Exit Polls and Voter Turnout*

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March 6, 2012
(First Version: September 28, 2010)

Abstract

After the 2009 referendum on a proposed change to the Danish Law of Succession it was widely claimed that the early publication of exit poll results changed the rate of turnout and eventually the outcome. We investigate this claim and contribute to the wider debate on the implications of exit polls by setting up and analyzing a formal model. We find that the introduction of an exit poll influences the incentive to vote both before and after the poll is published, but the signs of the effects are generally ambiguous. The observation that exit polls influence the incentive to vote even before they are published is often overlooked. We show that this can lead to premature conclusions about the impact of exit polls on electoral outcomes. In particular, in cases like the Danish referendum where it clearly looks like exit polls changed the outcome, it could well be that the outcome would have been the same had there been no exit polls.

*We thank David Dreyer Lassen and participants in the 2011 Annual Meeting of the Southern Political Science Association for valuable comments. All remaining errors are ours. Thomas Jensen gratefully acknowledges funding from The Danish Council for Independent Research | Social Sciences (grant number 09-066752).
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1 Introduction

On June 7, 2009, Danish voters went to the polls to decide on a proposed change to the Danish Law of Succession, the law that governs the succession to the Danish throne. The proposed change would imply that sons would no longer have precedence over daughters in the line of succession, establishing so-called equal primogeniture.

The referendum was subject to the procedure that governs changes to the Danish constitution. In order to pass, the proposal therefore had to overcome two obstacles: One, a majority of the votes cast in the referendum had to be in favor of the proposal. And two, at least 40% of all eligible voters had to vote in favor of the proposal.² In the weeks preceding the referendum, there was no doubt that only the latter of these requirements had the potential to become binding. In a Gallup poll released a week before the election, 84% of respondents indicated that they approved of the proposal to change the law. However, only 40.2% responded that they would show up at polls and vote in favor of the proposal.

On the afternoon of the election day, TV2, a major Danish TV channel, published the results of an exit poll, which predicted that 37.9% of all eligible voters would cast a vote in favor of the proposal to change the law. However, during the evening the situation turned around with pollsters reporting a considerable increase in turnout. In the end, the official result was that 45.1% of all eligible voters had voted in favor of the proposal, which corresponded to 85.4% of all votes cast. Thus, the proposal passed with a comfortable margin.

The discrepancy between the early exit poll and the final result sparked a lively public debate in the days following the referendum. The fact that voter turnout rose in the final hours before the polling stations closed led many observers to conclude that it was the publication of the exit poll itself, and the prospect of the proposal failing, that got additional supporters of the proposal to the polling stations. Without the exit poll, the proposal would not have passed, the argument went.² The Social Democrats, the major opposition party, took the opportunity to propose that Denmark follow a number of other countries in prohibiting the

¹Such requirements, known as approval quorum requirements, are common in European referenda. See Aguiar-Conraria and Magalhães (2010) for a recent overview of quorum rules in national referendums in EU countries.

²For example, on June 9, the Danish newspaper Politiken ran a front page story under the headline "Equal rights in the Royal family were saved by exit poll." Professor Jørgen Elklit of Aarhus University was quoted for saying that “the prospect of a no made more people vote.” A related article from the same day (p. 4) brought the following quote from Professor Johannes Andersen of Aalborg University: “It appears that 5 percent of voters changed their behavior in the last hours. They would not have voted if they had not been made aware that votes were lacking.” (All translations by the authors.)
publication of exit polls before polling stations are closed.\(^3\)

In this article we present a theoretical model to analyze whether and how exit polls influence voter turnout and outcomes in referendums (or elections) with two alternatives. We assume that voting takes place over two stages. Voters can choose to vote in either stage, or to abstain. Before stage two, an exit poll reveals how voters voted in the first stage. Any remaining potential voters then use this information to refine their decision rule for stage two.

Voters face heterogeneous costs of participating in the referendum. Costs are net of any direct benefits of participation and may thus be positive or negative.\(^4\) Each voter knows his own cost, but has only probabilistic knowledge about others’ costs (as in Palfrey and Rosenthal 1985). Thus, when the exit poll reveals all early votes it provides a remaining potential voter with information about, loosely speaking, how close the race is. But it also allows him to update his beliefs about the realized costs of the other remaining voters, i.e., those that did not vote early. The number of voters preferring each of the two alternatives is public information, for example because of widely reported pre-election opinion polls.

We restrict attention to referendums in which the outcome is determined not only by simple majority; one of the alternatives, typically a proposal to change a status quo, must also receive a certain number of votes in order to beat the other (an approval quorum requirement). Moreover, we simplify our model by assuming that all voters prefer the same alternative. This may appear overly restrictive, but it closely mimics the situation in the Danish referendum: Since more than 80\% of voters were in favor of changing the law, it was clear that the proposal would pass if and only if at least 40\% of the electorate voted for it. In that case, the opponents of the proposal would have no chance of getting a majority against it. This effectively made their behavior irrelevant for the outcome. Our simple model can also be seen as a building block that can be used in models of exit polls in more general electoral settings. For example, suppose that there are two alternatives and the electoral rule is simple majority. Then the supporters of each alternative are facing a problem that is similar to the one in our model: Taking the number of votes for the other alternative as given, they have to coordinate on beating this number of votes. Of course, since the required turnout for each group is endogenous, the general model becomes much more difficult to analyze. Still, the analysis in this paper can be seen as one step in this direction.

Our analysis shows that the introduction of an exit poll influences potential voters’ incentive to vote both before and after the release of the poll. For individ-

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\(^3\)Countries that prohibit the publication of exit polls before voting has ended include Germany, India, Norway, and United Kingdom.

