Energy Distribution and Economic Growth*

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Abstract. This research examines the physical constraints on the growth process. In order to run, maintain and build capital energy is required to be distributed to geographically dispersed sites where investments are deemed profitable. We capture this aspect of physical reality by a network theory of electricity distribution. The model leads to a supply relation according to which feasible electricity consumption per capita rises with the size of the economy, as measured by capital per capita. Specifically, the relation is a simple power law with an exponent assigned to capital that is bounded between 1/2 and 3/4, depending on the efficiency of the network. Together with an energy conservation equation, capturing instantaneous aggregate demand for energy, we are able to provide a metabolic-energetic founded law of motion for capital per capita that is mathematically isomorphic to the one emanating from the Solow growth model. Using data for the 50 U.S. states 1960-2000 we test a dual representation of the model structurally. Specifically, we examine the determination of growth in electricity consumption per capita. The model fits the data well. Moreover, we find evidence in favor of a scaling parameter, between energy and capital per capita, of about 2/3.

Keywords: Economic Growth, Energy, Power Laws, Networks.

JEL: O11, O13, Q43.

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1. Introduction

The work of Domar (1946) and Solow (1956) initiated the formal analysis of the growth process, where physical capital accumulation is seen a key growth engine. This research has unveiled key structural characteristics which impinge on the determination of labor productivity in the long run: savings, population growth, technological change. These factors share the common feature that they importantly affect the ability of an economy to mobilize resources for the purpose of capital accumulation.

At the same time, the basic neoclassical growth theory abstracts from a potentially important aspect of the growth process: it makes little sense to acquire a piece of machinery, at a particular time and place, unless the machine can be supplied with electricity and put to use. That is, it is of first order importance that an economy is able to *distribute* electricity across the economy. The present paper attempts to model the latter aspect and study its implications for growth.[1]

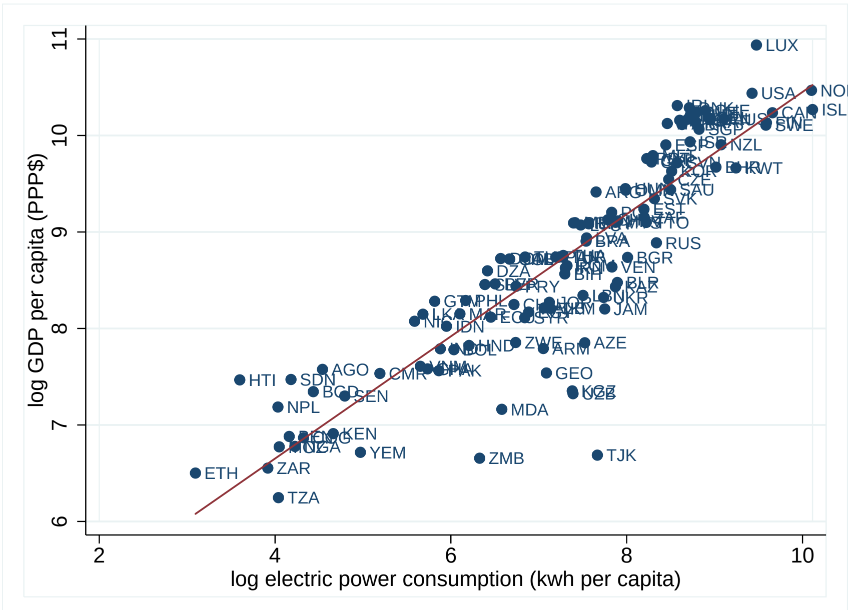
The importance of modeling electricity distribution lies in the need to understand the potential physical constraints on the process of capital accumulation and economic growth. Such physical constraints will have bearing on whether growth can be sustained in the long run. In addition, they may also influence comparative economic development: for resources given, some countries may be able to grow faster than others (attain higher levels of prosperity in the long run) because they are more efficient in distributing electricity to the areas where accumulation is deemed worthwhile. Accordingly, whereas neoclassical growth theory so far focusses on the importance of mobilizing (financial or natural) resources to the benefit of growth, the present research highlights the importance of complementary constraints which relate to physically enabling economic growth to occur by making electricity available for final use; consumption and accumulation of capital.

To be sure, the increasing use of electricity is a key characteristic of the modern growth regime within which we live. Figure 1 shows a scatter plot of electric power consumption per capita

[1] The present analysis is therefore related to the literature on infrastructure and economic growth. The traditional approach essentially consists of adding another input into the production function, thus capturing infrastructure capital (e.g., Arrow and Kurz, 1970 and many others since). Our approach to model the influence from the electricity infrastructure will differ in that we will provide a model for the network itself, and derive the critical associations between electricity use and capital. At the same time we abstract from other dimensions of infrastructure, like roads, ports etc. Also, we abstract from the problems associated with the production of electricity, which in practise requires the use of natural resources. See e.g. Stiglitz (1974) and Suzuki (1976) for a discussion of sustainable growth in the presence of exhaustible natural resources.

and per capita PPP GDP. The high positive correlation is immediately obvious. Indeed, electric power consumption can account for 78% of the variation in GDP per capita in 2000.

Figure 1: Log Electric Power Consumption per Capita vs. Log GDP per Capita in 2000

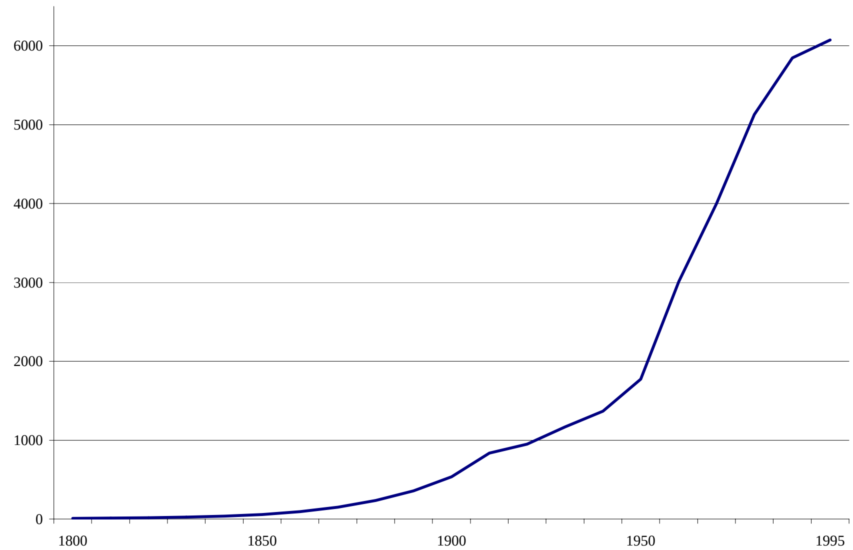


Data source: World Development Indicators 2005. Notes: a) The sample include 117 countries. b) The line in the figure is estimated by OLS; the slope estimate is 0.63 with an associated t-value of 24.

