The History Augmented Solow Model*

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Abstract. Unified growth theory predicts that the timing of the fertility transition is a key determinant of contemporary comparative development, as it marks the onset of the take-off to sustained growth. Neoclassical growth theory presupposes a take-off, and explains comparative development by variations in (subsequent) investment rates. The present analysis integrates these two perspectives empirically, and shows that they together constitute a powerful predictive tool vis-a-vis contemporary income differences.

Keywords: Comparative Development; Unified Growth Theory; Neoclassical Growth Theory.
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1. Introduction

One of the biggest questions in social science is why some countries are more developed than others. A good place to start thinking about an answer, from an economics angle, remains the seminal Solow (1956) model. As shown by Mankiw, Romer and Weil (1992), and by many since, the basic Solow model holds considerable explanatory power vis-a-vis contemporary income differences. Yet, it leaves much income variation unexplained, which has led to various "augmentations" involving human capital (Mankiw, Romer and Weil, 1992; Ram, 2007), R&D (Nunnemann and Vanhout, 1996) and health and longevity (Knowles and Owen, 1995).

Another strand of literature argues, however, that in order to better understand comparative development one must also understand the take-off to sustained growth, which separates an epoch of stagnation from the current regime where (many) countries grow persistently in terms of income per capita (Galor and Weil, 2000; Lucas, 2002; Hansen and Prescott, 2002; Galor and Moav, 2002). Hence, this literature – often referred to as "unified growth theory" – provides the mechanics of economic development before and after the take-off to growth (see Galor, 2011 for a survey). From the perspective of unified growth theory, neoclassical growth theory, which only analyzes the growth process *once it has begun*, is therefore an incomplete basis for an understanding of comparative development. A fuller understanding is arguably achieved by recognizing that a large fraction of contemporary income differences is due to the differential timing of the take-off itself (See Galor, 2010a). Theoretically, this critical historical turning point is hypothesized to be narrowly connected to the demographic transition; the *fertility transition*, specifically.[1] In theory, the onset of the fertility transition stimulates subsequent growth in labor productivity for two major reasons: (*i*) by lowering population (labor force) growth it reduces the extent of capital dilution, (*ii*) by stimulating human capital accumulation it drives growth directly, but also indirectly via technological innovation.[2]

The present paper demonstrates empirically that the combination of these two literatures delivers a powerful predictive tool. In its simplest version, the basic Solow model along with a control for the timing of the fertility transition accounts for about 2/3 of the cross-country variation in GDP per worker in 2000; up from about 40% without the control for the timing of the fertility decline.

[1] The demographic transition generally involves two separate transitions: a mortality transition and a fertility transition. The mortality transition often (but not always) precedes the fertility transition generating the familiar hump shaped trajectory for population growth.

[2] The fertility transition also leads to temporarily faster growth of the labor force relative to the total population which stimulates growth in income *per capita* (but not *per worker*). See Bloom and Williamson (1998) for more on this effect.

Moreover, the well known “anomaly” when estimating the parameters of the empirical Solow model – that the capital share is estimated to be exceedingly high – is eliminated.

The link between the year of the fertility transition and contemporary labor productivity is robust to a range of controls for early institutional developments, cultural variables, geography and climate as well as early levels of technological sophistication. However, we also demonstrate that the partial correlation between the year of the onset of fertility transition and labor productivity is eliminated if we control for the channels through which the transition (in theory) should affect the growth process: post take-off capital dilution and human capital accumulation.

The paper proceeds as follows. In the next Section we lay out the empirical framework, and explain how we integrate insights from unified growth theory with the basic neoclassical growth framework in a simple way. Section 3 provides the empirical analysis, and Section 4 concludes.