\(^4\)In terms of the standard rational choice voting model (see, for example, Blais 2000 or Dowding 2005), a negative cost means that the D-term is greater than the C-term.
uals who have not yet voted when the exit poll is released, the poll’s effect on the incentive to cast a late vote depends crucially on what it reveals. An exit poll that reveals a close race increases the probability of being pivotal, which stimulates late voting, while an exit poll revealing that the race is far from close does the opposite. While this is a rather basic insight, the exit poll’s effect on the incentive to cast an early vote is more subtle: Intuitively, voters who face a positive participation cost may find it worthwhile to await the result of the poll before they decide whether to abstain or vote. This *wait-and-see effect* discourages early voting. But the exit poll may also stimulate early voting. For example, by voting early, a voter may use the exit poll actively to convince other supporters of the same alternative that victory is within reach, and this could induce them to participate in the referendum. As a result, the exit poll’s effect on the incentive to vote early is generally ambiguous.

In relation to the debate on the implications of exit polls, the most important insight from our analysis is the following: It may well happen that the incentive to vote increases after the revelation of an exit poll. Furthermore, the extra turnout generated after the release of the poll could be sufficient to ensure that voters’ preferred alternative is implemented, whereas it would not have been implemented if voters had continued to behave as before the release of the exit poll. This is essentially the situation that led several commentators, politicians, and academics to conclude that exit polls changed the outcome of the Danish referendum. However, our model reveals that this conclusion could well be wrong. The problem is that it is not based on the correct counterfactual, which is what would have happened had there been no exit poll, not what would have happened if voters continued to behave as before the release of the exit poll. Some of the voters showing up at the polls after the release of the exit poll may just have postponed voting because of the wait-and-see effect. Thus, they might well have voted anyway had there been no exit poll. If so, the outcome would have been no different, the proposal would still have passed. A stylized version of such a situation is depicted in Figure 1.

Another possible example of a wait-and-see effect was seen during the Polish referendum on entering the European Union on June 7-8, 2003 (see Markowski and Tucker 2005 for a detailed study of this referendum). The referendum was subject to a participation quorum: it would only be valid if at least 50% of voters went to the polls. After the first day of voting, turnout was below 18%. However, the final turnout was 59% (and 77% voted in favor of joining the EU). Thus, it is at least a possibility that some voters awaited reports on turnout after the first day and then, when reported turnout was low, went to the polls on the last day of the referendum.
Figure 1: Turnout increases after the publication of an exit poll, but total turnout would have been the same had there been no exit poll.

The existing literature on the effect of exit polls is, to our knowledge, limited. In a US context, some researchers have explored the consequences of exit polls being used by the media to make early predictions in eastern states while the polls are still open for several hours on the west coast. For example, even if the results of exit polls from New York are not published before the end of voting there, they may still influence voting behavior in California. The evidence on this possible effect is not conclusive, but some studies suggest that in the 1980 presidential election, where pre-election polls showed a close race, early predictions suggesting a landslide for Reagan had a negative effect on late turnout in California (Delli Carpini 1984; Jackson 1983; Sudman 1986). See also Thompson (2004) for a discussion of how exit polls and early projections change the temporal structure of elections.

Pre-election opinion polls in two-candidate majoritarian elections have been studied theoretically by Goeree and Großer (2007) and Taylor and Yildirim (2010). They reach similar conclusions: Polls stimulate supporters of the alternative favored by a minority of the population to participate in the election, while individuals who support the alternative favored by a majority participate less frequently because of free riding incentives. Since we study exit polls rather than pre-election polls, voters in our model have the opportunity to vote before the result of the

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For a recent experimental study on the effects of public opinion polls, see Großer and Schram (2010).
poll is revealed. This makes our model dynamic and thus opens up for strategic interactions among potential voters that are absent in the models of pre-election polls.

Because we restrict attention to a specific electoral setting where we only need to consider the behavior of one voter group, our model is a dynamic version of a threshold public good game with binary contributions. Static versions of such games has been extensively studied in the literature, see for example Palfrey and Rosenthal (1984; 1988) and Bagnoli and Lipman (1989). Furthermore, there is also a substantial literature on dynamic public goods provision. For example, while the set-up is clearly different from ours, Yildirim (2006) also highlights the tradeoff between the incentive to wait and see in order to free ride on others and the incentive to contribute early to encourage others to contribute as well.

2 The Model

We consider a referendum with two alternatives, A and B. The electoral rules are such that alternative A is implemented if the number of votes for A is at least M and higher than the number of votes for B. Otherwise alternative B is implemented. Thus, letting $V_z$ denote the number of votes for alternative $z$, the outcome is A if and only if $V_A \geq \max\{M, V_B + 1\}$. We will only consider the special case where the number of B supporters in the population is commonly known (e.g., from pre-election polls) to be strictly smaller than M. So the behavior of the B supporters is irrelevant and alternative A is implemented if and only if it receives at least M votes.

The number of voters who prefer alternative A over alternative B is denoted $N$. They all receive a utility of 1 if A is implemented and a utility of 0 if B is implemented. Each voter $i$ faces a cost $c_i$ of participating in the election. The voting costs are drawn independently from a common probability distribution on $\mathbb{R}$, characterized by a cumulative distribution function $F$. The realized cost $c_i$ is private information to voter $i$.

Each voter can either vote early, vote late, or abstain. If there is no exit poll then no information is revealed during the referendum and thus the situation is equivalent to simultaneous voting. If there is an exit poll then early votes are revealed before the second stage of voting and this is common knowledge at the beginning of the game. So we model an exit poll as complete revelation of all early votes. This is clearly stylized, but we see it as a reasonable starting point.

