the UK. See Gabriel and Marsden (1990) for an empirical investigation of the interaction effects resulting from the bettors' option to select in which system to participate.

odds markets. The behavioral and risk loving explanations instead predict the presence of the bias regardless of the market structure, but do not account for the varying extent of the bias under different market institutions.

The paper proceeds as follows. In Section 2 we formulate the information structure of the model. In Section 3 we analyze the equilibrium with fixed odds. In Section 4 we turn to parimutuel betting. Section 5 compares the outcomes of the two market structures.

2. Model

We consider a race between two horses. The outcome that horse x wins the race is identified with the *state*, $x \in \{-1, 1\}$.

Unmodelled *outsiders* place bets on the two horses without responding to the market conditions. For simplicity, we assume that the same amount $a \geq 0$ is bet on either horse.

There is a continuum $[0, N]$ of privately informed bettors (or *insiders*). Insiders (as well as the bookmakers in the specification of Section 3) have a common *prior belief* $q = \Pr(x = 1)$, possibly formed after the observation of a common signal. In addition, each insider i privately observed *signal* s_i.[2] The signals are assumed to be identically and independently distributed across insiders, conditional on state x. Since there are only two states, the likelihood ratio $f(s|x = 1)/f(s|x = -1)$ is monotone without further loss of generality. For simplicity, we further assume that the likelihood ratio is strictly increasing in s.

Upon observation of signal s, the prior belief q is updated according to Bayes' rule into the *posterior belief*, $p = \Pr(x = 1|s)$. The posterior belief p is distributed according to the continuous distribution function G with density g on $[0, 1]$. By the law of iterated expectations, the prior must satisfy $q = E[p] = \int_0^1 pg(p)\,dp$. Bayes' rule yields $p = qg(p|x = 1)/g(p)$ and $1 - p = (1 - q)\,g(p|x = -1)/g(p)$, so that the conditional densities of the posterior are $g(p|x = 1) = pg(p)/q$ and $g(p|x = -1) = (1 - p)\,g(p)/(1 - q)$. Note that $g(p|x = 1)/g(p|x = -1) = (p/(1 - p))((1 - q)/q)$, reflecting the property that high beliefs in outcome 1 are more frequent when outcome 1 is true. Strict monotonicity of the likelihood ratio in p implies that $G(p|x = 1)$ first-order stochastically dominates $G(p|x = -1)$ on the support, i.e., $G(p|x = 1) - G(p|x = -1) < 0$ for all p such that $0 < G(p) < 1$.

[2] Private (or inside) information is believed to be pervasive in horse betting. See e.g., Crafts (1985).

It is convenient to state assumptions on the signal structure in terms of their implications for the conditional distributions of the posterior belief. The signal distribution is said to be *symmetric* if the chance of posterior p conditional on state $x = 1$ is equal to the chance of posterior $1 - p$ conditional on state $x = -1$, i.e., $G(p|x = 1) = 1 - G(1 - p|x = -1)$ for all $p \in [0, 1]$. The signal distribution is said to be *unbounded* if $0 < G(p) < 1$ for all $p \in (0, 1)$.

Example. To illustrate our results we use the *linear signal* example, which can be derived from a binary signal with precision distributed uniformly. In this example, the conditional densities are $f(s|x = 1) = 2s$ and $f(s|x = -1) = 2(1 - s)$ for $s \in [0, 1]$, with corresponding distribution functions $F(s|x = 1) = s^2$ and $F(s|x = -1) = 1 - (1 - s)^2$. The posterior odds ratio is

$$\frac{p}{1-p} = \frac{q}{1-q}\frac{f(s|x=1)}{f(s|x=-1)} = \frac{q}{1-q}\frac{s}{1-s}. \tag{2.1}$$

Inverting $p/(1-p) = qs/(1-q)(1-s)$, we obtain $s = p(1-q)/(p(1-q)+(1-p)q)$. The conditional distribution functions for p are

$$G(p|1) = \left(\frac{p(1-q)}{p(1-q)+(1-p)q}\right)^2 \tag{2.2}$$

and

$$G(p|-1) = 1 - \left(\frac{(1-p)q}{p(1-q)+(1-p)q}\right)^2. \tag{2.3}$$

3. Fixed-Odds Betting

We begin by considering fixed-odds betting. After introducing the rules of the market (Section 4.1), we characterize the equilibrium (Section 4.2) and the resulting favorite-longshot bias (Section 4.3).

