Optimal Provision of Public Goods: A Synthesis*

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Abstract

This paper considers two competing approaches in the literature on the optimal provision of public goods. The standard approach highlights the importance of distortionary taxation and distributional concerns. The new approach neutralizes distributional concerns by adjusting the non-linear income tax, and finds that this reinvigorates the simple Samuelson rule when preferences are separable in goods and leisure. We provide a synthesis by demonstrating that both approaches derive from the same basic formula. We further develop the new approach by deriving a general, intuitive formula for the optimal level of public goods without imposing strong assumptions on preferences. This formula shows that distortionary taxation may have a role to play as in the standard approach. However, the main determinants of optimal provision are different and the traditional formula with its emphasis on MCF may very well lead to underprovision. (JEL: H41, H23, H11)

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1 Introduction

Cost-benefit analysis is an important tool in everyday government decision making on public projects. When carried out in practice, the dominating view seems to be that the costs of a tax-funded project should be adjusted according to the marginal cost of funds (MCF), as a close reflection of the deadweight loss that will materialize if the project is added to the budget.[1] Today, the theoretical foundation for such a practice is less clear.

The simple view described above originates from Pigou (1947) and was developed formally by Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974). They argued that the famous Samuelson rule—which equates the sum of the marginal willingness to pay for the public good of all citizens to the marginal rate of transformation (MRT)—relies on an unrealistic first-best setting where individual lump sum taxes are available. Instead, they base their analyses on distortionary taxation and arrive at a modified Samuelson rule where the effective cost of public goods is identified as MCF times MRT. This 'standard approach' has been very influential and also underlies the excellent survey of Ballard and Fullerton (1992).

The contributions above rely on a representative agent setting that does not justify the use of distortionary taxation. The standard approach has since been extended to allow for multiple households with heterogenous earnings abilities and by integrating the government spending side more thoroughly in the analysis (Dahlby, 1998; Sandmo, 1998; Slemrod and Yitzhaki, 2001; Gahvari, 2006; Kleven and Kreiner, 2006). Two important conclusions emerge from these extensions. First, the evaluation of public projects should

[1] See, for example, Boardman et al. (2006) p. 104. Evaluation of tax-funded public projects in Denmark assumes that the cost of financing is 1.2 times the actual expenditures, corresponding to the official Danish marginal cost of funds (the Danish Ministry of Transportation and Energy, 2003). Similarly, Parry and Small (2009) in their conclusion suggest an MCF correction of 1.15.

take account, not only of the distortionary effect of taxation as reflected by the MCF, but also of government revenue effects stemming from behavioral responses generated by the expenditure side of the projects. For example, a government investment in infrastructure or child care may increase working hours, and thereby tax revenue. Second, distributional concerns become important for the optimal level of public goods. It matters how benefits and costs are distributed across households.

In its most general form, the standard approach makes no assumptions about the initial level of public goods and the initial tax system but simply examines whether a marginal reform improves social welfare. In particular, this implies that the set of government instruments need not be optimized initially. An alternative approach—which we label the 'new approach'—also considers marginal reforms starting from any given equilibrium but argues that distributional concerns are irrelevant to the evaluation of public projects which should instead be based on the more powerful Pareto criterion.[2] This line of research, initiated by Hylland and Zeckhauser (1979) and Christiansen (1981), and further developed by Kaplow (1996), holds that unintended distributional effects can be undone by the income tax. Underlying their analyses is the *benefit principle*, which, relying on the flexibility of the non-linear income tax, implies that each individual contributes to the financing of public goods corresponding to her own marginal willingness to pay.[3] Formally, Kaplow (1996) has shown that this principle restores the original

[2] Another important strand of literature provides an alternative to these two reform-based approaches by explicitly considering a full government optimum in which all available instruments are optimized at the same time (Sandmo, 1998; Boadway and Keen, 1993; Jacobs, 2009). The conclusions from this literature resemble those obtained by the 'new approach', which is, however, more general because it only relies on the Pareto criterion and because initial instruments are not optimized.

[3] The benefit principle does not imply that a given public project must be financed by a scheme that keeps everyone's utility unchanged. It simply asserts that a public project should be completed whenever a Pareto improvement is possible. This occurs if the reform studied raises government revenue.

Samuelson rule when preferences are separable in leisure and goods (including public goods). This somewhat surprising result arises because the effects on individual behavior from the benefit side and from the cost side of a government project cancel each other out, implying that a change in government consumption has no indirect effects on tax revenue.

The divergent results of the traditional approach and of the new approach have created a state of confusion as illustrated by the debate in the wake of Kaplow's (2004) survey (see Goulder et al., 2005, and the reply by Kaplow). One reason for this confusion may simply be that the underlying analyses appear to be very different (Christiansen, 2007). Another likely reason is that the new approach has been inextricably linked to the restrictive separability assumption on preferences, although the underlying benefit principle applies generally.

The fundamental difference between the two approaches lies in the assumption made about the financing of the public good. In principle, the standard approach makes no general assumptions about the way the project is financed. However, when applied in practice, some exogenous tax reform is imposed, often a proportional tax change, implying that there is no direct link between the distribution of the benefits from the public good and the distribution of the financing burden of the project. This decoupling of the financing scheme from the distribution of benefits has the drawback of leading way to distributional concerns that are unrelated to the public goods problem itself. As a result, government consumption may become a means to compensate for a lack of appropriate tax instruments. In contrast, the new approach follows the tradition in analyses of optimal taxation by assuming away exogenous restrictions on the instruments available to the government, except the restriction that innate abilities cannot be observed and taxed directly. This eliminates any distributional concerns due to the specifics of the financing scheme.

This paper contributes in different ways to the literature on optimal provision of public goods. First, we generalize previous results by considering a general framework where individuals with different abilities in market production may also differ with respect to abilities in home production through Beckerian type household consumption technologies or with respect to their tastes for consumption goods and leisure.

Second, we use the framework to reconcile the results of the two approaches. The traditional approach addresses the problem of optimal provision by examining whether a budget-neutral expansion of government consumption raises social welfare. The new approach, on the other hand, considers an expansion of government consumption together with an adjustment of the non-linear income tax that keeps everybody at the same utility level (the benefit principle). The optimality criterion then becomes whether government revenue increases or not. We demonstrate, using a simple duality property, that both approaches derive from the same basic formula, requiring that a public project is completed only when the social marginal benefit of the project (SMBP) exceeds the social marginal cost of public funds (SMCF).

Third, and most importantly, we contribute to the new approach by deriving a general, intuitive formula for the optimal level of public goods without imposing strong assumptions on preferences. The formula shows that distortionary taxation may have a role to play as in the standard approach. However, the main determinants of optimal provision are different, and we demonstrate that the traditional formula with its emphasis on MCF only obtains in a very special case where the willingness to pay for the public good is (linearly) increasing in ability.