Another way to appreciate the narrow connection between external energy consumption and the growth process is by examining the historical record. Figure 2 sketches the evolution of world energy use (including that pertaining to electric power consumption) from 1800 to 1995. The overall increase in energy use which has taken place over the last 200 years is staggering: from 1800 to 1995 world use of energy increased 630 fold. According to McNeil (2001) the human species likely consumed 10 times more energy during the 20th century than during the entire millennium preceding 1900 CE. This increase has been made possible by way of human ingenuity, i.e. is due to technological change. The expansion of the power grid is one such innovation that has made the trajectory in Figure 2 feasible; it is also likely to have bearing on the dramatic cross-country differences displayed in Figure 1.

Our approach to modelling electricity distribution is highly influenced by research in the field of biology. Ever since Kleiber (1932) it has been known to biologists that a strong correlation, referred to as "Kleiber's law", is found between energy intake of biological organisms (basal

Figure 2: World Energy Use: 1800-1995



Data source: Netherlands Environmental Assessment Agency, History Database of the Global Environment. Data is available at http://www.mnp.nl/en/themasites/hyde/index.html. *Notes*: Units are million tons of oil equivalents

metabolism) and their energy requirements (body mass). Specifically, the two are related (are scaled) as follows: $B = B_0 \cdot m^b$, where B is basal metabolism, m is mass, B_0 is a constant, and $b = 3/4$. Remarkably, this association holds across biological organisms spanning 27 orders of magnitude in mass; from the molecular level up to whales (West and Brown, 2005). Recently, biologists and physicists in collaboration have started to provide microfoundations for this power law (West et al., 1997, 1999; Banavar et al., 1999, 2002). The common denominator of these theories is that they fundamentally seek to explain Kleibers law as a manifestation of how energy is diffused and absorbed in biological systems, viewed as energy transporting networks. Inspired by these theories one might hypothesize that something similar holds for man-made networks.

In the next section we develop a model of an economy viewed as a transportation network for electricity. The model predicts that electricity consumption per capita (e) can, loosely speaking, be seen as the economic counterpart to metabolism, and capital per capita (k) as the counterpart to body size; the association between electricity and capital is concave, and log-linear as Kleiber's law. The relevant interpretation of this power law association is as a *supply relation*: it captures the ability of an economy to make electricity available at geographically dispersed sites for the purpose of final use.

The model delivers the above mentioned power law association between e and k and offers predictions regarding the size of the key elasticity linking capital and electricity consumption. Specifically, it is demonstrated that depending on the efficiency of the economy in the context of electricity distribution (in a sense to be made precise below), the elasticity should fall in a bounded interval ranging from 1/2 to 3/4; the more efficient the economy the larger the elasticity. By implication, economies that are more efficient in energy distribution will be able to make more electricity available for final use. *Ceteris paribus*, such economies will be able to grow faster than less energy efficient economies.

We subsequently add a representation of electricity demand. From an accounting perspective electricity can be viewed as being used for three basic purposes: running and maintaining existing capital and creating new capital. The notion of capital is broad, including both electricity consuming capital used by households (e.g., an air conditioner in a living room) as well as capital used by firms (e.g., an air conditioner in a factory hall).[2] If a "characteristic" machine consumes a certain amount of electricity in use, and requires a certain amount of electricity to be created, total electricity demand (at an instant in time) is simply the sum of the electricity requirements related to use, maintenance, and construction of capital. Assuming the demand for electricity (thus determined) equals supply (as reflected in the power law association) we can derive a simple first order differential equation governing the evolution of capital per capita over time. We can then proceed to study the implied dynamics, and characterize the steady state.

The law of motion for capital is mathematically isomorphic to the one emanating from a Solow (1956) model, where the aggregate production function is assumed to be Cobb-Douglas. But in contrast to the Solow model the structure developed below holds predictions for the amount of capital (usable in consumption or production) that an economy can sustain in the long run, from a physical perspective. This level of capital is determined by the efficiency of the electricity distributing network, as well as the energy cost associated with running, maintaining, and creating capital.

[2] As a result, our concept of capital is not the same as the national accounts concept, which only classifies an air conditioner as capital if it is used by firms; if the (same) air conditioner is placed in a household it is classified as a durable consumption good in national accounts. In many contexts this distinction is important. In present situation, however, it makes no difference to electricity requirements whether the machine is placed in a home, or in a factory. As a result, we abstract from the distinction in the formal analysis below.

Although we abstract from savings and investment behavior in the analysis below, it should be understood that behind the scenes households are allocating resources towards either consumption (some of which involve capital and thus electricity) or savings (investment) which influences the determination of GDP per worker. Hence, the essential parts of the neoclassical growth theory are fully compatible with the present framework as long as the economy is operating within the boundaries of what is physically feasible. That is, as long as the total amount of capital (equipment required by firms and durable consumption goods demanded by households) is within the bounds of what the economy can sustain from a physical perspective.

Below we argue that technological change is the key driver behind changes in this physical limit to capital per capita in the long run. Hence, one should not view the steady state derived below as an absolute boundary for capital, but rather, as a constraint which continually is being modified due to technological change.