3.1. Market Rules

We consider a market with competitive bookmakers operating as follows. First, the bookmakers simultaneously quote odds. We denote by $1 + \rho_x = 1/\pi_x$ the return to every dollar bet on horse x offered by the bookmaker(s) making the most advantageous offer. Second, the bettors place their bets. The insiders are allowed to bet one dollar on either horse, or abstain from betting. In addition, there is a given amount of bets placed by unmodelled outsiders, and these outside bets are equal to $a > 0$ on each of the two horses regardless of the state. Third, the state x is realized, and bookmakers pay out the promised returns on the winning tickets.

Note the difference of our model with Shin's (1991 and 1992) model. Shin assumes that the odds are set by a monopolist bookmaker, who is allowed to set odds on the different horses at the same time and so cross-subsidize across the two markets. In his setting, bookmarkers compete ex-ante for the monopoly position, and so make zero profits in equilibrium of the full game, even if they do make non zero positive profits on the market of each individual horse. We instead consider a separate competitive market for each horse, as in Glosten and Milgrom's (1985) market making model.

3.2. Equilibrium Characterization

When odds ρ_1 are offered on horse 1, every insider with beliefs above the cutoff belief p_1 prefers to bet on horse 1 rather than to abstain, where p_1 is defined by the indifference $p_1\rho_1 = 1 - p_1$. Thus, the solution $p_1 = 1/(1+\rho_1) = \pi_1$ is precisely the implied market probability.

By Bertrand competition, in equilibrium each bookmaker make zero expected profits in the market corresponding to each horse. Taking into account the optimal response of the informed bettors, the bookmaker makes zero expected profits on horse 1 when

$$q\left(a + N\left(1 - G\left(\frac{1}{1+\rho_1}|1\right)\right)\right)\rho_1 = (1-q)\left(a + N\left(1 - G\left(\frac{1}{1+\rho_1}|-1\right)\right)\right). \quad (3.1)$$

To understand this, note that the bookmaker believes that horse 1 wins with probability q, in which case the bookmaker makes a net payment equal to ρ_1 to a outsiders and to the insiders with a belief above π_1. If instead horse -1 wins, the informed place a lower amount of bets on the horse 1, since $1 - G(\pi_1|-1) < 1 - G(\pi_1|1)$ by the stochastic dominance property of beliefs.[3]

An equilibrium on the market for horse 1 is defined by any $\rho_1 > 0$ that solves equation (3.1). Observe that for $\rho_1 = 0$, the left-hand side is strictly lower than the right hand side, since $0 < (1-q)\,a$. As $\rho_1 \to +\infty$, the left-hand side increases without bound and so

[3] In Glosten and Milgrom's model, the bookmakers earn zero expected profits conditional on the news that the next bettor wants to bet on horse 1. Indeed, (3.1) can be rewritten to confirm that π_1 is the posterior probability of state 1 given a bet on horse 1.

exceeds the right-hand side, equal to the bounded $(1-q)(a+N)$. Thus there exists an equilibrium, that below is shown to be unique.

Equation (3.1) may be rewritten as

$$\rho_1 = \frac{1-q}{q} \frac{a+N\left(1-G\left(\frac{1}{1+\rho_1}|-1\right)\right)}{a+N\left(1-G\left(\frac{1}{1+\rho_1}|1\right)\right)} < \frac{1-q}{q} \tag{3.2}$$

where the inequality is due to $G(p|1) < G(p|-1)$. The left-hand side is the market odds, while the right-hand side gives the prior odds. Systematically, bookmakers quote market odds shorter than the prior odds in order to protect their profits against the informational advantage of the insiders. This difference may be loosely interpreted as a bid-ask spread. It implies that a bet based on the prior belief q results in a negative expected return.

3.3. Favorite-Longshot Bias

Note that the implied market probability $\pi_1 = 1/(1+\rho_1)$ that result in equilibrium departs systematically from the prior belief q that horse 1 wins. The corresponding empirical average return to a bet on horse 1 is then $q/\pi_1 - 1 < 0$. In accordance with Shin's (1991 and 1992) definition, we say that the favorite-longshot bias arises if the ratio π_1/q is a decreasing function of q, in which case the empirical average return to a bet on horse 1 is increasing in π_1.