Each of the set-ups (with or without exit poll) represents a private information game and we use standard solution concepts. When there is no exit poll the game is static, so we use Bayesian Nash Equilibrium. With an exit poll the game
is dynamic and we therefore use Perfect Bayesian Equilibrium. We will restrict
attention to symmetric equilibria, i.e., we assume that all agents use the same
strategy in equilibrium. Furthermore, we will assume that all agents use cut-off
strategies. That is, we assume that there is a critical cost level, such that each
voter \( i \) votes if and only if \( c^i \) is below this level. So in the simultaneous voting
game an equilibrium can simply be described by a single cut-off cost \( \bar{c} \). In the exit
poll game an equilibrium can be described by a vector
\[
c^* = (c_1^*, c_2^*(0), c_2^*(1), \ldots, c_2^*(M - 1)),
\]
where \( c_1^* \) is the cut-off cost in stage one and \( c_2^*(n) \) is the cut-off cost in stage two if
exactly \( n \leq M - 1 \) voters voted in stage one (if \( M \) or more voters voted in stage
one then stage two is obviously trivial).

If there is no exit poll then it is easily seen that the cut-off cost \( \bar{c} \) is an equilib-
rium if and only if it is equal to the probability of a voter being pivotal given that
all other voters use the \( \bar{c} \) strategy (i.e., the probability that exactly \( M - 1 \) out of
the other \( N - 1 \) voters have a voting cost below \( \bar{c} \)):
\[
\bar{c} = \left( \frac{N - 1}{M - 1} \right) F(\bar{c})^{M-1}(1 - F(\bar{c}))^{N-M}.
\] (1)

Then consider the exit poll game. We want to find the conditions for \( c^* \) to be
an equilibrium. First, suppose we are in stage two and that exactly \( n \leq M - 1 \)
voters voted in stage one. Agent \( i \) can then infer that the other remaining \( N - n - 1 \)
potential voters must have costs in excess of the stage one cut-off, \( c_1^* \). Then, by
analogy with the no exit poll game, \( c_2^*(n) \) must satisfy
\[
c_2^*(n) = \left( \frac{N - n - 1}{M - n - 1} \right) \Pr(c^i \leq c_2^*(n) | c^i > c_1^*)^{M-n-1}.
(1 - \Pr(c^i \leq c_2^*(n) | c^i > c_1^*))^{N-M}.
\] (2)

The condition that must be satisfied by the stage one cut-off \( c_1^* \) is more com-
plicated. For now, we simply note that at the cut-off \( c^i = c_1^* \), the expected utility
of voting early must be equal to the expected utility of waiting.

3 Analysis of Special Cases

In this section we will solve the model described above for \((N, M) = (2, 1), (2, 2), \) and
(3, 2). In each case we will compare the no exit poll solution to the exit
poll solution. Clearly, the \( N = 2 \) cases are extremely simplistic. However, they
are illustrative because each of them contains in isolation one of two collective action issues that are relevant in real world elections. When \( M = 1 \) there is a free riding problem: Each voter (with a positive voting cost) wants alternative \( A \) to be implemented but prefers to stay home while the other person votes. Obviously, the free riding problem is not present when \( M = 2 \). In that case we instead have a coordination problem: Each voter (with a voting cost between zero and one) wants to vote if and only if the other voter votes as well. The case of \( N = 3, M = 2 \) is the simplest case in which both problems are present. So, even though this case is obviously still very simplistic as a model of real world elections, it takes us a substantial step in the right direction.

As described earlier, the voting cost of each voter is drawn from a common distribution on the real axis represented by a distribution function \( F \). We will assume that \( F \) is twice differentiable on \([0, 1] \) with \( F' > 0 \) and either \( F'' \leq 0 \) or \( F'' \geq 0 \) on this interval. The most restrictive of these assumptions is that the second derivative of \( F \) is either non-positive or non-negative. This means that, for example, \( F \) cannot be given by a density function \( f \) with mode in \((0, 1) \). We will also assume that \( F(0) > 0 \) and \( F(1) < 1 \). This means that there is a positive probability that a given voter will always vote and also a positive probability that he will never vote. All of these assumptions are made for existence and uniqueness purposes. In some cases they are substantially stronger than we need, but for simplicity we keep them throughout the section.

As an example of a voting cost distribution we will use the uniform distribution on some interval \([\varepsilon_L, 1 + \varepsilon_H] \), where \( \varepsilon_L, \varepsilon_H > 0 \). Such a distribution clearly satisfies all the assumptions above. Note that we can write \( F \) as \( F(c) = F(0) + (F(1) - F(0))c \) for all \( c \in [\varepsilon_L, 1 + \varepsilon_H] \). In fact, under the assumption of a uniform distribution we will treat the values \( F(0) \) and \( F(1) \) (with \( 0 < F(0) < F(1) < 1 \)) as parameters. When we do so it is important to remember that by changing \( F(0) \) or \( F(1) \) we change the support of the distribution.\(^6\)

### 3.1 \( N = 2 \) and \( M = 1 \): Free riding

First note that \( \bar{c} \) is an equilibrium in the no exit poll game precisely if

\[
\bar{c} = 1 - F(\bar{c}).
\]  

\(^6\)Of course, \( \varepsilon_L \) and \( \varepsilon_H \) can be expressed in terms of \( F(0) \) and \( F(1) \) (and vice versa). By simple calculations we get

\[
\varepsilon_L = \frac{F(0)}{F(1) - F(0)} \quad \text{and} \quad \varepsilon_H = \frac{1 - F(1)}{F(1) - F(0)}.
\]
The function $1 - F(c)$ is positive at $c = 0$, below one at $c = 1$, and decreasing. From these observations it easily follows that there exists a unique equilibrium $\bar{c} \in (0, 1)$.