From an empirical perspective, however, the law of motion for capital is less directly applicable, because the definition of capital in the formal analysis differs from the national accounts concept (Cf footnote 2). However, we demonstrate below that there is a *dual* representation of the dynamics. Namely, in terms of electricity consumption per capita. This is a directly observable variable. Moreover, as argued below, this representation *will* have bearing on the empirically observed trajectory for electricity consumption per capita. Accordingly, with this representation in hand we can take the model to the data so as to examine its key predictions.

In terms of electricity consumption per capita the model holds several strong predictions. First, conditional on structural characteristics of an economy (notably population growth) one would expect to see β-convergence in electricity consumption per capita. Second, the model can be structurally estimated, which allows for the identification of the networks parameter for which we have a theoretical prior. Using data on electricity sales across the 50 U.S. states we find strong support for the model. Over the period 1960-2000 there is marked tendency for conditional β-convergence in electricity consumption per capita. Moreover, the 95% confidence interval for the networks parameter conforms with the theoretical predictions: 1/2 to nearly 3/4. The point estimate is about 2/3.

The remaining part of the paper proceeds as follows. Section 2 and 3 lay out the model of the economy, seen as an electricity distributing network, and derive the power law association between electricity consumption and capital per capita. We then proceed, in Section 4, to add electricity demand, and derive the law of motion for capital and energy, respectively. Section 5 contains a discussion of the models' predictions, and Section 6 contains the empirical analysis. Finally, Section 7 concludes.

2. The Economy as a Network

In the biological context, the power law (i.e., Kleiber's law mentioned above) is derived from the notion of living organisms as energy transporting networks whereby the size of the organism is given by the number of energy consuming units, i.e. the number cells that have to be fed.[3] By way of analogy we view an economic "organism" as an electricity transporting network whereby the size ultimately is related to the amount of capital which needs to be "fed". This section therefore extends the model due to Banavar et al. (1999) so as to analyze an economy distributing electricity.[4]

Fundamentally, the purpose of the network is to deliver (non-human) energy to the electricity consuming units of the economy. Specifically, electricity is assumed to originate from a source (a power plant), and is diffused across the economy via a power grid to the sites at which it is used. In keeping with the established terminology (Banavar et al., 1999), we refer to each site as "a transfer site"; this is where energy is converted into work effort. Each transfer site is assumed to be scale-invariant; as the network expands the geometric size of the individual transfer site does not change. A reasonable way to think about the transfer sites is as electricity outlets, which arguably fulfill this requirement.[5] Moreover, all transfer sites are locally connected, and thus linked to the source either directly or indirectly via transmission lines.

The size of the network is defined by the geometric size of the shape which defines its outer contours. In biology the outer contour of the network is tangible - the body. In the present case it is abstract; i.e., the geometric shape which *would* be able to engulf the network. The fact that this geometric shape is not tangible is immaterial for the argument.

[3] West et al., 1997; West and Brown, 2005; Banavar et al., 1999, 2002.

[4] For other applications of the theory to drainage basins of rivers, see Maritan et al. (2002) and Rinaldo et al. (2006).

[5] The size of electricity outlets are in practise independent of the size of the associated building, and the surrounding network. They also tend to have the same size across countries; electricity outlets are no bigger in rich countries than in poor countries (though they surely are more plentiful). As a result, maintaining scale invariance of the transfer site may not be unreasonable in an economic context.

Let the linear size of the network be denoted by L so that the total size of the network is proportional to L^D, where D is the dimension of the network.[6] The distance between a transfer site and the source i is defined by the number of transfer sites one will have to pass to get from i to the source. A bigger network nests more transfer sites. Hence, given the distance convention it is therefore inevitable that the mean distance between the transfer sites and the source rises as the network becomes larger. However, the specific nature of the network matters for how large the increase is, as will be seen momentarily.[7]

Generally, assuming a space-filling network and scale-invariant transfer sites, the number of transfer sites must rise with the (geometric) size of the network. As each transfer site uses energy, one may anticipate an association between the size of the network and the total energy consumed at the transfer sites, E. More specifically, we assume that for a given size of the network L^D total energy consumption is linear in population size P.

$$E \propto L^D \cdot P. \tag{1}$$

As a result, a change in *per capita* electricity consumption requires a proportional change in the size of the network, $e \propto L^D$. That is, per capita electricity consumption, e, is ultimately attributable to the number of devices (e.g. television sets, washing machines, computers and so on), which a given population utilizes. The notion is that every time a new piece of equipment is connected to an electricity outlet, a new transfer site emerges, and the network expands allowing for more electricity consumption per capita.

Empirically, strong support has been found for such a linear association between E and P for a cross section of German cities (Kühnert et al. 2006) and Chinese urban administrative units (Bettencourt et al., 2006). In the empirical section we provide additional support using cross-state data for the US.

A key result in Banavar et al.'s network theory is the proof of an association between *total* flow of energy in a network, F, and the size of the network given by $F \propto E \cdot L^x$, where x depends on the efficiency of the network. Specifically, $x = 1$ in directed networks, which minimize total energy requirements needed to fuel the economy (or the organism in biology) subject to the

[6]For example, suppose the network could be contained within a geometric shape in form of a sphere. The linear size of this object is the diameter. The volume of a sphere (i.e. a 3 dimensional object) with radius $(1/2)\cdot L$, is $(4/3) \cdot \pi \cdot ((1/2) \cdot L)^3$, which therefore is proportional to L^3.

[7]See Banavar et al. (1999) for sketches of networks.

requirement that all sites are served. In the most inefficient network, the space-filling spiral, $x = D$. Inserting (1) we get $F \propto L^{D+x} \cdot P$. In other words, when the size of the network rises the energy flow per capita (F/P) expands at least in proportion to L^{D+1}, and at most in proportion to L^{2D}.

The basic intuition for this result the following. With a directed network, the average distance between the transfer sites and the source is minimized, which implies that the total amount of energy needed to serve the system (F) of a given size is minimized. By contrast, with the space filling spiral, energy needs to be transported (physically speaking) father to serve each transfer site in the network. Consequently, the amount of energy needed to serve the system in its totality, at any given instant in time, is larger. Hence, the organization of the network may be more or less efficient, in the sense of the total amount of energy needed relative to the size of the network (or the number of appliances that needs to be fed with electricity).