We now establish that π_1 is an increasing function of q and π_1/q is a decreasing function of q:

Proposition 1 *There exists a unique constant factor $\mu > 1$, such that the market implied probability can be written as $\pi_1 = \mu q/[\mu q + (1-q)]$. There is a favorite-longshot bias.*

Proof. Defining $\alpha = a/N$, and using the variable $\pi_1 = 1/(1+\rho_1)$, the equilibrium condition (3.1) is

$$\frac{(1-q)\pi_1}{q(1-\pi_1)} = \frac{\alpha+1-G(\pi_1|1)}{\alpha+1-G(\pi_1|-1)}. \tag{3.3}$$

Here, the left hand side is continuous and strictly increasing in π_1, being 0 at $\pi_1 = 0$ and tending to infinity as $\pi_1 \to 1$. The right-hand side is continuous with value 1 at both ends $\pi_1 = 0, 1$. Thus, for every given $q \in (0,1)$ there exists a solution π_1 to (3.3).

This solution is unique, for whenever equation (3.3) holds, the right-hand side intersects the left hand side from above. This follows, since the slope of the right-hand side at an intersection is

$$\frac{g(\pi_1|-1)(\alpha+1-G(\pi_1|1))-g(\pi_1|1)(\alpha+1-G(\pi_1|1))}{(\alpha+1-G(\pi_1|-1))^2}=\frac{g(\pi_1|-1)-g(\pi_1|1)\frac{(1-q)\pi_1}{q(1-\pi_1)}}{(\alpha+1-G(\pi_1|-1))}$$

where the equality followed from (3.3). By $g(\pi_1|1)/g(\pi_1|-1)=(\pi_1/(1-\pi_1))((1-q)/q)$, we see that this slope is in fact zero. Since the left hand side is increasing, the intersection is from above, as claimed.

So far we have established that for every q there exists a unique solution π_1 to (3.3). We now describe how π_1 depends on q. Recall that the distribution G of the posterior belief depends on q, so we aim first to rewrite (3.3) in terms of the fixed signal distribution F. Bayesian updating gives $\pi_1(1-q)/[(1-\pi_1)q] = f(s|1)/f(s|-1)$, and since the likelihood ratio $f(s|1)/f(s|-1)$ is strictly monotone, we can recover s as a function σ of the variable $\mu=\pi_1(1-q)/[(1-\pi_1)q]$. Using the relationship between s and π_1, for given q we defined $G(\pi_1|x)=F(s|x)$. Now (3.3) reduces to

$$\mu=\frac{\alpha+1-F(\sigma(\mu)|1)}{\alpha+1-F(\sigma(\mu)|-1)} \tag{3.4}$$

The existence of the unique solution for π_1 given q to equation (3.3) implies that there exists a likewise unique solution in μ to equation (3.4). Namely, consider $q=1/2$ and take the π_1 that solved (3.3), let $\mu=\pi_1/(1-\pi_1)$ and note that this is a solution to (3.4). On the other hand, if $\mu'\neq\mu$ also solved (3.4), then $\pi_1'=\mu'/(1+\mu')\neq\pi_1$ would also solve (3.3) for $q=1/2$, in contradiction to its uniqueness.

As noted in (3.2), the market odds are shorter than the prior odds, so $\mu>1$. Rewriting, we see that $\pi_1=\mu q/[\mu q+(1-q)]$. Thus π_1 is an increasing function of q, while $\pi_1/q=\mu/[\mu q+(1-q)]$ is a decreasing function of q since $\mu>1$. Thus, there is a favorite-longshot bias. □

As the chance q that horse 1 wins is increased, the bookmaker naturally sets shorter odds for horse 1. This drives away some of the informed bettors, but we have assumed that the outsiders keep betting the same amount. Thus, the bookmaker's adverse selection problem is reduced, and in equilibrium the implied market probability π_1 is brought closer to the prior probability q. Yet, we showed that the ratio π_1/q falls. The subtlety of the result is underlined by the fact that the ratio of market odds to prior odds is constant with respect to q and equal to $1/\mu$.

The factor μ is a decreasing function of the ratio a/N of outsider to insider population sizes, since the bookmaker's adverse selection problem is smaller when the outsiders place more bets in comparison to the insiders. This implies that the favorite-longshot bias is more pronounced when there are more insiders.