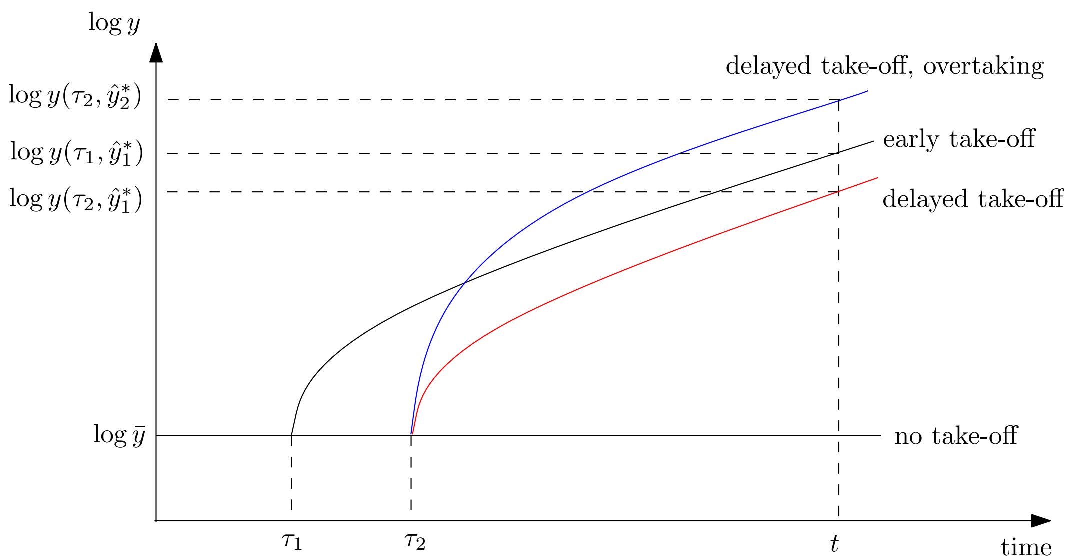
2. Empirical Framework

Figure 1 provides a stylized account of contemporary income differences. One reason why income differences may arise is found in the timing of the take-off, as described by unified growth theory. The figure illustrates a situation were one country experiences an early take-off at time τ_1, which puts it on a positive growth trajectory. Meanwhile another country has not taken off at all (as yet); the “no take-off” scenario. Naturally, if we compare the income difference between these two countries at (say) time τ_2 the difference in income would mainly be due to the differential timing of the take-off.

But the timing of the take-off is probably not the full story. For instance, Japan did not move onto a sustained growth trajectory until sometime during the 20th century, and yet it is today richer than many Western European countries that went through the transition in the 19th century. The combination of high investment rates, and the process of catch-up is a compelling explanation. Indeed, it is the standard textbook account, which involves the use of the neoclassical growth model. Figure 1 illustrates this possibility; a country that takes off at $\tau_2 > \tau_1$ may either partially catch up with the early take-off group, or even overtake them. The overtaking phenomenon would reflect different (investment) behavior *post* take-off.

Hence, whereas unified growth theory explains how comparative development is affected by the difference in timing of take-off (i.e., $\tau_1 - \tau_2$), neoclassical growth theory provides a complementary explanation as to the growth process after the take-off, which relates to the location of the steady

Figure 1: Stylized Comparative Development



The figure illustrates how the relative timing of the take-off (τ_1,τ_2 or "no-take-off") and post take-off investment behavior, implicit in relative steady state paths ($\hat{y}_1^*$, $\hat{y}_2^*$), jointly explain GDP per worker differences at time t.

state growth trajectory.[3] One might expect that the combination of the two theories would be a powerful vehicle for an understanding of comparative development.

To put these considerations slightly more formally, suppose *observed* GDP per worker stagnated prior to some point in time τ, after which the growth process takes off.[4] Observed GDP per worker at time $t > \tau$ can then be written (as a matter of pure accounting, in a continuous time setting) as

$$y(t) = y(\tau) \cdot e^{g_y(t-\tau)}, \tag{1}$$

in which g_y it the observed average growth rate between τ and t. In order to say something about how y_t is determined, we need to make statements about how the average growth rate between τ and t, g_y, is determined. This requires the use of theory.

Consider a standard continuous time Solow (1956) model, where production is Cobb-Douglas. Accordingly, output is given by $Y = K^{\alpha} (AL)^{1-\alpha}$, where Y is GDP, K is physical capital, A

[3] Of course, unified growth theory *also* speaks to the post take-off period. But from an empirical angle the Solow model (so far) provides more structure, which is useful for present purposes.

[4] Naturally, in practice, there is probably no such thing as "the year" of the take-off. The process is gradual and initially accelerating (e.g., Phillips and Sul, 2009). Hence, this is a simplistic representation designed to deliver a regression equation, the results from which are amendable to interpretation.

represents technology and L is labor; α, is the capital share in production and is bounded between zero and one. Total savings in the economy is determined as a constant share, s, of total income, and total investments, I, equal total savings. Physical capital accumulates in accordance with $\dot{K} = I - \delta K$, where δ is the rate of capital depreciation. Finally, technology grows at a constant rate, g, and the labor force at the rate n. In the long-run steady state of the model, GDP per efficiency unit of labor, $\hat{y} \equiv Y/AL$, is given by

$$\hat{y}^* = \left(\frac{s}{n+\delta+g}\right)^{\frac{\alpha}{1-\alpha}}. \tag{2}$$