Finally, we assume proportionality between the total capital stock and the total energy $F \propto K$. This assumption is thought to capture that capital is nested at the transfer sites *and* in the network itself, in the form of the transmission lines that connects the transfer sites. Hence capital is needed to transfer energy (and "hosts" energy in the process) to the sites where capital uses energy. Energy conservation in the system at large (at any given instant in time) would then suggest proportionality between the capital stock and the total flow of energy in the system.[8]

3. A Power Law for Energy Consumption of Economies

The established associations between the capital stock, size of the network, and energy consumption per capita, $F \propto K$, $F \propto L^{D+x} \cdot P$, $e \propto L^D$ can be summarized as a log-linear association between energy use per capita and the amount of capital per capita k, determined up to a constant ϵ. It represents the reduced form of the economy as a network:

$$e = \epsilon k^a, \qquad a \equiv \frac{D}{D+x}. \tag{2}$$

[8]We impose *exact* proportionality (unit elasticity) based on the following long-run consideration. Suppose we instead assumed $F \propto K^{\phi}$, where ϕ may differ from 1. If $\phi < 1$ this would mean that as the capital stock gets larger, F/K drops, which implies that capital in the limit can be applied without the use of energy. This seems like an undesirable property. Conversely, if $\phi > 1$ it implies that energy can be diffused throughout the system, in the limit, without the use of capital. This does not seem plausible either (currently at least). That is not to deny that there could be periods during which F/K rises or declines, which could be captured by allowing for a specification such as $F \propto K^{\phi}$, where ϕ differs from 1. However, we doubt such a state of affairs could be maintained in the long run. As a result, we opt for the specification where $\phi = 1$.

The intuition for the concave scaling association is the following. When K rises, new transfer sites emerge and the size of the network expands. As a result, the mean distance between the source and the transfer sites increases. A greater mean distance between the sites and the source implies that a greater fraction of total energy supply (F) is used to "fuel" the system, as opposed to being available for consumption at the sites (E). Accordingly, the concavity reflects the difficulty in delivering increasing amounts of energy to machines connected to the power grid, when the size of the network expands. In essence it implies that the *supply* of electricity inevitably will run into "diminishing returns" as the capital stock is expanded. This will ultimately be the reason why the physically sustainable stock of capital per capita is bounded from above, *absent* technological progress.

Notice that the scaling coefficient, a, should fall in a $[1/2,\ D/(D+1)]$ interval. For a more precise prior, we need to pin down D. The most natural notion is probably that $D = 3$, i.e. a three-dimensional network. In this case the scaling exponent a should fall in the interval $[1/2,\ 3/4]$, depending on the efficiency of the system.

4. A Theory of Capital Accumulation and Energy Consumption

Energy is used to run, maintain, and create capital. Here we use a notion of capital that is broader than the national accountants' definition. It includes all energy consuming appliances. That is, the k appearing here also includes durable consumption goods. As a result, we do not distinguish whether, for example, an air-conditioning system is placed in a firm or a private household.

Assume time is continuous, and let μ be the energy requirement to operate and maintain the generic capital good while ν is the energy costs to create a new capital good. In that case energy conservation implies $E(t) = \mu K(t) + \nu \dot{K}(t)$.

We may think of this equation as capturing *demand* for electricity at any given instant in time; from an accounting perspective the right hand side of the equation summarizes the instantaneous electricity requirements. As noted in the Introduction: behind the scenes households and firms are deciding on whether to build up capital or not. The outcome of their deliberations is a given change in K, which, in conjunction with historically accumulated capital, instigates a certain amount of electricity demand as captured by the conservation equation stated above.[9]

[9]One may object to the conservation equation from the perspective that human energy is also used to run, maintain and build capital. This would call for an additional term on the left hand side, capturing metabolic energy supply by the labor force. In per capita terms, however, the human contribution is in contemporary societies minuscule

For future reference, observe that if we were to shut off energy use, the capital stock would be declining over time at the rate μ/ν, due to lack of maintenance and replacement. Hence, the ratio μ/ν captures the physical phenomenon of *capital depreciation*.

Dividing through by population size P in the energy conservation equation we get

$$e(t) = \mu k(t) + \nu \frac{\dot{K}(t)}{P(t)}.$$

Assume that the population grows at a constant rate of n. Then $\dot{K}(t)/P(t) = \dot{k}(t) + nk(t)$. Inserting this and the power law association (1) into the above equation provides the law of motion for capital:

$$\dot{k}(t) = \frac{\epsilon}{\nu} k(t)^a - \left(\frac{\mu}{\nu} + n\right) k(t). \tag{3}$$

Observe that by inserting the power law we are in effect assuming that demand for electricity equals electricity supply, at any given instant in time. The meaningful interpretation of the above law of motion is that it captures physical *feasibility*. In practise, of course, households and firms decide on how much capital to accumulate. Standard neoclassical growth theory provides a basis for analyzing this process. However, the above equation represents the upper contour of what an economy at most can sustain if the conservation law is to be fulfilled, and, electricity is to be distributed to serve the (increasing number of) machines. In general, therefore, the above equation may not hold positive implications. But in some instances the above equation could represent a binding constraint on the process of capital accumulation. If the network is sufficiently inefficient, and the energy costs of running and maintaining capital is sufficiently large, an economy may not be physically able to accumulate at the "desired rate", as determined by households and firms. Under this scenario the above equation could provide a positive description of capital accumulation.[10]

The dynamical analysis is straight forward. Formally, the model shares the technical properties with the Solow model, where technology is assumed to be Cobb-Douglas. In particular, there exists a unique globally stable steady-state, to which the economy adjusts.

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compared to the non-human energy use. Moses and Brown (2003) puts it nicely into perspective (p. 296): : "*The per capita energy consumption in the United States is 11.000 W ... which is approximately 100 times the rate of biological metabolism and, ...* [it] *is the estimated rate of energy consumption of a 30.000-kg primate*". From this perspective nothing much lost by ignoring the human *physical* (as opposed to intellectual) contribution to the growth process.