Proposition 2 *The factor μ, and thus the extent of the favorite-longshot bias, is a decreasing function of a/N.*

Proof. An increase in $\alpha = a/N$ serves to decrease the right-hand side of (3.3) for every $\pi_1 \in (0,1)$. This follows from the first-order stochastic dominance property $1 - G(\pi_1|1) > 1 - G(\pi_1|-1)$. The strictly increasing left-hand side is not affected by the change in α, so the equilibrium value of π_1 must be smaller than before for every q. This implies that $\mu = \pi_1(1-q)/[(1-\pi_1)q]$ is smaller, and closer to one, than before. □

Example. In the special symmetric signal distribution example, G is defined by (2.2) and (2.3). Defining $\alpha = a/N$, and using the variable $\pi_1 = 1/(1+\rho_1)$, the equilibrium condition (3.1) is then

$$\frac{(1-q)\pi_1}{q(1-\pi_1)} = \frac{\alpha + 1 - \left(\frac{\pi_1(1-q)}{\pi_1(1-q)+(1-\pi_1)q}\right)^2}{\alpha + \left(\frac{(1-\pi_1)q}{\pi_1(1-q)+(1-\pi_1)q}\right)^2}. \tag{3.5}$$

Using $\mu = (1-q)\pi_1/[q(1-\pi_1)]$, this equation further reduces to the cubic $\alpha\mu^3 + \alpha\mu^2 - (1+\alpha)\mu - (1+\alpha) = 0$. Eliminating the useless root $\mu = -1$, this reduces to the quadratic $\alpha\mu^2 - (1+\alpha) = 0$ which is solved by $\mu = \sqrt{(1+\alpha)/\alpha} > 1$, decreasing in α. Figure 3.1 displays the expected return $q/\pi_1 - 1$ against market odds $(1-\pi_1)/\pi_1$, for five values of α. Notice the similarity of this figure with Jullien and Salanié's (2002) Figure 1, derived from bookmakers' odds data for horse races run in Britain between 1986 and 1995 (see also Jullien and Salanié 2000).

4. Parimutuel Betting

This section reviews how the favorite-longshot bias arises from simultaneous informed betting, as derived in Ottaviani and Sørensen (2004). We first introduce the rules of the mar-

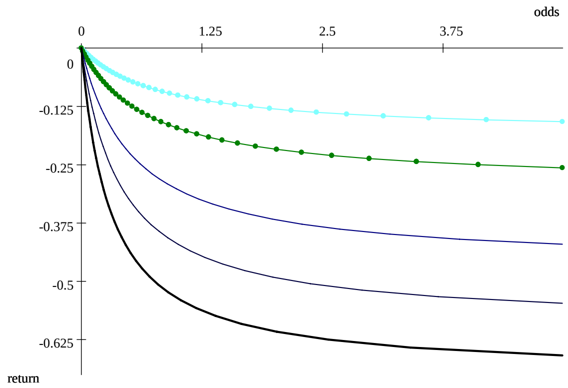

Figure 3.1: The expected return to a bet on outcome 1 in the fixed-odds market is plotted against the market odds ratio in the linear signal example. The five curves show the cases $a/N = 2, 1, .4, .2, .1$, in progressively thicker shade.

ket (Section 4.1). We then characterize the equilibrium and derive the bias (Section 4.2). Finally, we illustrate how the bias varies with the number of insiders (Section 4.3).

4.1. Market Rules

We assume that the informed bettors can bet only a limited amount 1, and bet in order to maximize their expected return. The total amount bet by these insiders on outcome y in state x is denoted by $b_{y|x}$. All bets on both outcomes are placed in one common pool, from which is subtracted the *track take* $\tau \in [0, 1)$. The remaining pool is returned to the winning bets. Thus, if x is the winner, each unit bet on outcome x yields

$$W(x|x) = (1 - \tau) \frac{2a + b_{x|x} + b_{-x|x}}{a + b_{x|x}}. \quad (4.1)$$

Notice the important difference to the case of fixed-odds betting, that each horse cannot be studied in isolation — the return on horse x is influenced by the amount bet also on horse $-x$. With a continuum of small informed bettors, we assume each of them takes the returns $W(x|x)$ as given.[4]

4.2. Equilibrium Characterization

With several bettors possessing private information, we solve the model for a Bayes-Nash equilibrium. Each bettor takes the correct equilibrium numbers $W(x|x)$ as given, and

[4]Ottaviani and Sørensen (2005) consider finitely many insiders.

chooses a best reply. Naturally, the greater is the belief in an outcome, the more attractive it is to bet on that outcome. This leads to the following result.