Then consider the exit poll game. For $c^* = (c_1^*, c_2^*(0))$ to be an equilibrium, $c_2^*(0)$ must satisfy

$$c_2^*(0) = 1 - \Pr(c^j \leq c_2^*(0)|c^j > c_1^*).$$

If $c_2^*(0) \leq c_1^*$ then we get $c_2^*(0) = 1 \leq c_1^*$ and then $c^*$ is obviously not an equilibrium. Thus, we must have $c_2^*(0) > c_1^*$ and then the equation above can be written

$$c_2^*(0) = 1 - \frac{F(c_2^*(0)) - F(c_1^*)}{1 - F(c_1^*)} = \frac{1 - F(c_2^*(0))}{1 - F(c_1^*)}.$$

Now consider stage one. We first claim that $c_1^* \leq 0$. Suppose that we had an equilibrium with $c_1^* > 0$ and consider a voter $i$ with $c^i \in (0, c_1^*)$. He would then vote in stage one. However, abstaining in stage one would actually ensure him a higher expected payoff because it may save him a costly vote (if the other voter votes in stage one). Thus we must have $c_1^* \leq 0$.

A voter with $c^i \leq 0$ will clearly always vote, but is indifferent between voting early and voting late. However, by voting early he may save the other voter a costly vote. So if we assume that a voter who is indifferent with respect to his own payoff will maximize the payoff of the other voter then we have $c_1^* = 0$. And then the conditions for $c^*$ to be an equilibrium are

$$c_1^* = 0 \text{ and } c_2^*(0) = \frac{1 - F(c_2^*(0))}{1 - F(0)}. \quad (4)$$

It is easy to see that there is a unique $c_2^*(0) \in (0, 1)$ satisfying (4) (mimicking the argument from the no exit poll situation) and that $c_2^*(0) > \bar{c}$. The intuition behind this result is straightforward: Consider the problem facing voter $i$ in stage two. He can infer that the other voter does not have a negative voting cost, since he did not vote in stage one. Thus, for a given cut-off strategy of the other voter, the probability that he votes is lower than in the no exit poll game. This makes free riding less attractive and voter $i$ will therefore vote for higher realized voting costs.

Table 1 summarizes the total number of votes for all combinations of realized voting costs in both the exit poll game and the no exit poll game.

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7 Even if $c_1^* < 0$ (i.e., if voters with negative costs do not care about saving the other voter a costly vote) we get the same conclusion as long as $F(c_1^*) > 0$. However, the closer $F(c_1^*)$ is to zero, the closer is $c_2^*(0)$ to $\bar{c}$. 
We see that there is no general conclusion as to whether turnout is higher with or without an exit poll. For example, turnout is higher with an exit poll when \( c < c^1 \leq c^2 \) and \( c^2 > 0 \), while it is the other way around when \( 0 < c^1 \leq c^2 \). However, note that if the realized voting cost of each voter is positive then the exit poll always leads to a level of turnout that is at least as high as without an exit poll and sometimes higher. Also note that the exit poll game dominates the no exit poll game with respect to implementation of \( A \): If \( A \) is implemented without an exit poll then it is also implemented with an exit poll and sometimes it is implemented with an exit poll but not without it.

3.2 \( N = M = 2 \): Coordination

In this case a voter is pivotal if the other voter votes. So the condition for \( c \) to be an equilibrium in the no exit poll game is

\[
\bar{c} = F(\bar{c}).
\]  

Since \( F(0) > 0, F(1) < 1 \), and \( F' \) is either non-increasing or non-decreasing in the interval \([0, 1]\), it easily follows that we have a unique equilibrium \( \bar{c} \in (0, 1) \).

Then consider the exit poll game. In stage two, if one of the voters contributed in stage one then the other voter knows he is pivotal. Thus we have \( c^2(1) = 1 \). If no votes were cast before the exit poll, then the stage two game is very similar to the no exit poll game. The only difference is that each voter can infer that the realized cost of the other voter is above \( c^*_1 \). So \( c^*_2(0) \) must satisfy

\[
c^*_2(0) = \Pr(c^j \leq c^*_2(0)|c^j > c^*_1) = \max\{0, \frac{F(c^*_2(0)) - F(c^*_1)}{1 - F(c^*_1)}\}. \tag{6}
\]

Now consider stage one. Voter \( i \)'s payoff from voting in this stage is (let \( j \) denote the other voter)

\[1 - \Pr(c^j > 1) - c^i.\]

His payoff from not voting early is (assuming that \( j \) follows the strategy \( c^* \) and
that \( i \) follows \( c^* \) in stage two:

\[
1 - \Pr(c^j > \max\{c^*_1, c^*_2(0)\}) - c^i \quad \text{if } c^i \leq c^*_2(0) \\
\Pr(c^j \leq c^*_1)(1 - c^i) \quad \text{if } c^*_2(0) < c^i \leq 1 \\
0 \quad \text{if } c^i > 1
\]

In any equilibrium we must have \( c^*_1, c^*_2(0) < 1 \) (since \( F(1) < 1 \)). And then it immediately follows from the expressions above that voting early is better than not voting early for voter \( i \) if \( c^i \leq c^*_2(0) \). Thus we must have \( c^*_1 \geq c^*_2(0) \), and it then follows from equation (6) that \( c^*_2(0) = 0 \). So we have that, in equilibrium, if no votes are cast in stage one then no votes will be cast at all. We also see that the equilibrium condition for \( c^*_1 \) is

\[
1 - c^*_1 - \Pr(c^j > 1) = \Pr(c^j \leq c^*_1)(1 - c^*_1),
\]

which is equivalent to

\[
c^*_1 = \frac{F(1) - F(c^*_1)}{1 - F(c^*_1)}.
\]

So, summing up, \( c^* = (c^*_1, c^*_2(0), c^*_2(1)) \) is an equilibrium if and only if

\[
c^*_1 = \frac{F(1) - F(c^*_1)}{1 - F(c^*_1)}, \quad c^*_2(0) = 0, \quad \text{and } c^*_2(1) = 1.
\] (7)

By standard arguments we see that there is a unique solution \( c^*_1 \in (0, 1) \) to the first equation, so there is a unique equilibrium in the exit poll game.