Our analysis identifies the *partial* correlation between ability and the marginal willingness to pay for the public good as the driving force behind any deviations from the Samuelson rule. That is, public goods provision should only be less (more) than the Samuelson rule predicts if high ability individuals have a higher (lower) marginal willingness to pay for the public good—*when evaluated at a given earnings level*. We may observe that high earning, high ability individuals have a higher willingness to pay for the public good. However, if this correlation is driven entirely by the effect of income on the willingness to pay (as is the case with a standard normal good) the Samuelson rule still applies. Only a partial effect directly from ability to the willingness to pay leads to a departure from the Samuelson rule since any correlations with income can be made distributionally neutral through appropriate adjustments of the income tax. The basic insight that correlations between ability and individual characteristics should affect public policy is not new.[4] In the context of public goods, Boadway and Keen (1993) show in a two-type optimal tax framework that the Samuelson rule should be modified according to the degree of complementary/substitutability between leisure and the marginal willingness to pay. Our result relies only on the Pareto criterion and does not require that the income tax system is optimal. Further, their main result does not generalize to our setting, but we show that our general formula may be reformulated to a relationship between leisure and the marginal willingness to pay for the public good when we confine

[4] The pioneering paper by Akerlof (1978) considers tagging as a general means to increase redistribution. Nichols and Zeckhauser (1982) and Blackorby and Donaldson (1988) apply this logic in the context of in kind transfers and Saez (2002) and Boadway and Pestiau (2003) in the context of optimal commodity taxation.

the analysis to a labour-leisure framework with homogeneous preferences.

The paper is organized as follows. Section 2 presents our model with a continuum of agents and preference heterogeneity. Section 3 derives a general formula for the optimal level of public goods when the financing scheme is not linked to the benefit distribution as in the standard approach. Section 4 shows the relationship between the standard approach and the new approach, and derives a general, intuitive formula for the optimal level of public goods when marginal tax changes are governed by the benefit principle. In Section 5 we provide a special case where the two approaches lead to identical results, and where the simple, traditional formula with its emphasis on MCF applies. Finally, Section 6 discusses policy implications.

2 The Framework

This section presents a general framework to analyze the optimal provision of public goods. The model has a continuum of agents, each characterized by an innate ability $n \in N$, which is also our index of identification. The distribution of abilities across the population is given by the non-degenerate density function $f(n)$. Each agent derives utility from private consumption c and from public goods g provided by the public sector. Gross earnings or, more generally, taxable income is denoted z, and acquiring income imposes a utility loss on the agent. The utility of a type n individual equals

$$u(c, g, z, n), \tag{1}$$

where $u_c \equiv \partial u / \partial c > 0$, $u_g > 0$, $u_z < 0$, and $u(\cdot)$ is C^2 and quasiconcave. This utility specification embodies preference heterogeneity across individuals of different abilities. It also encompasses the traditional Mirrleesian specification, $u(c, g, z/n)$, as a special case.

The term z/n builds on the notion that more able persons must exert less effort to attain a given income level. If this logic is extended to other domains of everyday life, as in Becker (1965), it seems natural that ability also has an impact on the utility of consuming, as long as the skills of home production are correlated with market productivity.[5] The formulation in (1) captures both innate preference differences between individuals of different abilities and preference differences due to the technology of home production.

Since the government cannot condition taxes on the unobservable ability, it is forced to operate a (possibly) non-linear income tax function $T(z)$. To simplify matters, we consider an initial equilibrium where the marginal tax rate $m \equiv \partial T(z)/\partial z$ is a smooth non-decreasing function of income.[6] Consumption equals $c = z - T(z)$ which, together with the utility function (1), give

$$\mathrm{MRS}_{cz}(z,n) \equiv -\frac{u_z(z - T(z), g, z, n)}{u_c(z - T(z), g, z, n)} > 0, \tag{2}$$

$$\mathrm{MRS}_{cg}(z,n) \equiv \frac{u_g(z - T(z), g, z, n)}{u_c(z - T(z), g, z, n)} > 0, \tag{3}$$

which measure the marginal rates of substitution between, respectively, c and z and c and g for a type n individual at the income level z. Notice that an increase in the ability level affects the MRS's both directly and indirectly through an impact on the earnings level z. This distinction turns out to be important for the results.

[5] The theory of household production views market goods as an input in a production process, which along with individual skills determine the output that ultimately enters individual utility. Thus, persons of different skills may benefit differently from a given input of c or g. For instance, an individual's ability to cook determines the utility derived from a basket of groceries. Similarly, the utility derived from public goods such as the police or the judicial system depends on both the skill and the need to benefit from such institutions, which is likely influenced by individual ability.

[6] We make these simplifying assumptions in order to avoid having to deal with multiple solutions to the household maximization problem and the possibility of bunching where individuals with different abilities choose the same earnings level.

The first-order conditions for the optimal choices of c and z imply

$$\mathrm{MRS}_{cz}\left(z\left(n\right),n\right)=1-m, \tag{4}$$

where $z\left(n\right)$ denotes the optimal income level for a type n individual and $m\equiv\partial T\left(z\left(n\right)\right)/\partial z$ is the marginal tax rate at that income level. The indirect utility function is $v\left(n\right)\equiv u\left(c\left(n\right),g,z\left(n\right),n\right)$ and gives the utility level of individual n when consumption and labor supply are chosen optimally. We follow the standard approach in optimal income taxation and contract theory and assume (i) that utility is increasing in ability, $\partial u/\partial n>0$, and (ii) that the Spence-Mirrlees single-crossing condition is satisfied (e.g., Salanié, 2003):

$$\partial\mathrm{MRS}_{cz}\left(z,n\right)/\partial n<0. \tag{5}$$

The first assumption along with the Envelope Theorem ensures that indirect utility is increasing in ability, $dv/dn=\partial u/\partial n>0$. The second assumption on preferences is well-known from optimal non-linear income taxation. It ensures that the tax system is implementable, i.e., that higher ability individuals always choose (weakly) higher equilibrium earnings, implying that the government can use income as a signal of the underlying ability.

The government cares about redistribution as well as the provision of public goods. The preferences of the government are captured by a standard social welfare function

$$\Omega=\int_{n}\Psi\left(v\left(n\right)\right)f\left(n\right)dn, \tag{6}$$

where $\Psi\left(\cdot\right)$ is a concave function reflecting the distributional concerns of the policymaker. The marginal rate of transformation between private goods and public goods (MRT_{cg}) is normalized to one, without any loss of generality. The government budget constraint then becomes

$$R \equiv \int_n T(z) f(n) \, dn - g \geq 0,$$

where the public goods nature of g is seen from the fact that g enters only once in the government budget constraint but still appears in everyone's utility functions. We assume that the initial tax function is Pareto efficient to avoid Laffer effects but otherwise impose no restrictions on the tax function or the level of public good expenditures.