Proposition 3 *Assume that the private beliefs distribution is unbounded, and that* $0 \leq \tau < 1/2$*. There exists a unique equilibrium. An insider bets on* -1 *when* $p < \hat{p}_{-1}$*, abstains when* $\hat{p}_{-1} < p < \hat{p}_1$*, and bets on* 1 *when* $p > \hat{p}_1$*, where the thresholds* $0 < \hat{p}_{-1} < \hat{p}_1 < 1$ *constitute the unique solution to the two indifference conditions*

$$\hat{p}_1 = \frac{1}{1-\tau}\frac{a + N\left(1 - G\left(\hat{p}_1|1\right)\right)}{2a + N\left(1 - G\left(\hat{p}_1|1\right)\right) + NG\left(\hat{p}_{-1}|1\right)} \tag{4.2}$$

and

$$1 - \hat{p}_{-1} = \frac{1}{1-\tau}\frac{a + NG\left(\hat{p}_{-1}|-1\right)}{2a + N\left(1 - G\left(\hat{p}_1|-1\right)\right) + NG\left(\hat{p}_{-1}|-1\right)}. \tag{4.3}$$

Proof. See Proposition 1 of Ottaviani and Sørensen (2004). ☐

Once the bettors use threshold strategies, we obtain $b_{1|x} = 1 - G\left(\hat{p}_1|x\right)$ and $b_{-1|x} = G\left(\hat{p}_{-1}|x\right)$. The conditions (4.2) and (4.3) express that the threshold bettor is exactly indifferent among betting and abstaining, i.e., $\hat{p}_1 W\left(1|1\right) = \left(1 - \hat{p}_{-1}\right) W\left(-1|-1\right) = 1$.

4.3. Favorite-Longshot Bias

In the parimutuel market, by definition, the implied market probability for outcome x is $\left(1-\tau\right)/W\left(x|x\right)$, equal to the fraction of money placed on outcome x. The favorite-longshot bias claims that the greater is this implied market probability, the greater the expected return to a dollar bet on x. Conditioning on the realization of this implied market probability, the empirical researcher can estimate the expected return to a bet on x. In our model, with the continuum of privately informed bettors, strictly more bets are placed on outcome x when it is true than when it is false. Thus, the realized bets fully reveal the true outcome. We conclude that the equilibrium outcome exhibits the favorite-longshot bias. The insiders' bets $\left(b_{1|x}, b_{-1|x}\right)$ reveal the true winner, and although horse x is more of a favorite $\left(b_{x|x} > b_{x|-x}\right)$ when it wins, the market implied probability for the winner is less than one.[5]

When the insiders place a greater share of all bets, they have a greater impact on the market odds. Likewise, when τ decreases, the rational insiders are keener on betting, and

[5]Ottaviani and Sørensen (2005) investigate more generally the conditions for the occurrence of the favorite-longshot bias with a finite number of players. The sign and extent of the bias depends on the interaction of noise and information.

therefore have greater impact on the market odds. Either change has the effect of reducing the favorite-longshot bias.

Proposition 4 *Assume that the distribution of private posterior beliefs is symmetric and unbounded, and that* $0 < \tau < 1/2$. *The unique equilibrium of Proposition 3 satisfies* $\hat{p}_1 = 1 - \hat{p}_{-1} \in (1/2, 1)$. *The threshold* $\hat{p}_1$ *is increasing in* τ *and* N/a. *The favorite-longshot bias is reduced when either* a/N *or* τ *is decreased.*

Proof. See Proposition 4 of Ottaviani and Sørensen (2004). □

Example. In the linear signal example with fair prior ($q = 1/2$) and track take $\tau \leq 1/2$, the unique symmetric-policy Nash equilibrium has an explicit expression, with cutoff belief

$$\hat{p}_1 = \frac{(1-\tau)(1+a/n) - \sqrt{(1+a/N)\left(\tau^2 + (1-\tau)^2 a/N\right)}}{(1-2\tau)} \in [1/2, 1) .$$

5. Comparison of Market Structures

We have established that the favorite-longshot bias can arise as the result of informed betting in both parimutuel and fixed-odds markets. Figure 3.1 illustrates the extent of the favorite-longshot bias under fixed-odds betting. This plot can be easily compared with a plot generated with data obtained from fixed-odds betting. However, we have not produced any directly comparable plot for parimutuel betting, since the market odds have different meanings in the two market structures. In this section, we propose a method for comparing the extent of the favorite-longshot bias in these two market structures.

To verify the presence of the favorite-longshot bias in the two settings, we have studied the relation between the empirical odds and the market odds. But the market odds are different in the two settings. In fixed-odds betting, the bookmaker uses the prior probability q as a basis for quoting the market implied probability π_1, which is then associated with the empirical chance q. In parimutuel betting, however, the market implied probability $\pi = k/N$ results from the volume of bets placed on the two horses. Since this random volume reveals information, it is associated with the empirical (Bayesian) probability $1/(1+\beta)$.