Our next step is to compare the outcome of the exit poll situation with the outcome when there is no exit poll. Voter turnout with and without an exit poll as a function of the realized costs of the voters (\( c^1 \) and \( c^2 \)) is summarized in Table 2 (\( c^*_1 \geq \bar{c} \)) and 3 (\( c^*_1 < \bar{c} \)).

**Table 2: Turnout with [without] exit poll when \( N = M = 2 \) and \( c^*_1 \geq \bar{c} \)**

<table>
<thead>
<tr>
<th>Voter 1, ( \rightarrow ) Voter 2</th>
<th>( c^2 \leq \bar{c} )</th>
<th>( \bar{c} &lt; c^2 \leq c^*_1 )</th>
<th>( c^*_1 &lt; c^2 \leq 1 )</th>
<th>( c^2 &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c^1 \leq \bar{c} )</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{c} &lt; c^1 \leq c^*_1 )</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c^*_1 &lt; c^1 \leq 1 )</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c^1 &gt; 1 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 3: Turnout with [without] exit poll when \( N = M = 2 \) and \( c^*_1 < \bar{c} \)**

<table>
<thead>
<tr>
<th>Voter 1, ( \rightarrow ) Voter 2</th>
<th>( c^2 \leq c^*_1 )</th>
<th>( c^*_1 &lt; c^2 \leq \bar{c} )</th>
<th>( \bar{c} &lt; c^2 \leq 1 )</th>
<th>( c^2 &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c^1 \leq c^*_1 )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>( c^*_1 &lt; c^1 \leq \bar{c} )</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{c} &lt; c^1 \leq 1 )</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c^1 &gt; 1 )</td>
<td>1</td>
<td>1</td>
<td>0</td>
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</tr>
</tbody>
</table>
Clearly, the effects of an exit poll is highly dependent on whether $c_1^* \geq \bar{c}$ or $c_1^* < \bar{c}$. If the first inequality holds then the exit poll outcome dominates the no exit poll outcome with respect to both turnout and implementation of $A$. If the second inequality holds then the effects of an exit poll on turnout and implementation of $A$ are ambiguous, i.e., positive for some combinations of realized voting costs and negative for others. Therefore, it is important to find out when we have $c_1^* \geq \bar{c}$ and when it is the opposite inequality that holds. The proposition below states our main results for the $N = M = 2$ case. The proof of this and two later propositions can be found in the Appendix.

**Proposition 1 (The effects of an exit poll when $N = M = 2$)**

1. If the distribution function $F$ satisfies

$$F'' \geq 0 \text{ (on } [0, 1]) \text{ and } F(0) \leq (\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)})$$

then $c_1^* \geq \bar{c}$. This implies that an exit poll has unambiguous positive effects on the level of turnout and the implementation of $A$.

2. If the distribution function $F$ satisfies

$$F'' \leq 0 \text{ (on } [0, 1]) \text{ and } F(0) \geq (\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)})$$

then $c_1^* < \bar{c}$. This implies that an exit poll has ambiguous effects on the level of turnout and the implementation of $A$ (for some combinations of realized voting costs the effect is positive, for others it is negative).

Let us explore some of the intuition behind the results above. For simplicity, assume that $F'' = 0$, i.e., consider a uniform distribution of voting costs. We are especially interested in why $c_1^* \geq \bar{c}$ if $F(0)$ is sufficiently low, while we have the opposite inequality when it is not. Therefore, we hold $F(1)$ fixed. This of course means that a change in $F(0)$ corresponds to changes in both the upper and lower limits of the support of the distribution.

An exit poll influences a voter’s incentive to vote early in two ways. First, the exit poll gives rise to a *encouragement effect*, which stimulates early voting: If voter $i$ votes early then he might induce the other voter to vote as well. Intuitively, if the other player observes that player $i$ voted in stage one, he will know with certainty that he is pivotal in stage two, which makes voting more attractive. Second, the exit poll produces a wait-and-see effect that discourages early voting. This effect parallels the wait-and-see effect from the free riding case studied above, but now
the incentive to wait and see comes from a desire to avoid casting a costly but useless vote, not from a desire to free ride on the other voter.

How does the size of $F(0)$ affect the relative strengths of these two opposite effects? As $F(0)$ increases it becomes less likely that the encouragement effect from an early vote will kick in. Because, for any $c \in (0, 1)$, the probability that the other voter has a cost in the interval $(c, 1)$ becomes lower. Therefore, it becomes less likely that an early vote by $i$ will make $j$ switch from abstention to late participation. On the other hand, an increase in $F(0)$ makes it more attractive to wait and see in stage one, since the probability that the other voter votes early is now higher. In total, these observations imply that as $F(0)$ rises, not voting in stage one becomes relatively more attractive than voting.

The role of $1 - F(1)$ in the results is more subtle. An increase in $1 - F(1)$ (we now hold $F(0)$ fixed) makes it less likely that an early vote will make the other voter vote in stage two, thus weakening the encouragement effect. But the effect on the benefit of waiting is also negative because it becomes less likely that the other voter will vote early. And indeed, it follows from the proposition that the effect can go either way, the expression $(\sqrt{1-F(1)})(1-\sqrt{1-F(1)})$ is not monotone with respect to $1 - F(1)$.

### 3.3 $N = 3$ and $M = 2$: Free riding and coordination

Our final case combines the insights from the previous two cases. With three potential voters and two votes required for implementation of $A$, the free riding problem and the coordination problem are both relevant. A voter is now pivotal if precisely one of the other two voters votes. Thus $c$ is an equilibrium in the no exit poll game if and only if

$$
\bar{c} = 2F(\bar{c})(1 - F(\bar{c})).
$$

We will in this case assume that the distribution of voting costs is given by a uniform distribution on some interval containing $[0, 1]$. So we have $F(c) = F(0) + (F(1) - F(0))c$ for all $c$'s in the support of the distribution. With this assumption, the function on the right hand side of the equilibrium equation above becomes a second order polynomial with negative second derivative. This polynomial is positive at $c = 0$ and its maximum value is $\frac{1}{2}$. From these observations it easily follows that there is a unique equilibrium $\bar{c} \in (0, 1)$. It is straightforward to solve explicitly for the equilibrium. The solution, which is of course a function of $F(0)$ and $F(1)$, can be found in the appendix.