[10]It may be that firms and households accumulate machines, in spite of the lack of an ability to "feed" them with electricity. Needless to say, this would imply that all capital above that given by the law of motion stated above would lay idle.

Mechanically, the adjustment process works as follows. At any given instant in time k is predetermined. Given k a size of the underlying network is implied, and consequently the supply of energy (which is assumed to adjust), e, is determined. If e is sufficiently large, i.e. it exceeds the energy needs required to maintain and run existing capital, the stock of capital can expand further. However, as the network expands the amount of additional energy which can be made available for direct use starts to diminish (i.e. E/F declines). Eventually, therefore, the system settles down at a steady state level of $k = k^* \equiv [(\mu + n\nu)/\epsilon]^{1/(a-1)}$. This can be seen as the maximum sustainable stock of capital, given the parameters. Below we argue that ν, a and μ should be given a technological interpretation. Hence, if these parameters change then k^* changes as well. Accordingly, this upper boundary is to be viewed as a moving "target".

Observe that we can obtain a *dual* representation of the model in terms of energy consumption, by using $e = \epsilon \cdot k^a$ in (3):

$$\dot{e}(t) = \left[\frac{\epsilon^{1/a}}{\nu} \cdot e(t)^{-(a-1)/a} - \left(\frac{\mu}{\nu} + n \right) \right] \cdot a \cdot e(t) \tag{4}$$

Accordingly, this equation provides a description of the evolution of energy consumption per capita over time.

This representation will prove useful in the empirical analysis below. Partly because e can be directly observed (as opposed to the present notion of k), and partly because the above equation should hold from a positive perspective. The trajectory for k is in practise decided upon by households and firms, as noted above. Whatever the choices of households and firms may be in this regard, it will instigate a certain electricity demand, as captured by the conservation equation. Moreover, under the theory, supply needs to keep pace. The supply of electricity is captured by the power law. Accordingly, given these two elements the above equation should govern the evolution of electricity demand, if indeed supply equals demand. We hypothesize that this state of affairs is a reasonable characterization of the 50 U.S. states over the last 40 years. If so, the above law of motion for electricity demand should hold explanatory power *viz* observed state level electricity sales per capita.

5. Discussion

The model holds various implications on issues of whether growth can be sustained, the plausibility of neoclassical growth theory and on technological change and economic growth. These implications are discussed in turn.

5.1. **Neoclassical growth theory and its critics.** The model allows some reconciliation between neoclassical growth theory and the work of its staunchest critics (Daly, 1977, Georgescu-Roegen, 1976). The central charge is that energy is introduced into the standard models in an unsatisfactory way (if not ignored altogether). That is, by including energy in the aggregate production function as a separate input, which can be substituted for by capital. This approach is fundamentally flawed, the argument goes, because it does not consider that any capital good is itself produced by means of energy. Here, we have explicitly taken into account that all capital is created, run, and maintained through energy use. Interestingly, we nevertheless arrive at a law of motion for capital which is structurally identical to that implied by the Solow model. Hence, the structure of the Solow (1956) model is not at variance with fundamental physical principles, like energy conservation.

5.2. **Can growth be sustained?** In the last section we found that the maximum sustainable stock of capital per capita is bounded from above. For a proper assessment of the steady-state result note that it is not derived from the assumption of limited supply of energy. Instead, convergence towards a constant capital stock per capita is a consequence of energy demand, distribution, and the entailed decreasing returns from expanding networks. It is sometimes argued that economic growth is ultimately limited from above by energy availability (Daly, 1977). Interestingly, however, the present analysis demonstrates that – absent technological progress – economic growth is limited *even if* energy supply were unlimited. This brings us to the issue of how "technology" is said to be present in the model above.

5.3. **Technological Change.** In the context of the model, technological change will come in the shape of changes in the key parameters: ν, μ and a. That is, innovations which lower electricity requirements or improve the efficiency of the network. These innovations are critically important in facilitating capital accumulation. Without them, equation (3) would (eventually) become a binding constraint on the growth process implying that capital accumulation should come to a halt on purely physical grounds.

Innovations that map into these parameters are easy to think of. The fact that technological advances make appliances less energy-hungry because, for example, smaller sizes of computers or microchips are fulfilling the same task that required large, energy consuming machines in the past, is captured by the model as a reduction of μ. A one time reduction of μ leads to temporarily higher growth according to (3) and convergence towards a permanently higher steady-state k^*. A perpetual reduction of μ could be conceptualized as convergence towards "the weightless economy" (Quah, 1999).

A permanently lower value of ν captures an innovation that allows to produce new machines at less energy costs. Such a parametric change seems to fit quite easily with the sort of innovations growth theories usually refer to as "general purpose technologies" (GPT). GPT innovations are viewed as "fundamental" innovations which tend to "reset" the economy, and instigate (ultimately) a growth "spurt". The process, however, may involve a non-monotonous adjustment process, with an initial slump of productivity while the GPT forces a replacement of old machines with new ones that employ the new basic technology.

Bresnahan and Trajtenberg (1996) who where among those who initiated GPT research asked (p. 84): "Could it be that a handful of technologies had a dramatic impact on growth over extended periods of time? What is it in the nature of the steam engine, the electric motor, or the silicon wafer, that make them prime suspects of having played such a role?" They gave a very broad answer which is still used in the literature (see e.g., Jovanovich and Roussseau, 2005): The technology must be pervasive (spread to most sectors), there must be scope for improvement over time (lowering the costs of its use) and it must be innovation spawning, i.e. it enables the production of new products. The following "handful" of technologies are usually referred to as GPT's: the waterwheel, the steam engine, electricity, railways, motor vehicles, and IT.

Based on the theory developed above we can suggest a more precise answer to Bresnahan and Trajtenberg's question. A GPT must improve either the use of energy (waterwheel, steam), its delivery through a network (railways, cars) or both (electricity, IT). Interestingly, while not all proposals of GPT candidates available in the literature coincide perfectly, electricity and IT, the technologies that revolutionized both the use and distribution of energy, are always on the lists. Speculating about what could possibly be the next GPT experts usually come up with nano-technology; again a new system for distributing energy at a new (finer) level of network with less power loss. With our theory at hand it becomes intuitive why other seemingly equally fundamental innovations (e.g. the decoding of the DNA) are not GPT's: they do not (much) improve the distribution and use of non-human energy.