A reform is characterized by a marginal change in the supply of the public good dg and an associated adjustment of the tax function at each earnings level, $\{dT(z), dm(z)\}_{z \in Z}$ where $Z \equiv \{z(n) | \, n \in N\}$. Differentiating (6) and using the first-order condition (4) yields the effect on social welfare

$$\frac{d\Omega}{\lambda} = -\int_n \omega(n) \cdot dT(z(n)) \cdot f(n) \, dn + dg \int_n \omega(n) \frac{u_g}{u_c} f(n) \, dn, \tag{7}$$

where $\lambda \equiv \int_n \Psi'(\cdot) \, u_c(\cdot) \, f(n) \, dn$ is the average social marginal utility of income in society and $\omega(n) \equiv \frac{\Psi'(u(\cdot)) u_c(\cdot)}{\lambda}$ is the social marginal welfare weight of agent n. Similarly, the effect of a reform on government revenue is given by

$$dR = \int_n dT(z(n)) f(n) \, dn - dg + \int_n m \cdot dz(n) \cdot f(n) \, dn, \tag{8}$$

where the first two terms are the mechanical revenue effects while the last term captures the effect of behavioral responses on government revenue. These behavioral responses are driven both by changes to the tax schedule and by effects of government consumption on household utility.

3 The Standard Approach

The standard theory of optimal public goods supply is due originally to Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974) and has exerted a tremendous influence on the practice of cost-benefit analysis (Ballard and Fullerton, 1992). This approach to deriving a formula for the optimal public goods supply considers a small change in public good expenditures dg financed by some exogenously given tax reform that affects the tax burden at each earnings level $dT(z)$ as well as the marginal tax rate $dm(z)$ and keeps revenue unchanged, $dR = 0$. This reform is desirable if it increases social welfare, $d\Omega \geq 0$. By isolating dg in eq. (8), using $dR = 0$, and inserting the result into (7), we see that the reform is optimal if

$$\int_n \omega(n) \frac{u_g}{u_c} f(n)\, dn \geq \frac{\int_n \omega(n) \cdot dT(z(n)) \cdot f(n)\, dn}{\int_n [dT(z(n)) + m \cdot dz(n)] f(n)\, dn}. \tag{9}$$

The earnings choice of the household, determined by eqs (2) and (4), may be written as a function $\hat{z}((1-m), y, g, n)$, where $(1-m)$ is the marginal net-of-tax rate and $y \equiv mz - T(z)$ is virtual income. The uncompensated elasticity of taxable income with respect to the net-of-tax rate may then be defined as $\varepsilon \equiv \frac{1-m}{z} \frac{\partial \hat{z}}{\partial(1-m)}$. From the Slutsky-equation, it may be decomposed into a compensated elasticity and an income effect, that is $\varepsilon = \varepsilon^c - \eta$ where ε^c is the compensated elasticity and $\eta \equiv -(1-m) \frac{\partial \hat{z}}{\partial y}$ is the income effect. Further, let

$$\Phi \equiv dm(z) / da(z), \quad s(n) \equiv dT(z) / \left(\int_n dT(z) f(n)\, dn \right), \tag{10}$$

where a is the average tax rate. The parameter Φ captures the progressivity of the implied tax reform, and $s(n)$ is the share of the direct tax changes that is borne by agent n. Using this we can rewrite (9) in terms of behavioral elasticities to arrive at:

Proposition 1 *A marginal expansion of a public good, dg, financed by some exogenously given tax reform, $\{dT(z), dm(z)\}_{z \in Z}$, increases social welfare iff*

$$\frac{\int_n \omega(n) \cdot \mathrm{MRS}_{cg}(\cdot) f(n)\, dn}{1 - \int_n m \frac{dz}{dg} f(n)\, dn} \geq \frac{\int_n \omega(n) s(n) f(n)\, dn}{\int_n \left(1 - \frac{m}{1-m}(\Phi \cdot \varepsilon^c - \eta)\right) s(w) f(n)\, dn}. \tag{11}$$

Proof: See Appendix A. □

Expression (11) combines the results of Dahlby (1998), Slemrod and Yitzhaki (2001), Gahvari (2006), and Kleven and Kreiner (2006). Intuitively, a marginal expansion of the public good is desirable when the social marginal benefit of the project (SMBP, the left-hand side) exceeds the social marginal cost of public funds (SMCF, the right-hand side). The expression for SMCF is equivalent to the social marginal cost of public funds derived in Dahlby (1998) with elasticities defined on taxable income rather than more narrowly on labor supply.[7]

Proposition 1 demonstrates the importance of tax distortions and distributional considerations for the optimal level of the public good. With no distributional weighting, $\omega(n) = \omega \; \forall n$, and no initial tax distortions, $m = 0 \; \forall z$, the Samuelson rule applies (independently of how a marginal expansion of the public good is financed). Introducing positive marginal tax rates implies that the optimal g may be lower or higher than prescribed by the Samuelson rule, depending on the sizes of the behavioral effects stemming from changes to the tax schedule (the RHS denominator) and from changes to the public goods supply (the LHS denominator). If we further assume that changes in the public good do not affect labor supply decisions (corresponding to the utility function

[7] The elasticity of taxable income captures hours-of-work responses as well as all other behavioral responses that are relevant to total tax payments, and the empirical evidence indicates that this elasticity may be significantly larger than the hours-of-work elasticity (e.g. Gruber and Saez, 2002). Kleven and Kreiner (2006) include also extensive labor supply responses. We have chosen to follow the tradition in analyses of the optimal provision of public goods and MCF by focusing on intensive responses only.

being additively separable in g such that $dz/dg = 0$), that the (uncompensated) earnings elasticities are homogeneous, and that a public good expansion is financed by raising the marginal tax rate m in a linear tax system (implying that $\Phi = 1$), the optimality condition (11) simplifies to the modified Samuelson rule often used in applied work

$$\int_n \mathrm{MRS}_{cg} \cdot f(n)\, dn \geq \frac{1}{1 - \frac{m}{1-m}\varepsilon} = \mathrm{MCF} \cdot \mathrm{MRT}_{cg}, \tag{12}$$

where MCF is the marginal cost of public funds reflecting the distortionary effect of raising the marginal tax rate (Browning, 1987; Ballard and Fullerton, 1992; Dahlby, 1998). This simple modification of the Samuelson rule focuses only on the distortionary effects of raising taxes and disregards distributional concerns.

Introducing distributional concerns may affect the optimal level of public goods, even in the absence of any tax distortions (implying that both denominators in eq. (11) equal one). Consider, for example, the case where the aggregate willingness to pay for a public project exceeds the total costs of the project. Such a project should be implemented according to the original Samuelson rule but not necessarily according to the policy rule (11), which depends on the financing scheme. If, for example, high-income people receive most of the benefits and the public project is financed by a poll tax, the project might be discarded because the distribution of welfare is worsened. However, such a conclusion ignores the flexibility of the non-linear income tax, and thereby assigns a role to distributional considerations that are unrelated to the problem of public goods provision (see also Auerbach and Hines, 2002). This approach may have merit when there are exogenous constraints that limit the adjustment of the tax schedule as emphasized by Slemrod and Yitzhaki (2001) and Gahvari (2006). On the other hand, without any specific justification for constraining the tax function, it is natural to consider a financing scheme where those who benefit from the public good also pay the extra taxes, thereby neutralizing any distributional effects. This is the direction taken by the new approach.