Then consider the exit poll game. An equilibrium is written $c^* = (c_1^*, c_2^*(0), c_2^*(1))$. The following proposition establishes existence of an equilibrium with $c_2^*(0) = 0$. 13
Proposition 2 (Existence of exit poll equilibrium when \( N = 3, M = 2 \))

There exists an equilibrium of the exit poll game with \( c_2^*(0) = 0 \) and \( c_1^*, c_2^*(1) \in (0,1) \). For all such equilibria it holds that \( c_1^*, \bar{c} < c_2^*(1) \).

If we have an equilibrium of the exit poll game with \( c_1^* \geq \bar{c} \) then the turnout will always (i.e., for all cost realizations) be at least as high as in the no exit poll equilibrium and sometimes higher. Furthermore, we also have that the exit poll outcome dominates the no exit poll outcome with respect to implementation of \( A \). On the other hand, if we have an equilibrium with \( c_1^* < \bar{c} \) and \( c_2^*(0) = 0 \) then there are cost realizations such that the turnout is zero in this equilibrium while it would be at least two without an exit poll (for example, let \( c_i^j \in (c_1^*, \bar{c}) \) for all voters \( i \)). In Proposition 3 below we find existence conditions for these two types of exit poll equilibria.

To state the proposition, we need the following definition. Let \( c_2(\bar{c}) \in (0,1) \) be the unique number given by the equation

\[
c_2(\bar{c}) = \frac{1 - F(c_2(\bar{c}))}{1 - F(\bar{c})}.
\]

This is the equation for the cut-off strategy \( c_2(\bar{c}) \) to be optimal in stage two after one vote in stage one, given that all voters used the cut-off strategy \( \bar{c} \) in stage one and that the other remaining voter also uses the \( c_2(\bar{c}) \) strategy in stage two.

Proposition 3 (The effects of an exit poll when \( N = 3, M = 2 \))

1. Suppose the following condition is satisfied:

\[
(1 - F(\bar{c}))^2 - (1 - F(c_2(\bar{c})))^2 \geq \bar{c}(1 - \bar{c}).
\]

Then there exists an equilibrium of the exit poll game with \( c_1^* \geq \bar{c} \), i.e., an equilibrium that dominates the no exit poll equilibrium with respect to both turnout and implementation of \( A \).

2. Suppose the condition from part one is not satisfied. Then there exists an equilibrium of the exit poll game with \( c_1^* < \bar{c} \) and \( c_2^*(0) = 0 \). In such an equilibrium there are cost realizations such that turnout is zero while it would be at least two if there were no exit poll.

3. For any equilibrium of the exit poll game there exist cost realizations such that the turnout is at least two while it would be at most one in the no exit poll equilibrium.
Because of our assumption about a uniform distribution of voting costs it is straightforward to explicitly write $c_2(\bar{c})$ as a function of $\bar{c}$:

$$c_2(\bar{c}) = \frac{1 - F(0)}{1 - F(0) + (F(1) - F(0))(1 - \bar{c})}.$$ 

And thus, given that we have already found an explicit expression for $\bar{c}$ in terms of $F(0)$ and $F(1)$ (see the appendix), we can in principle write the condition from part one of Proposition 3 as a condition on $F(0)$ and $F(1)$. However, the expressions on either side of the inequality are complicated and it seems impossible to analytically derive insights from them. So instead we numerically calculate each side of the inequality. And then, treating $F(0)$ and $1 - F(1)$ as parameters, we plot whether the condition from part one holds or not. See Figure 2 for the plot. Note that we use $1 - F(1)$ instead of $F(1)$ as a parameter because then our parameters are, respectively, the probability that a voter will always vote and the probability that a voter will never vote.

**Figure 2**: The effects of an exit poll when $N = 3$ and $M = 2$ (see Proposition 3)

We see that in a large region of the $(F(0), 1 - F(1))$ parameter space we can only conclude that the introduction of an exit poll can have both positive and
negative effects on turnout and the implementation of $A$. Only in a much smaller subset of the parameter space, in which $F'(0)$ is below ten percent and $1 - F'(1)$ is above thirty percent, can we guarantee the existence of an equilibrium such that the introduction of an exit poll has unambiguous positive effects on the level of turnout and the implementation of $A$. This dovetails nicely with the results from the pure coordination case ($N = M = 2$) and the intuition is similar.

We end our analysis with an important observation, especially in relation to the debate following the 2009 Danish referendum. Suppose there is an exit poll and that the inequality in part one of Proposition 3 is not satisfied (the large region in Figure 2). Then there exists an equilibrium with $c_1^* < \bar{c} < c_2^*(1)$ and $c_2^*(0) = 0$. Suppose that the three voters face cost realizations such that $c^1 \leq c_1^*$, $c^2 \in (c_1^*, \bar{c})$, and $c^3 > c_2^*(1)$. With this combination of costs, voter 1 will vote early, voter 2 will vote late, and voter 3 will abstain. Alternative $A$ is therefore implemented with a minimal margin of victory. To the casual observer, this outcome may suggest the following interpretation: “The release of the exit poll raised the incentive to vote. This led voter 2, whose vote was pivotal to the outcome of the election, to cast a late vote. Without the exit poll, alternative $A$ would therefore not have been implemented.” This interpretation is wrong. In the absence of an exit poll, voter 1 and voter 2 would both have voted (since $c^1, c^2 < \bar{c}$) and the outcome of the election would thus have been the same. The problem with the erroneous interpretation is that it presupposes that voter 2’s behavior before the release of the poll is indicative of how she would have behaved if there were no exit poll. In doing so, it ignores that it is in fact the exit poll itself that, through the wait-and-see effect, causes voter 2 to abstain in stage one.