An *ad hoc* way to mimic the arrival of a GPT within the standard Solow model is to simultaneously vary general productivity (A) and the depreciation rate (δ). The first parameter change is meant to capture the long-run increase in productivity, and the second captures the initial slump, originating from obsolescence of machines embodying the old technology (see Aghion and Howitt, 1998, Ch. 8.4). The problem is that both measures move the steady-state in opposite directions and some fine-tuning is needed to create the desired transitional and long-run effects.

In the current model, we have a – while admittedly equally *ad hoc* – somewhat more elegant way to produce the desired growth trajectory: a decrease of ν. A lower value for ν means that machines can be produced at lower energy costs, which, for example, could have been initiated through the transistor replacing the energy-intensive vacuum tube in electronic devices. From inspection of the reduced form of the model (3) we see that a lower ν has a double effect. It raises both the first term, "productivity", and the second term, "depreciation". It is easy to see from the steady state solution for k that a lower value for ν unambiguously raises the steady-state level. Starting at the original steady-state (using tube technology) $\dot{k}$ equals zero initially. Evaluating the RHS of (3) after the fall of ν, we see (since $a < 1$) that initially the negative effect through the depreciation channel dominates. In conclusion, energy saving technological progress causes GPT-like adjustment dynamics with an initial slump, recovery, and convergence towards a higher steady-state level.

6. Empirical Evidence

A central purpose of this section is to obtain an estimate for the scaling parameter a; in theory a should fall in an interval between 1/2 and 3/4. Hence, a central question is whether data supports this prediction. In addition, the model holds predictions about the nature of the growth process with regard to electricity consumption per capita: conditional β-convergence should prevail. In this section we examine these predictions.

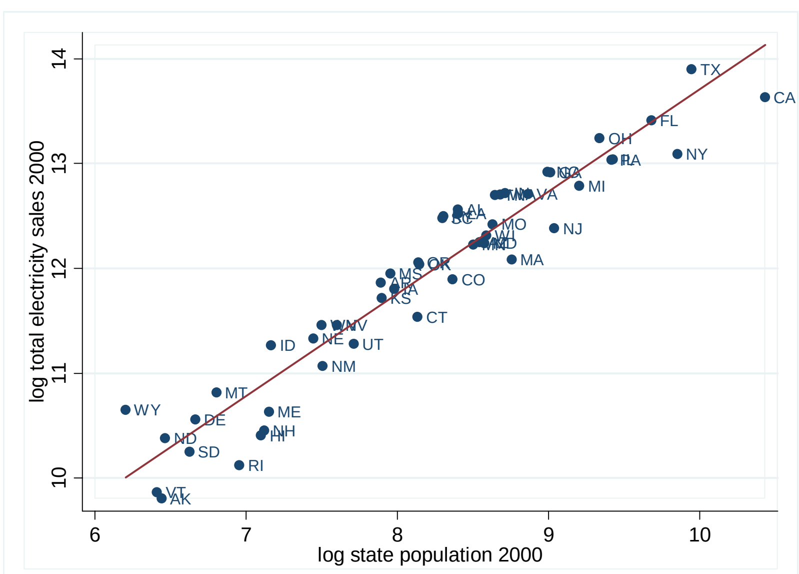
6.1. **Testing a key assumption.** As a preliminary test, however, we check the validity of the crucial underlying assumption of the model, the postulated linear association between total electricity consumption and total population (1). It is based on the following specification

$$\log(E_i) = a_0 + a_1 \cdot \log(P_i) + \varepsilon_i, \tag{5}$$

where i denotes the unit of observation. The variable ε_i is a noise term. That is, structurally $L_i^D = a_0 + \varepsilon_i$, which allows the size of the network to vary across units of observation. Our assumption, for which we require validation, is that $a_1 = 1$. The identifying assumption, for ordinary least squares (OLS) to deliver an unbiased estimate for a_1 is that $E(\varepsilon \cdot P) = 0$, where $E(\cdot)$ is the expectation operator. In terms of the model, this means that the size of population is uncorrelated with the capital-labor ratio, since the latter determines L. Under the null (the model is correct), this assumption is plausible.

We obtained data for electricity sales across the U.S. states from the US energy information administration. Specifically, electricity sold to end users: the residential sector, the commercial sector, and the industrial sector. The data is available free of charge at http://www.eia.doe.gov/emeu/states/ _seds.html. From this source we also obtained data for state populations.

Figure 3: Population Size and Energy Consumption Across US States



Electricity consumption vs. Population size: 50 US States in 2002. The line illustrated is estimated by OLS, and yields the following result: log(Electricity) $= 3.95 + 0.98 \cdot$ log(Population), $R^2 = 0.92$, 95%, CI for population coefficient is (0.87,1.08), Observations =50.

Figure 3 provides a visual impression of the results from estimating equation (5) by OLS and reports the parameter estimates; the regression relates to the year 2000.

As is visually obvious, the cross-state sample corroborates previous findings for German cities and Chinese urban administrative units. The coefficient for $\log(P)$ is close to 1, with a fairly narrow 95% confidence interval. Hence, Figure 3 informs us that a potential empirical failure of the theory cannot be ascribed to a violation of equation (1).

6.2. **Testing the full theory.** The second test concerns the ultimate structure of the model. One possibility would be to test the "primal" equation (3). Unfortunately this is not a viable exercise for two reasons. First, k is not directly observable, as noted above. Second, the law of motion for k does not necessarily carry positive predictions; it captures feasible accumulation which is not (nessesarily) the same as actual accumulation.

Fortunately, instead of estimating the primal model we can use the fact that the model has a dual representation (4) in energy consumption. In this representation of the model we are dealing with a directly observable variable: electricity consumption per capita. Moreover, the law of motion for e ought to have positive implications, as argued above.