In order to compare the extent of the favorite longshot bias in the two models, we perform the following transformation of the odds generated in the parimutuel market. For

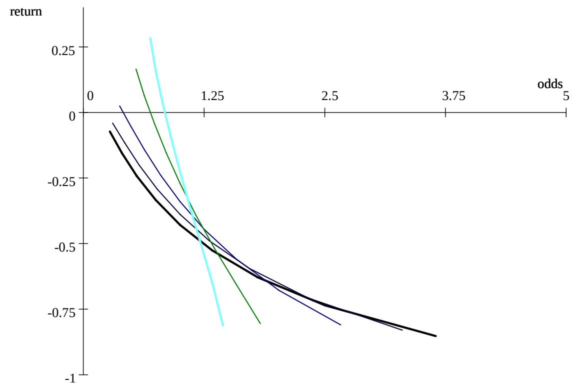

Figure 5.1: The average return to a parimutuel bet on horse 1 is plotted against the average market odds ratio in the linear signal example. All plots have $\tau = .15$, and show the results as q varies in the interval $(.1, .9)$. The five curves show the cases $a/N = 2, 1, .4, .2, .1$, in progressively thicker shade.

any value of the prior probability q, in the parimutuel market we determine the *average* market implied probability for horse 1 and the *average* return to an extra bet on horse 1. Varying $q \in (0, 1)$ parametrically, we then obtain a plot of returns against market implied probabilities, directly comparable to Figure 3.1 obtained for the fixed-odds market.

The amount of bets on horse 1 is $a + N(1 - G(\hat{p}_1|x))$ when x is the winner. The average market implied probability for horse 1 is then

$$q\frac{a + N(1 - G(\hat{p}_1|1))}{2a + N(1 - G(\hat{p}_1|1)) + NG(\hat{p}_{-1}|1)} + (1 - q)\frac{a + N(1 - G(\hat{p}_1|-1))}{2a + N(1 - G(\hat{p}_1|-1)) + NG(\hat{p}_{-1}|-1)}.$$

Ex ante, the expected payoff of a marginal extra bet on horse 1 is

$$qW(1|1) - 1 = q\frac{2a + N(1 - G(\hat{p}_1|1)) + NG(\hat{p}_{-1}|1)}{a + N(1 - G(\hat{p}_1|1))} - 1.$$

Using the equilibrium conditions (4.2) and (4.3), we can solve for $(\hat{p}_1, \hat{p}_{-1})$ as a function of q and compute these quantities. Figure 5.1 shows the resulting plot in our linear example, when the track take is $\tau = .15$. We have solved the system numerically, using Maple.[6]

In the parimutuel system, the amount bet on the longshot is almost completely lost. This is because the limited interest obtained by longshots is bad news about their chance of winning. In comparison, Figure 3.1 reveals that the loss to a bet on a longshot is bounded below. In fixed-odds betting, an ex-ante longshot is unlikely to attract insiders, so competition among bookmakers leads them to set more attractive odds on longshots.

[6] Our Maple worksheet is available upon request.

A related striking feature of Figure 5.1 is that favorites present a strictly positive return, when the number of insiders is small. This arbitrage opportunity arises since the cash-constrained insider population cannot correct the mis-pricing inherent in the fair noise bets.

Finally, we observe in Figure 5.1 that in parimutuel betting the expected loss to a given long market odds ratio is decreasing in the population share of insiders. To intuitively understand this, notice that a long odds ratio $(a + b_{-1}) / (a + b_1) > 1$ arises for a smaller information ratio b_{-1}/b_1 when the total amount of insiders bets (both b_{-1} and b_1) are larger. Thus, the same ratio is actually less informative news against the longshot, when the insider population is larger. So, for relative longshots, more insider betting serves to limit the losses to uninformed bettors (who bet on the longshot). In contrast, in fixed-odds markets the bookmaker always increases the spread in the presence of more insiders.

The comparative statics of the favorite-longshot bias with respect to the prevalence of insider betting works differently depending on the market structure. In parimutuel markets an increase in the number of uninformed bettors results in more of a bias for any given market odds realization, but at the same time drives market odds towards greater extremes where the bias is smaller. In fixed-odds markets instead, an increase in the fraction of uninformed bettor results in a reduction of the markup and so in less favorite-longshot, due to the fact that the adverse selection is reduced if there are more uninformed bettors.

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