4 Conclusion

In this article we have studied the impact of exit polls. We have shown that the introduction of an exit poll influences the incentive to vote both before and after the results of the poll are released. Before the exit poll is released, potential voters may find it worthwhile to await its result before they decide on whether to stay home or go to the polls. That way, they may be able to to free ride on other voters who support the same alternative, or they may avoid wasting time and effort on voting if their preferred alternative is bound to lose anyway. This effect thus discourages early voting. On the other hand, supporters of one alternative may also use the exit poll actively to coordinate their efforts: By influencing the result of the exit poll, early voters can induce fellow supporters to vote after the result of the poll is revealed, and this may stimulate early voting. In sum, the total effect of an exit poll on the incentive to vote early is ambiguous. Once the
results of the exit poll is released, the effect on remaining potential voters’ incentive to participate depends on the information revealed by the poll. Voting becomes more attractive if the poll reveals a close race, but less attractive if it reveals the opposite. As a result of these opposite effects, we find that an exit poll’s effect on voter turnout and election outcomes is in general ambiguous.

Much of the skepticism towards exit polls comes from the belief that they may change the outcome of elections and referendums. While we certainly agree that exit polls have the potential to change electoral outcomes, we also believe that the empirical case for this hypothesis is sometimes overstated. For example this was the case, we believe, in the debate following the 2009 Danish referendum. The problem with this debate was that it did not recognize the possibility that voter behavior was influenced by exit polls not only after, but also before the release of the first results. Hence, the low early turnout was interpreted as an indicator for what would have happened later in the day, had the results of the exit poll not been published. However, the low early turnout could be a consequence of voters knowing that exit polls would be published later in the day. Therefore the outcome might well have been the same had there been no exit polls. Our modelling highlights this possibility.
Appendix

Proof of Proposition 1.
We first make the following claim:

\[ c^*_1 > \bar{c} \iff \bar{c} \leq 1 - \sqrt{1 - F(1)}. \]

Proof of claim: First note that the functions \( F(c) \) and \( \frac{F(1) - F(c)}{1 - F(c)} \) intersects at precisely one cost \( \xi \in (0, 1) \). By solving a second order equation we get that \( F(\xi) = 1 - \sqrt{1 - F(1)}. \)

Suppose \( c^*_1 > \bar{c} \). Since \( \frac{F(1) - F(c)}{1 - F(c)} \) is decreasing and \( F(c) \) is increasing this is equivalent to these two functions intersecting below the 45\(^\circ\) line, which is obviously equivalent to \( F(\xi) < \xi \). Further, since \( F \) is increasing and intersects the diagonal precisely once, this is equivalent to

\[ \bar{c} = F(\bar{c}) < F(\xi) = 1 - \sqrt{1 - F(1)}. \]

Thus we have shown that \( c^*_1 > \bar{c} \iff \bar{c} < 1 - \sqrt{1 - F(1)}. \) The rest of the claim follows by completely analogous arguments.

Now suppose the distribution of voting costs is uniform on some interval containing \([0, 1]\) such that \( F(c) = F(0) + (F(1) - F(0))c \) for \( c \in [0, 1] \). Then, using the claim above, we get a condition on \( F(0) \) and \( F(1) \) that determines whether \( c^*_1 \geq \bar{c} \) or \( c^*_1 < \bar{c} \). Because, with the uniform distribution, a straightforward calculation shows that the equilibrium in the no exit poll game is

\[ \bar{c} = \frac{F(0)}{1 - (F(1) - F(0))}. \]

And then, by a bit of algebra, we see that \( \bar{c} \leq 1 - \sqrt{1 - F(1)} \) is equivalent to

\[ F(0) \leq (\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)}). \]

Finally, if \( F'' > 0 \) (on \([0, 1]\)) then \( F \) must intersect the diagonal at a lower cost than if we had a uniform distribution with the same \( F(0) \) and \( F(1) \) values. Thus, \( \bar{c} \) must be lower when \( F'' > 0 \) than when \( F'' = 0 \). Therefore, it follows that if

\[ F'' \geq 0 \text{ and } F(0) \leq (\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)}) \]

then \( c^*_1 \geq \bar{c} \). Part two of the proposition follows analogously. \( \Box \)

Equilibrium in the no exit poll game when \( N = 3, M = 2. \)
The unique equilibrium of the no exit poll game is

$$
\bar{c} = \frac{2(F(1) - F(0))(1 - 2F(0)) - 1}{4(F(1) - F(0))^2} + \frac{\sqrt{(2(F(1) - F(0)) - 1)^2 + 8F(0)(F(1) - F(0))}}{4(F(1) - F(0))^2}.
$$

**Proof.** Remember that the equilibrium condition is $\bar{c} = 2F(\bar{c})(1 - F(\bar{c}))$. Since the cost distribution is uniform, we have $F(c) = F(0) + (F(1) - F(0))c$ for all $c \in [0, 1]$ and thus the equilibrium condition becomes

$$
\bar{c} = 2(F(0) + (F(1) - F(0))\bar{c})(1 - F(0) - (F(1) - F(0))\bar{c}).
$$

This equation can be rewritten as

$$
2(F(1) - F(0))^2\bar{c}^2 + (4F(0)(F(1) - F(0)) - 2(F(1) - F(0)) + 1)\bar{c} - 2F(0)(1 - F(0)) = 0.
$$