In order to facilitate estimation we log-linearize (4) around the steady state where $e = e^* \equiv \epsilon[(\mu + n\nu)/\epsilon]^{a/(a-1)}$:

$$\ln[e(t)] = \ln[e(0)]\, e^{-\lambda t} + \left(1 - e^{-\lambda t}\right)\left[\frac{1}{1-a}\ln(\epsilon) - \frac{a}{1-a}\ln(v) - \frac{a}{1-a}\ln(n + \mu/\nu)\right],$$

where $\lambda \equiv (1-a)(n + \mu/\nu)$.

This equation is, from a formal perspective, very similar to that implied by the Solow model. The conventional approach to taking this model to the data consists of estimating the convergence equation by OLS, assuming the rate of convergence (λ) is a parameter (e.g., Mankiw et al., 1992). However, the equation is *non-linear* in the parameter of interest (a). As a result, the model should not be estimated by OLS. Instead we will have to resort to an iterative procedure. Below we therefore report the results from estimating a by non-linear least squares (NLS).[11]

Unfortunately, some of the variables entering the estimation equation are unobservable: the energy costs of running and maintaining capital (μ), the costs associated with creating new capital (ν), and the efficiency parameter ϵ. From the primal model, however, we have a prior

[11] To our knowledge Dowrick (2004) is the first to make this point in the context of cross-country growth empirics, and to estimate the (augmented) Solow model by NLS. We essentially follow his approach when estimating equation (6).

for the *ratio* μ/ν; it should reflect the rate of capital depreciation. Hence, a reasonable number for μ/ν can be imposed from the literature; a typical finding for the US is a rate of capital depreciation of 6% (Nadiri and Prucha, 1996, McQuinn and Whelan, 2007), which we therefore impose *a priori*.

In sum, the equation we estimate is:

$$\ln\left[e_i\left(T\right)\right] - \ln\left[e_i\left(0\right)\right] = -\left[1 - e^{-b_1\cdot(n_i+0.06)\cdot T}\right]\left[b_2 + b_3\ln\left(n_i + 0.06\right) + \ln e_i\left(0\right)\right] + u_i, \quad (6)$$

where the predictions of our theory are: $b_1 > 0$, $b_3 > 0$ and $1 - b_1 = b_3/(1+b_3) = a \in (1/2, 3/4)$. The parameter b_2 captures the influence from ϵ and ν; its constancy reflects the belief that these variables are constant across the 50 states. We find this identifying assumption plausible. The parameters ν, ϵ and μ are arguably of a technological nature, as discussed above, and the U.S. states are undoubtedly fairly homogenous in this respect. Omitting these parameters is therefore likely to be only a minor problem. In any case, that is an identifying assumption of the empirical analysis.

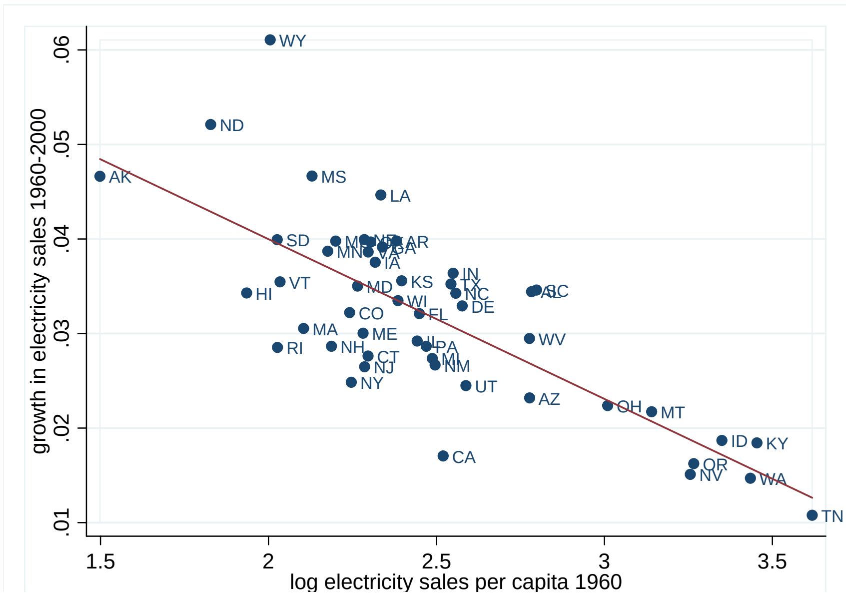
We test the model over the period 1960-2000; data on n and e derive from the US energy information administration. Since we are examining the period 1960-2000, $T = 40$; u_i is a noise term, which is assumed to be uncorrelated with the right hand side variables.

Before we turn to the NLS estimates it is worth showing the basic correlations in the data. Figure 4 shows the unconditional association between growth in electricity consumption per capita 1960-2000 versus initial electricity consumption per capita. The straight line is estimated by OLS. The partial correlation is highly significant, as is obvious. Figure 5 shows the unconditional correlation between growth in electricity consumption per capita and the log of average state population growth 1960-2000 (plus 0.06). In keeping with the theory, the association is negative, and significant. A simple OLS regression, where the nonlinearities are ignored, can account for 60 % of the variation growth of electricity sales per capita across the 50 states.

Turning to formal regressions, Table 1 reports the results from estimating equation (6) by NLS.

In panel A we report the results from estimating the model freely. That is, without imposing the restriction $1 - b_1 = b_3/(1+b_3)$. As can be seen from the R^2, the model accounts well for the cross-state variation in log changes in energy consumption per capita over the 1960-2000 period. Moreover, the signs of the parameters are as predicted: $b_1 > 0$, $b_3 > 0$; both are significant at