4 The New Approach

The new approach evaluates the benefits of an expansion of public goods by use of the *benefit principle*, introduced by Hylland and Zeckhauser (1979) and applied by Kaplow (1996, 2004). When applying this principle, we consider a (marginal) expansion of g and an associated adjustment of the tax function, $dT(z)$ and $dm(z)$, that keep everyone's utilities unchanged. In this reform, each individual's share of the additional tax burden corresponds to their individual benefit from the public good, and the reform is therefore distributionally neutral. The public good expansion is then desirable if the total effect on government revenue is positive in which case it is possible to make a Pareto improvement. Vice versa, if the impact on government revenue is negative, it is possible to generate a Pareto improvement by reducing public good expenditures.

Testing whether a marginal expansion of g that keeps everyone's utility unchanged can raise government revenue, $dR \geq 0$, is incompatible with the method used to derive the optimal level of g in the standard approach of the previous section. Indeed, condition (9) is derived by considering whether a budget-neutral reform, $dR = 0$, raises social welfare, $d\Omega \geq 0$. Instead, we use an alternative approach that keeps social welfare unaffected and determines the desirability of a marginal expansion of g by calculating the effect of the reform on government revenue. If the effect is positive, the reform is socially desirable. We show in Appendix B that the requirements $d\Omega = 0$ and $dR \geq 0$ are equivalent to

$$\int_n \omega(n) \frac{u_g}{u_c} f(n)\, dn \geq \frac{\int_n \omega(n) \cdot dT(z(n)) \cdot f(n)\, dn}{\int_n [dT(z(n)) + m \cdot dz(n)] f(n)\, dn}, \tag{13}$$

which is similar to condition (9). The fact that we arrive at the same formula as in the standard approach is not surprising since we have merely applied a dual approach to determine the optimal level of g. Importantly, the equivalence of conditions (9) and (13) provides a simple link between the two approaches. It shows that all results within the two approaches may be derived from the same basic formula. The differences in results stem entirely from the different assumptions made regarding the associated tax change.

Whereas the standard approach considers an exogenously given tax reform, the benefit principle makes the change to the entire tax schedule endogenous. Indeed, at every income level both the direct change to the tax burden and the change in the marginal tax rate are determined endogenously by the requirement that the utility of each individual $v(n)$ is unchanged. Thus, we consider a reform that affects g and the tax function $T(\cdot)$ such that

$$dv(n) = u_c(\cdot)\,dc + u_g(\cdot)\,dg + u_z(\cdot)\,dz = 0 \quad \text{for all } n. \tag{14}$$

Notice that this equation alone does not characterize a unique post-reform equilibrium (i.e., we have for each individual one equation with two unknowns dc and dz).

The reform also has to be implementable, i.e., the post-reform allocation must be incentive compatible. In Appendix C, we address the issue of implementability leading to the conclusion that the allocation must satisfy the constraint

$$v_n(n) = u_n(c(n), g, z(n), n), \tag{15}$$

which is well-known from optimal non-linear income taxation. This relationship is fulfilled in the initial equilibrium where the optimal allocations satisfy the individual first-order conditions. However, we need to impose (15) to ensure that the first-order conditions are also met by the post-reform allocation. In the special case of separable preferences studied by Kaplow (1996, 2004) and others, the above requirement is trivially met because individual choices are unaffected by the reform (Jacobs, 2009). This line of reasoning does not carry over to our general framework where the reform induces agents to make earnings and consumption adjustments.

From (15) and the benefit principle, which implies that $dv_n(n) = 0$, we obtain

$$dv_n(n) = u_{cn}(\cdot)\, dc + u_{gn}(\cdot)\, dg + u_{zn}(\cdot)\, dz = 0 \quad \text{for all } n. \tag{16}$$

The impact of the public good expansion on the incentives to supply earnings will depend on the implicit effect on the marginal tax rate implied by the financing scheme and on the direct effect of the public good on work incentives. The total effect on earnings may be derived by combining eqs (14) and (16). This gives

$$dz(n) = \frac{u_{gn}(\cdot) - u_{cn}(\cdot)\, u_g(\cdot)/u_c(\cdot)}{-u_{zn}(\cdot) + u_{cn}(\cdot)\, u_z(\cdot)/u_c(\cdot)} dg = \frac{\partial \mathrm{MRS}_{cg}(z(n), n)/\partial n}{\partial \mathrm{MRS}_{cz}(z(n), n)/\partial n} dg, \tag{17}$$

where the last equality follows by differentiating the definitions in eqs (2) and (3) w.r.t. n. The partial derivatives in this expression measure the effect of ability on the marginal rates of substitution between, respectively, c and g in the numerator and c and z in the denominator. Notice that the single-crossing condition (5) implies that the denominator is negative and therefore that the sign of the effect is determined by $\partial \mathrm{MRS}_{cg}(z, n)/\partial n$.

The benefit-offsetting expansion of g adjusts the tax function to capture the benefits of the additional g from each individual n. To see this, differentiate the relationship $c = z - T(z)$ in order to get $dc = (1 - m)\, dz - dT(z)$. This expression and the first order condition (4) enable us to write condition (14) as

$$dT(z(n)) = \frac{u_g(\cdot)}{u_c(\cdot)} dg = \mathrm{MRS}_{cg}(z(n), n) \cdot dg. \tag{18}$$

This equation shows that the increase in the tax burden of an individual with earnings z is exactly equal to the extra benefit from the expansion of government consumption.

The application of the benefit principle implies that the expansion of g and the accompanying change in the tax function keeps everyone's utility, and thus social welfare, unchanged. Now eq. (18) gives $\int_n \omega(w) \frac{u_g}{u_c} dg \cdot f(n)\, dn = \int_n \omega(w)\, dT(z(n))\, f(n)\, dn$ implying that condition (13) is equivalent to

$$\int_n [dT(z(n)) + m \cdot dz(n)]\, f(n)\, dn \geq dg. \tag{19}$$

From eqs (17), (18), and (19), it is now possible to establish our main result:

Proposition 2 *A fully financed marginal expansion of a public good, dg, may induce a Pareto improvement (thereby increasing social welfare) iff*

$$\int_n \left(\mathrm{MRS}_{cg}(z(n), n) + m \cdot \frac{\partial \mathrm{MRS}_{cg}(z(n), n) / \partial n}{\partial \mathrm{MRS}_{cz}(z(n), n) / \partial n} \right) f(n)\, dn \geq \mathrm{MRT}_{cg}. \tag{20}$$