Finally, solve for the positive solution to get the unique equilibrium. □

**Proof of Proposition 2.**

First we derive the conditions for $c^* = (c^*_1, 0, c^*_2(1))$ with $c^*_1, c^*_2(1) \in (0, 1)$ to be an equilibrium. The condition for $c^*_2(1)$ to be optimal is

$$
c^*_2(1) = \Pr(c^j > c^*_2(1)|c^j > c^*_1).
$$

If $c^*_2(1) \leq c^*_1$, the right-hand side is equal to 1. Thus, since $c^*_2(1) < 1$, we must have

$$
c^*_2(1) > c^*_1 \text{ and } c^*_2(1) = \frac{1 - F(c^*_2(1))}{1 - F(c^*_1)}. \quad (9)
$$

To derive the condition for $c^*_1$ to be optimal, first note that if voter $i$ votes in stage one then his expected payoff is

$$
1 - \Pr(c^j > c^*_2(1))^2 - c^j = 1 - (1 - F(c^*_2(1)))^2 - c^j = F(c^*_2(1))(2 - F(c^*_2(1))) - c^j.
$$

If $0 < c^j \leq c^*_2(1)$ then his payoff from not voting in stage one is

$$
\Pr(c^j \leq c^*_1)^2 + 2\Pr(c^j \leq c^*_1)(1 - \Pr(c^j \leq c^*_1))(1 - c^j) = F(c^*_1)^2 + 2F(c^*_1)(1 - F(c^*_1))(1 - c^j)
$$

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(since we have that \(c_2^*(1) > c_1^* > 0\), we do not need to consider the cases \(c^i > c_2^*(1)\) and \(c^i \leq 0\). Thus \(c_1^*\) must satisfy

\[
F(c_2^*(1))(2 - F(c_2^*(1))) - c_1^* = F(c_1^*)^2 + 2F(c_1^*)(1 - F(c_1^*))(1 - c_1^*).
\]

This equation is equivalent to

\[
c_1^* = \frac{(1 - F(c_1^*))^2 - (1 - F(c_2^*(1)))^2}{F(c_1^*)^2 + (1 - F(c_1^*))^2}.
\]  \hspace{1cm} (10)

Thus, if we can find a solution \(c_1^*, c_2^*(1) \in (0, 1)\) to (9) and (10) then we have proved the first statement of the proposition \((c_2^*(0) = 0\) is obviously optimal in stage two when no votes were cast in stage one).

For each \(c_1 \in [0, 1]\), let \(c_2(c_1)\) be the unique solution to the equation

\[
c_2(c_1) = \frac{1 - F(c_2(c_1))}{1 - F(c_1)}.
\]

Then \(c_2\) is a continuous function of \(c_1\) on \([0, 1]\) and we have \(c_2(c_1) > c_1\) for all \(c_1 \in [0, 1]\) (note that \(c_2(1) = 1\).

Now consider the equation

\[
c_1^* = \frac{(1 - F(c_1^*))^2 - (1 - F(c_2(c_1^*)))^2}{F(c_1^*)^2 + (1 - F(c_1^*))^2}.
\]

The right hand side of this equation (considered as a function of \(c_1^* \in [0, 1]\)) is positive at \(c_1^* = 0\), zero at \(c_1^* = 1\), and continuous. Thus it must intersect the diagonal at least once between zero and one, so we have at least one solution in \((0, 1)\) to this equation. Pick a solution \(c_1^*\) and let \(c_2^*(1) = c_2(c_1^*)\). Then we have a solution to (9) and (10) and the solution obviously satisfies \(c_1^* < c_2^*(1)\).

Finally, it remains to be shown that \(c_2^*(1) > \bar{c}\). Since \(\bar{c} = 2F(\bar{c})(1 - F(\bar{c})) \leq \frac{1}{2}\), it suffices to show that \(c_2^*(1) > \frac{1}{2}\). We have that \(c_2^*(1)\) satisfies

\[
c_2^*(1) = \frac{1 - F(c_2^*(1))}{1 - F(c_1^*)}.
\]

Thus, \(c_2^*(1)\) is given as the intersection between the function \((1 - F(\cdot))/(1 - F(c_1^*))\) and the diagonal. This function is a straight line (because \(F\) is linear) with a negative slope, is above one at zero, and above zero at one. So it is easy to see that it intersects the diagonal at a cost above one half. □

\textit{Proof of Proposition 3}.
1. Since \( \bar{c} = 2F(\bar{c})(1 - F(\bar{c})) = 1 - F(\bar{c})^2 - (1 - F(\bar{c}))^2 \), the inequality can be rewritten as

\[
\bar{c} \leq \frac{(1 - F(\bar{c}))^2 - (1 - F(c_2(\bar{c})))^2}{F(\bar{c})^2 + (1 - F(\bar{c}))^2}.
\]

Consider, for a moment, the right hand side as a function on \([0, 1]\). At 1 it is equal to zero and thus below the diagonal so (because of continuity) there must exist a \( c_1^* \in [\bar{c}, 1) \) with

\[
c_1^* = \frac{(1 - F(c_1^*))^2 - (1 - F(c_2(c_1^*)))^2}{F(c_1^*)^2 + (1 - F(c_1^*))^2}.
\]

And thus (see the proof of Proposition 2) we have an exit poll equilibrium with \( c_1^* \geq \bar{c} \) (and \( c_2^*(0) = 0 \)).

2. Analogous to the proof of part one.

3. Suppose \( c^* = (c_1^*, c_2(0), c_2(1)) \) is an exit poll equilibrium. Analogously to the proof of Proposition 2 it can be shown that \( c_2^*(1) > \bar{c} \). Any cost realization satisfying, for example, \( c_1 < c_1^* \) and \( \bar{c} < c^2 \leq c_2^*(1) \) would then result in at least two votes if there is an exit poll and at most one vote if there is not (one if \( c_1 \leq \bar{c} \), zero if \( c_1 > \bar{c} \)). \( \square \)
References