Figure 4: Convergence



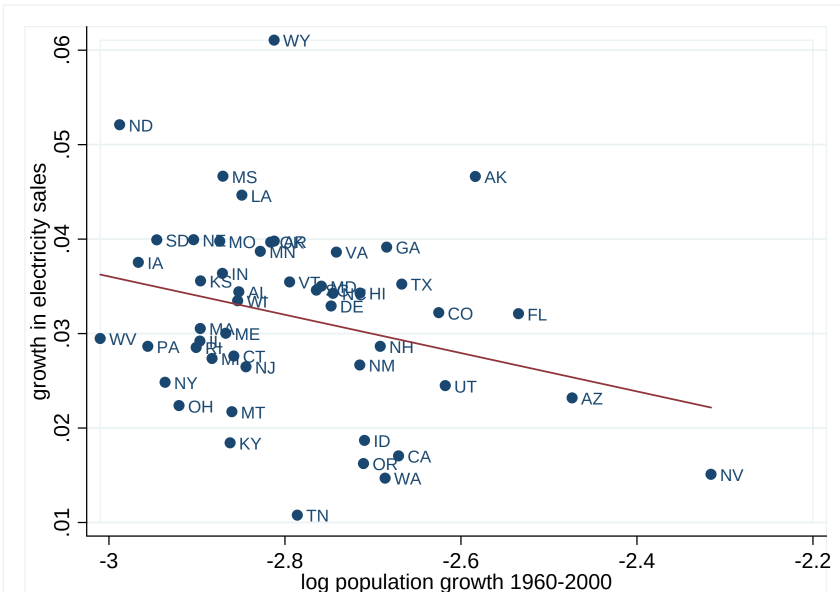
The figure shows the correlation between initial (log) energy consumption per capita and subsequent growth in electricity consumption per capita: 50 US states, 1960-2000. The straight line is estimated by OLS

the 1% level of significance. In contrast, the parameter for b_2 comes out insignificant. Taken at face value this suggests $\frac{1}{1-a}\ln(\epsilon) = \frac{a}{1-a}\ln(v)$. The *structure* of the model is not rejected by the data either. That is, the implied equality of $1 - b_1$ and $b_3/(1 + b_3)$ is not rejected.

Consequently, we also estimated the model where $a = 1 - b_1 = b_3/(1 + b_3)$ is imposed. The results are shown in Panel B. The point estimate of the scaling exponent a is 0.62; close to 2/3. Interestingly, the confidence interval for a coincides almost perfectly with the predicted range for a, as suggested by theory (i.e. $a \in [1/2,\ 3/4]$). As a result, we can reject a being smaller than the lowest admissible value under the theory: 1/2. At the same time, "full efficiency" can be rejected: $a < 3/4$ at the 5% level. In biology it is reasonable to argue that nature ensures full efficiency, by way of natural selection, which implies a scaling coefficient between basal metabolism and body size of 3/4 . In the context of man-made networks it is perhaps unsurprising to find evidence in favor of some inefficiencies.

In sum, the above results support a model of energy consumption which builds on two elements: (i) log-linear scaling between capital per capita and electricity consumption per capita, and, (ii) an accounting equation relating electricity consumption to capital use, maintenance

Figure 5: Growth and state population growth



The figure shows the correlation between log state population growth (+0.06) and growth in electricity consumption per capita: 50 US states, 1960-2000. The straight line is estimated by OLS

and accumulation. Moreover, the state-level data suggests that a scaling coefficient around 2/3 is the best approximation.

7. Conclusion

The fundamental notion that economic growth originates from (and is limited by) energy has a long intellectual history, going back to Herbert Spencer's (1862) *First Principles*. According to Spencer the evolution of societies depends on their ability to harness increasing amounts of energy for the purpose of production. Differences in stages of development can be accounted for by energy: the more energy a society consumes the more advanced it is. Chemist and Nobel prize winner Wilhelm Ostwald (1907) developed the Spencerian ideas further. Ostwald emphasized that it is not the sheer use of energy, but the degree of efficiency by which raw energy is made available for human purposes that defines the stage of economic (and according to Ostwald also cultural) development of society.[12]

[12] Further refinements were made by several natural and social scientists, among them Frederick Soddy, Alfred Lottka, and Fred Cottrell.

Table 1: NLS estimates

Panel A Unrestricted Model	
Dependent variable: log change in energy consumption per capita 1960-2000	
b1	0.36 (0.07)
b2	-0.38 (0.89)
b3	1.55 (0.37)
1-b1=(b3/(1+b3) (p-value)	0.60
R^2	0.96
Obs.	50
Panel B Restricted model; 1-b1=(b3/(1+b3) imposed	
1-b1	0.62 (0.05)
b2	-0.05 (0.88)
95 % CI for a	(0.52,0.73)
R^2	0.96
Obs.	50
Notes: Heteroskedasticity robust standard deviations in parenthesis.	

The theory developed above demonstrates that this notion of development, when given a modern network interpretation, is compatible with neoclassical growth theory. Indeed, it coincides with the structural form of the economist's core model of economic growth, the Solow growth model. The central element of the theory, the network equation for electricity supply, receives support in US state-level data.

The theory has bearing on the fundamental "limits to growth" debate. In particular, while conceding the importance of energy for growth, the theory also highlights the crucial importance of human ingenuity. As shown above, absent technological change, growth will come to a halt even with unlimited supplies of energy, since energy dissipation increases as the economic network (appliances and machines connected) becomes larger. This result therefore implies that technology, associated with the harnessing and use of energy, is as important for growth prospects as the supply of energy itself; energy and technology are equal partners in development. Indeed, as argued above, "major" innovations (which usually are referred to as GPTs) can be seen as rare instances of progress, which in a profound way improve the harnessing, transformation, and/or distribution of energy. Integrating the literature on endogenous technological change, with the present model of capital accumulation, would therefore seem like a useful topic for future research.

The framework could also be adapted to the study of growth in the very long run. It seems widely conceded that human societies at large enjoy income and consumption levels of historically unprecedented magnitudes (e.g. Galor, 2005). A key implication of the model above is that such increases is inescapably linked to the ability of human societies to expand energy supply, which requires technological innovations. In particular then, such a long-run growth model would suggest that the recent harnessing of electricity during the 19th century should sow the seeds of a dramatic change in human societies. First, it is the period during which the modern day energy transport network is created. That is, this period represents the genesis of $e(t) = \epsilon k(t)^a$. Second, as a result, these innovations allowed for investment growth, and thus income growth, of unprecedented scale, by removing the constraint on capital accumulation previously imposed by energy supply in ways of the metabolism of humans and animals. Accordingly, integrating the framework above with the unified growth literature also seems like a fruitful avenue for future research.

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