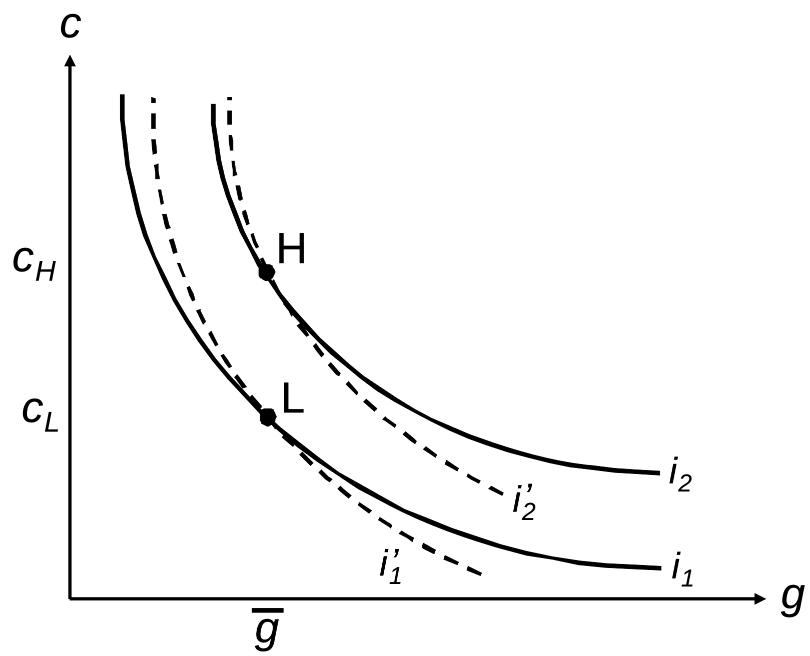
Proof: The inequality follows by inserting eqs (17) and (18) in condition (19). When inequality (20) is satisfied, it is possible to raise government revenue while keeping individual utilities fixed through an expansion of g and an associated tax reform $\{dT(z), dm(z)\}_{z \in Z}$ that satisfy the benefit principle and implementability. The increase in government revenue may then be used to raise individual utilities, thus attaining a Pareto improvement. To prove necessity, consider an expansion of g and an associated tax reform that give rise to a Pareto improvement. Because of the assumption that the initial tax function is Pareto-efficient, the tax reform may be decomposed into two distinct tax changes: first, taxes are increased to satisfy the benefit principle and implementability. Second, taxes are reduced, giving rise to the Pareto improvement. The second part is only possible if the first part gives rise to an increase in government revenue which implies that condition (20) has to be satisfied. $\square$

Proposition 2 shows that the Samuelson rule must be amended by a term that is affected by the *partial* correlation, i.e., conditional on income, between ability and the marginal willingness to pay for the public good. The additional term corrects for the revenue implications of the behavioral responses to the reform. The optimal level of g is affected by correlations with the unobservable n because the tax function is constrained to depend on the imperfect signal that is income. It is important to note that the partial effects on the MRS's in (20) are evaluated at a given income level. To see this, note that the total effect of higher ability on the marginal willingness to pay for the public good is given by

$$\frac{d\mathrm{MRS}_{cg}\left(z\left(n\right),n\right)}{dn}=\underbrace{\frac{\partial\mathrm{MRS}_{cg}\left(z\left(n\right),n\right)}{\partial z}\frac{dz\left(n\right)}{dn}}_{\text{Term 1}}+\underbrace{\frac{\partial\mathrm{MRS}_{cg}\left(z\left(n\right),n\right)}{\partial n}}_{\text{Term 2}}.$$

This is illustrated on Figure 1, which displays indifference curves and the marginal rate of substitution between private consumption and public goods. A low-ability person who has low earnings/private consumption is at point L, while a high-ability person with high earnings/private consumption is at point H. Assume first that the preferences of both agents are given by the same solid indifference curves i_1 and i_2. In this case, the high-income person has a higher willingness to pay for the public good (MRS_{cg} is larger at H than at L), which is only natural when g is a normal good because both agents receive the same level of public good consumption $\bar{g}$. This effect works entirely through earnings, dz/dn (term 1 in the above expression), and does not affect the optimal level of g according to Proposition 2. Intuitively, when the marginal willingness to pay increases with income, the benefit principle implies that marginal tax rates increase as a result of the reform. However, because the higher marginal tax rate directly reflects a higher individual benefit from the public good as income increases, the high-ability individual does not find it attractive to reduce earnings in order to mimic the low-ability person. Thus, earnings are unchanged and variations in MRS due entirely to variations in z do not affect the optimal public goods supply.

Figure 1: High versus low ability



With an assumption of weak separability between consumption goods and work effort in individual preferences—as studied by Christiansen (1981), Kaplow (1996) and others—term 2 in the above expression vanishes and Proposition 2 simplifies to the original Samuelson rule.[8] Without this assumption, term 2 is either positive or negative reflecting that the slope of the indifference curves differ across individuals with different

[8]The original Samuelson rule would apply if the individual utility function satisfies the separability assumption $u(c, g, z, n) = \tilde{u}\left[w^1(c, g, z), w^2(z, n)\right]$. The utility function $\tilde{u}(w(c, g), l)$, studied by Kaplow (1996), is a special case where $w^1(c, g, z) = \tilde{w}^1(c, g)$ and $w^2(z, n) = z/n$. More generally, Gauthier and Laroque (2009) and Jacobs (2009) consider separability in second-best environments and establish when first-best and second-best policy rules coincide.

abilities *when evaluated at a given income/consumption level.* This situation arises if the preferences of the high ability person are instead represented by the dashed indifference curves i_1' and i_2'. In this case, the high-ability person has a higher willingness to pay at any given point, implying that the supply of public goods should be reduced relative to the Samuelson rule. Intuitively, the benefit principle implies that higher incomes must contribute more to the financing of the public good. However, part (or all) of the additional benefit enjoyed by persons with higher incomes stems from their innate ability and is realized independently of the chosen income level. Thus, the additional taxes implied by the reform reduce the incentive to work.

The size of the additional distortion depends on the responsiveness of earned income as captured by the denominator of the second term in (20). Also, the stronger is the influence of ability on the marginal willingness to pay, the more difficult it is for the government to finance g in a non-distortionary fashion. Obviously, a reversal of the above argument explains why the public goods supply should be higher than advocated by the Samuelson rule when there is a negative correlation between ability and the marginal willingness to pay for the public good. An alternative intuition would stress that any direct correlation between the marginal willingness to pay for the public good and ability implies that the public good effectively redistributes based on the unobserved ability.

Proposition 2 is in line with Boadway and Keen (1993) who show in a two-type optimal tax framework that the partial correlation between leisure and the marginal willingness to pay determines deviations from the Samuelson rule.[9] In our general framework, this

[9] Williams (2005) analyzes different public policy problems with non-separable preferences. His main goal, however, is to compare representative-agent and multiple-agent models. His result on public good provision with multiple agents (eq. 24M) does not provide much guidance on whether non-separability implies more or less provision compared to the Samuelson rule. In particular, he does not identify the crucial distinction between income and ability in determining deviations from the Samuelson rule.

result does not apply. However, Proposition 2 may be reformulated to a relationship between leisure and the marginal willingness to pay for the public good when we confine the analysis to a labour-leisure framework with homogeneous preferences:

Corollary 1 *With the individual utility specification $u(c,g,z,n) = \tilde{u}(c,g,l)$ where $l \equiv z/n$, a fully financed marginal expansion of the public good, dg, may induce a Pareto improvement (thereby increasing social welfare) iff*

$$\int_n \left(\mathrm{MRS}_{cg}(z,l) + m \cdot \frac{\partial \mathrm{MRS}_{cg}(z,l)/\partial l}{\partial \mathrm{MRS}_{cz}(z,l)/\partial l} \right) f(n)\, dn \geq \mathrm{MRT}_{cg}.$$

Proof: With this utility function, we can use the relation $z = n \cdot l$ to express the change in n as a function of the dependence of MRS on l instead. Indeed,

$$\frac{\partial \mathrm{MRS}}{\partial n} = \frac{\partial \mathrm{MRS}}{\partial l}\frac{\partial l}{\partial n} = -\frac{\partial \mathrm{MRS}}{\partial l}\frac{z}{n^2} \Rightarrow \frac{\partial \mathrm{MRS}_{cg}(z,n)/\partial n}{\partial \mathrm{MRS}_{cz}(z,n)/\partial n} = \frac{\partial \mathrm{MRS}_{cg}(z,l)/\partial l}{\partial \mathrm{MRS}_{cz}(z,l)/\partial l}.$$

By inserting this expression in eq. (20), we arrive at the above inequality. □

When ability is restricted to affect utility only through l, the evaluation of a public project departs from the Samuelson rule if the marginal willingness to pay for the public good depends on individual labor supply. Thus, if MRS_{cg} displays a *negative correlation* with l—for a given income z—the optimal level of the public good is less than predicted by the Samuelson rule (notice that the denominator in the second term under the integral is now positive) and vice versa.

5 A Special Case: Simple MCF Correction

When deriving the simple modified Samuelson rule (12) from the standard approach, we focused only on the distortionary effects of raising taxes and disregarded distributional concerns, although the reform under consideration would most likely affect the well-being of the agents in different ways. We now show that there is one special case where this simple formula obtains using the new approach. Assume utility is given by

$$u = c + n \cdot w(g) - n \cdot h(z/n), \tag{21}$$

where the functional form of the disutility of labor and the normalization assumption $h'(1) = 1$ imply that n reflects potential earnings, i.e., without any tax system the individual chooses $z = n$ (see Saez, 2001). The crucial feature of this preference specification is that the marginal utility from the public good rises linearly with ability. Proposition 2 now implies that a marginal expansion of g generates scope for a Pareto improvement iff (see Appendix D)

$$\int_n \mathrm{MRS}_{cg} \left(1 - \frac{m}{1-m}\varepsilon\right) f(n)\, dn \geq \mathrm{MRT}_{cg}, \tag{22}$$

where ε is the elasticity of taxable income with respect to the net-of-tax rate. This formula identifies MCF as a central determinant of the optimal g. If, in addition, the income tax system is linear initially and the elasticity of taxable income is constant across individuals, the condition simplifies to

$$\int_n \mathrm{MRS}_{cg} \cdot f(n)\, dn \geq \frac{1}{1 - \frac{m}{1-m}\varepsilon} \cdot \mathrm{MRT}_{cg} = \mathrm{MCF} \cdot \mathrm{MRT}_{cg},$$

which is again a modified Samuelson rule. However, only when utility from the public good is linearly increasing in ability and the initial tax system is proportional is this traditional MCF correction valid according to the new approach. If, instead, utility from the public good is increasing in earnings—and thereby merely indirectly increasing in ability—the original Samuelson rule would apply.

6 Policy Implications

The standard approach provides the most general answer to the question of the optimal public goods supply in an economy with distortionary taxation. However, for practical policy making Proposition 1 is probably of limited relevance. A cost-benefit analysis based on that proposition requires information about the social welfare weights of different income groups, knowledge of earnings responses to changes in taxation and to changes in the public goods supply, as well as a specification of the underlying tax reform used to finance the public good.

Instead, many real-world cost-benefit analyses are based on the simple MCF-correction of the Samuelson rule in formula (12). Section 5 showed that this approach implicitly puts very strong assumptions on the utility function. In particular, it is assumed that the willingness to pay is (linearly) increasing in ability, conditional on income. As the examples below illustrate, this hardly seems a natural benchmark for general decisions on public goods provision. In the absence of a direct, strong positive relationship between ability and willingness to pay, the MCF-correction leads to underprovision of public goods.

The new approach has been suggested as a way to simplify the problem of optimal provision. As long as the income tax is sufficiently flexible, we can neutralize any distributional effects and use the Pareto criterion to evaluate whether the public good should be expanded. This resembles the idea of Musgrave (1959) that the redistributive and allocative branches of government may be dealt with separately. The allocative branch ensures that we are located at the (second-best) Pareto frontier, while the redistributive branch, by adjusting tax policy, chooses the specific location.

The main insight of the new approach is that the Samuelson rule is restored but only if preferences are weakly separable in consumption goods and leisure. This is a powerful result because the Samuelson rule is relatively easy to apply to practical policy problems. It only requires information about the price willingness of households and the cost of the public good. However, the strong assumption of separability is crucial for the result.

Our Proposition 2 shows that in a general setting without separability, the optimal supply of public goods follows a modified Samuelson rule with an additional term representing the correlation between ability and the willingness to pay for the public good, *conditional on income*. It is very difficult to identify this additional term empirically since correlations between the marginal willingness to pay and, respectively, ability and income are observationally equivalent but have vastly different policy implications as first noted by Hylland and Zeckhauser (1979). For instance, are wealthy people overrepresented among opera audiences because they are wealthy, or because they are of higher ability? For some purposes, casual observation may be sufficient to decide on the desirability of a public project but in general it is difficult to distinguish empirically between innate ability and income—a feature that also underlies the main assumption behind optimal income taxation.[10]

For instance, police protection and the safety it provides might be an example of a public good where the willingness to pay is increasing in income/wealth but where there is no clear relationship between willingness to pay and ability (for a given income level). Then the original Samuelson rule provides the best benchmark for the optimal level of expenditures on public safety.

[10] The identification problem is not easily solved. Basically, we need to identify the marginal willingness to pay for the public good of a high-ability type when he mimics the income choice of a low-ability type. This requires knowledge about out-of-equilibrium outcomes.

In contrast, education seems to be an example of a good that is valued higher by the more able, even conditional on income. Presumably, people of higher innate ability are better equipped to benefit from educational training. If so, the optimal public financial support for education is less than the Samuelson rule predicts because such support effectively redistributes income towards the more able.[11] Correspondingly, public transportation is likely to benefit persons of lower ability more for a given income. Efficient public transportation reduces the travel time to and from the workplace, leaving more time for other activities. A low ability individual must work longer hours to uphold a given income and therefore, presumably, values her sparetime more compared to a higher ability individual with the same workload. Thus, subsidies to public transportation effectively redistribute income towards the less able, over and above what is attainable through the income tax. Importantly, consumption patterns across incomes do not necessarily reveal the desirability of public transport subsidies. If low income individuals choose public transport because they cannot afford a car, not because they are of low skill, the Samuelson rule still applies.

The new approach requires that the income tax system to be sufficiently flexible. But in practice there may be several constraints that limit the adjustment of the income tax as emphasized by e.g. Slemrod and Yitzhaki (2001). For example, the implementation of a progressive tax system in developing countries is hampered by the difficulty of taxing income directly (Gordon and Li, 2005). In this light, let us reconsider our previous example of police protection and safety which primarily benefits the rich. When exogenous

[11] Education is, of course, not a pure public good but the benefit principle—and therefore our results—may also be applied to externalities (see e.g. Kaplow and Shavell, 2002). Note also that redistributive policies may discourage private investments in education, creating a second-best argument in favor of subsidizing education (Bovenberg and Jacobs, 2005).

constraints prevent us from raising the desired extra revenue from the rich, we can no longer simply apply the Samuelson rule as advocated by the new approach. Instead, an expansion of the spending on public safety inevitably redistributes welfare from the poor to the rich. In this case, we need to apply the standard approach, which requires information on welfare weights etc.

A Proof of Proposition 1

The effect of the tax reform on government revenue is

$$dR = \int_n \left[dT\left(z\left(n\right)\right) + m \cdot dz\left(n\right)\right] f\left(n\right) dn,$$

which is identical to the denominator on the right-hand side of (9). Using the earnings function $\hat{z}\left(\cdot\right)$, we may write the earnings response as

$$dz\left(n\right) = \frac{\partial \hat{z}}{\partial y} - \frac{\partial \hat{z}}{\partial\left(1-m\right)} dm + \frac{\partial \hat{z}}{\partial g},$$

which decomposes the effect on earnings into an income effect, an effect from the change in the marginal tax rate, and an effect from a change in the public good. The change in virtual income is

$$dy = z\left(n\right) \cdot dm + m \cdot dz\left(n\right) - m \cdot dz\left(n\right) - dT\left(z\right) = z\left(dm - da\right),$$

where $a \equiv T\left(z\right)/z$ is the average tax rate and $da \equiv dT\left(z\right)/z$ is the change in the average tax rate at the pre-reform earnings level z. This implies that the earnings response above may be rewritten to

$$dz\left(n\right) = \left[\frac{\partial \hat{z}}{\partial y}\left(dm - da\right) - \frac{1}{1-m}\varepsilon \cdot dm\right] z + \frac{\partial \hat{z}}{\partial g},$$

where $\varepsilon \equiv \frac{1-m}{z}\frac{\partial \hat{z}}{\partial(1-m)}$is the uncompensated elasticity of taxable income. From the Slutsky-equation, it may be written as $\varepsilon = \varepsilon^c - \eta$ where ε^c is the compensated elasticity and $\eta \equiv -(1-m)\frac{\partial \hat{z}}{\partial y}$ is an income effect. Using this relationship, the above expression becomes

$$dz(n) = (\eta \cdot da - \varepsilon^c \cdot dm)\frac{1}{1-m}z + \frac{\partial \hat{z}}{\partial g},$$

which implies

$$dR = \int_n \left[dT(z(n)) + \frac{m}{1-m}\left(\eta - \frac{dm}{da}\varepsilon^c\right)dT(z(n)) + m\frac{\partial \hat{z}}{\partial g}\right]f(n)\,dn.$$

Insert this in the denominator on the right-hand side of (9) and use the definitions in (10) in order to obtain formula (11) in Proposition 1.

B Derivation of Equation (13)

From eq. (7) and the condition $d\Omega = 0$, we get

$$dg = \frac{\int_n \omega(n) \cdot dT(z(n)) \cdot f(n)\,dn}{\int_n \omega(n)\frac{u_g}{u_c}f(n)\,dn}.$$

We may rewrite eq. (8) as

$$dR = \int_n [dT(z(n)) + m \cdot dz]\,f(n)\,dn - dg.$$

Insert dg from above and apply the criterion $dR \geq 0$ to get (13).

C Implementability of Benefit-offsetting reforms

The revelation principle implies that we can restrict attention to direct revealing mechanisms (Laffont and Tirole, 1994). Thus, the government's (acting as the principle) problem amounts to a choice among all feasible allocations $\{c(n), g, z(n)\}_{n\in N}$ that induce each agent to truthfully reveal his own ability. For a type n agent, this gives rise to the following incentive compatibility constraints

$$u\left(c\left(n\right),g,z\left(n\right),n\right)\geq u\left(c\left(\hat{n}\right),g,z\left(\hat{n}\right),n\right)\ \text{ for all } n,\hat{n}, \tag{C-1}$$

stating that agent n prefers the allocation intended for him, $(c\left(n\right),g,z\left(n\right))$, and not the allocation intended for any other agent $\hat{n}$. We then have the following result:

Lemma 1 *If the Spence-Mirrlees condition (5) is satisfied then an allocation* $\left\{c\left(n\right),g,z\left(n\right)\right\}_{n\in N}$ *satisfies (C-1) iff it satisfies the conditions (i)* $v_n\left(n\right)=u_n\left(c\left(n\right),g,z\left(n\right),n\right)$ *in eq. (15) and (ii)* $z'\left(n\right)\geq 0$ *for all* n.

Proof: The utility of individual n if he chooses the allocation intended for agent $\hat{n}$ is

$$\hat{u}\left(n,\hat{n}\right)\equiv u\left(c\left(\hat{n}\right),g,z\left(\hat{n}\right),n\right). \tag{C-2}$$

For the mechanism to be revealing, $\hat{u}\left(\cdot\right)$ must have maximum at $\hat{n}=n$ such that it is optimal for agent n to truthfully reveal his own ability. Thus, the IC constraints (C-1) are fulfilled iff

$$\frac{\partial}{\partial\hat{n}}\hat{u}\left(n,\hat{n}\right)\left(n-\hat{n}\right)\geq 0 \text{ for all } n,\hat{n}. \tag{C-3}$$

This implies that the following first order necessary condition (FOC) and second order necessary condition (SOC) have to be fulfilled

$$\frac{\partial\hat{u}}{\partial\hat{n}}\left(n,n\right) = 0, \tag{C-4}$$

$$\frac{\partial^2\hat{u}}{\left(\partial\hat{n}\right)^2}\left(n,n\right) \leq 0. \tag{C-5}$$

The additional gain to agent n from mimicking agent $\hat{n}+d\hat{n}$ over mimicking agent $\hat{n}$ is given by $\frac{\partial}{\partial\hat{n}}\hat{u}\left(n,\hat{n}\right)$, which from eq. (C-2) equals

$$\frac{\partial}{\partial\hat{n}}\hat{u}\left(n,\hat{n}\right)=u_c\left(c\left(\hat{n}\right),g,z\left(\hat{n}\right),n\right)c'\left(\hat{n}\right)+u_z\left(c\left(\hat{n}\right),g,z\left(\hat{n}\right),n\right)z'\left(\hat{n}\right), \tag{C-6}$$

where $z'(\hat{n})$ and $c'(\hat{n})$ are movements along the equilibrium path. By evaluating the above equation for a type $\hat{n}$ agent (such that $n = \hat{n}$) and using eq. (C-4), these movements satisfy

$$c'(\hat{n}) = -\frac{u_z(c(\hat{n}), g, z(\hat{n}), \hat{n})}{u_c(c(\hat{n}), g, z(\hat{n}), \hat{n})} z'(\hat{n}). \tag{C-7}$$

By substituting this expression back into (C-6) and using the definition (2), we obtain

$$\frac{\partial}{\partial \hat{n}} \hat{u}(n, \hat{n}) = (\mathrm{MRS}_{cz}(z(\hat{n}), \hat{n}) - \mathrm{MRS}_{cz}(z(\hat{n}), n)) u_c(c(\hat{n}), g, z(\hat{n}), n) z'(\hat{n}).$$

Insert this expression into the requirement (C-3) to obtain

$$(\mathrm{MRS}_{cz}(z(\hat{n}), \hat{n}) - \mathrm{MRS}_{cz}(z(\hat{n}), n))(n - \hat{n}) \cdot u_c(c(\hat{n}), g, z(\hat{n}), n) z'(\hat{n}) \geq 0,$$

for all n, $\hat{n}$. The Spence-Mirrlees condition (5) ensures that

$$(\mathrm{MRS}_{cz}(z(\hat{n}), \hat{n}) - \mathrm{MRS}_{cz}(z(\hat{n}), n))[n - \hat{n}] \geq 0,$$

and since $u_c > 0$, it follows that the above condition is only fulfilled if $z'(\hat{n}) \geq 0$.

Now consider the second-order condition (C-5). By differentiating (C-4) w.r.t. n, we obtain

$$\frac{\partial^2 \hat{u}}{\partial \hat{n} \partial n}(n, n) + \frac{\partial^2 \hat{u}}{(\partial \hat{n})^2}(n, n) = 0,$$

which implies that (C-5) is equivalent to

$$\frac{\partial^2 \hat{u}}{\partial \hat{n} \partial n}(n, n) \geq 0. \tag{C-8}$$

Differentiate (C-6) to obtain

$$\frac{\partial^2 \hat{u}}{\partial \hat{n} \partial n}(n, \hat{n}) = u_{cn}(c(\hat{n}), g, z(\hat{n}), n) c'(\hat{n}) + u_{zn}(c(\hat{n}), g, z(\hat{n}), n) z'(\hat{n}), \tag{C-9}$$

which together with (C-7) imply that inequality (C-8) may be written as

$$\left[u_{zn}(c(\hat{n}), g, z(\hat{n}), n) - u_{cn}(c(\hat{n}), g, z(\hat{n}), n) \frac{u_z(c(\hat{n}), g, z(\hat{n}), n)}{u_c(c(\hat{n}), g, z(\hat{n}), n)}\right] z'(\hat{n}) \geq 0.$$

The term in square-brackets is simply the derivative of MRS_{cz} w.r.t. n implying that

$$\frac{\partial \mathrm{MRS}_{cz}\left(z\left(\hat{n}\right), n\right)}{\partial n} \cdot z'\left(\hat{n}\right) \geq 0,$$

which, because of the Spence-Mirrlees single-crossing condition (5), amounts to $z'\left(\hat{n}\right) \geq 0$. Thus, given the Spence-Mirrlees condition, if the equilibrium allocation $\left\{c\left(n\right), g, z\left(n\right)\right\}_{n \in N}$ satisfies $\frac{\partial \hat{u}}{\partial \hat{n}}\left(n, n\right) = 0$ and $z'\left(n\right) \geq 0$ for all n then it is incentive compatible. Finally, note that $\frac{\partial \hat{u}}{\partial \hat{n}}\left(n, n\right) = 0 \Leftrightarrow v_n\left(n\right) = u_n\left(c\left(n\right), g, z\left(n\right), n\right)$ since

$$\begin{aligned} v_n\left(n\right) &= u_c\left(c\left(n\right), g, z\left(n\right), n\right) c'\left(n\right) + u_z\left(c\left(n\right), g, z\left(n\right), n\right) z'\left(n\right) + u_n\left(c\left(n\right), g, z\left(n\right), n\right) \\ &= u_n\left(c\left(n\right), g, z\left(n\right), n\right) \end{aligned}$$

where we have used (C-7). QED.

The pre-reform allocation $\left\{c\left(n\right), g, z\left(n\right)\right\}_{n \in N}$ satisfies the first order conditions of the agents and thus, because of the envelope theorem, property (i) of Lemma 1. The Spence-Mirrlees single-crossing condition (5) and the assumption that the marginal tax rate m is a smooth non-decreasing function of income ensure that earnings is everywhere a strictly increasing function of ability, $z'\left(n\right) > 0$, in the pre-reform equilibrium, thereby satisfying property (ii) of Lemma 1. Thus, the initial allocation is incentive compatible. In the derivation of Proposition 2, we impose eq. (16) implying that property (i) of Lemma 1 also holds after the reform. The changes in the earnings levels $dz\left(n\right)$ are given by eq. (17) for all n, and because the utility function is C^2, we have that $dz\left(n\right)$ is a continuous function of n. Moreover, $dz\left(n\right)$ is infinitessimal implying that $z'\left(n\right) \geq 0$ for all n after the reform, i.e., property (ii) of Lemma 1 is fulfilled.

D Derivation of Equation (22)

We start by deriving dz from eq. (17). With the utility function (21), we have $u_{cn} = 0$, $u_{gn} = w'(\cdot)$, and the first-order condition for the choice of earnings (4) implies

$$h'(\cdot) = 1 - m \implies \frac{dz}{d(1-m)} = \frac{n}{h''(\cdot)},$$

which gives the elasticity of earned income w.r.t. the take-home rate as

$$\varepsilon \equiv \frac{dz/z}{d(1-m)/(1-m)} = \frac{nh'(\cdot)}{zh''(\cdot)}.$$

The cross-derivative u_{zn} then becomes

$$u_{zn} = h''(\cdot)\frac{z}{n^2} = (1-m)\frac{1}{n}\frac{1}{\varepsilon}.$$

By inserting this relationship and $u_{cn} = 0$ into (17), we obtain

$$dz = -\frac{MRS_{cg}}{1-m}\varepsilon \cdot dg,$$

where we have used $\mathrm{MRS}_{cg} = n \cdot w'(g)$. By substituting the above expression and eq. (18) into condition (19), we obtain the inequality (22).

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