In order to use the model to think about how observed GDP per worker is determined, we need to work out the predicted average growth rate of GDP per worker between τ and t (i.e, g_y in equation (1)). Seen through the lens of the Solow model the average growth rate, g_y, is approximately given by [5]

$$g_y \approx g + \frac{1}{t-\tau}\left[\log \hat{y}^* - \log \hat{y}(\tau)\right]. \tag{3}$$

That is, the average growth rate depends on the underlying - constant - trend growth rate, g, and on the contribution from "convergence", which is given by the distance between the level of GDP per worker in efficiency units at time τ, $\hat{y}(\tau) \equiv Y(\tau)/(L(\tau)A(\tau))$, and the steady state level of GDP per efficiency unit of labor, as predicted by the Solow model, $\hat{y}^*$.

If we insert equation (3) into the accounting equation (1), we obtain:

$$y(t) \approx A_\tau \left(\frac{s}{n+\delta+g}\right)^{\frac{\alpha}{1-\alpha}} \cdot e^{g(t-\tau)}, \tag{4}$$

in which A_τ is the level of technology at time τ, $A(\tau) = A_\tau$. Taking logs we are thus left with

$$\log(y(t)) \approx \log\left(A_\tau\right) + g\left(t-\tau\right) + \frac{\alpha}{1-\alpha}\log\left(s\right) - \frac{\alpha}{1-\alpha}\log\left(n+\delta+g\right). \tag{5}$$

The only difference between this expression and the one emanating from a standard Solow model is that the initial level of technology is given as of time τ rather than at some arbitrary "time zero".[6] While this difference superficially seems minute, it has a substantive implication in that it

[5] See Appendix for details on derivations. The adopted approximation assumes that the period $t-\tau$ is sufficiently long to admit the country in question to approximately reach its steady state path. This means our regressions presume that GDP per worker at the time of observation (in practise, the year 2000) is a point on the steady state path. This is a common assumption in the empirical literature, going back to the work of Mankiw et al (1992). For a recent contribution invoking this assumption, though in the context of studying historical levels of economic activity, see Ashraf and Galor (2011).

[6] See e.g. Mankiw et al (1992, equation 6), for the standard expression.

demonstrates how neoclassical growth theory as well as unified growth theory jointly contribute to an understanding of contemporary comparative development.

To see this, note that the last two terms on the right hand side of equation (5) are the neoclassical contribution to the understanding of contemporary levels of income, reflecting the level of steady state income. That is, these are the *post* take-off growth determinants.

The contribution from "history" is found in the two other terms on the right hand side of equation (5): time passed since the take-off, $t - \tau$, and the initial level of technological sophistication at the eve of take-off, A_τ (rather than at "time 0"). Accordingly, *conditional* on post take-off growth determinants (s and so forth) we expect countries to be richer the longer its been since the economy underwent the "industrial revolution", and the more sophisticated the country was technologically at the time.[7]

Before we move on, it is useful to observe how equation (5) speaks to the conceptualization of contemporary income differences in Figure 1. In the depicted illustration a collection of the countries are on their - parallel - balanced growth paths when t becomes "large", whereas another country (group of countries) is stuck in stagnation. It it clear that the implied income variation at this juncture is attributable to both factors that influence the location of the steady state growth path and factors that influence whether the process has begun at all. That is, the observed cross sectional variation in income $(\log(y[\tau_2, \hat{y}_2^*]) - \log(\bar{y}))$ is partly due to "neoclassical factors" (that influence $\hat{y}_2^*$) and partly due to the difference in timing of the take-off τ_2. More generally, the fact that A_τ enters into equation (5) implies that even though two countries may have identical $\hat{y}^*$ they do not necessarily attain the same growth path; if one were more technologically sophisticated initially this effect will linger, as shown in Figure 1. This is the "decomposition" that equation (5) captures.

In order to take equation (5) to the data a few issues need to be addressed up front. The first issue is how to date the "take-off"; τ in the derivations above. Based on the literature on unified growth theory, the year of the onset of the fertility transition should be a good proxy for τ (Galor, 2010b). While our preferred measure of τ thus is the year of the onset of the fertility transition, we also explore an alternative indicator of the take-off: the "year of industrialization", which is determined

[7] Considerable variation in technological sophistication, prior to the take-off, is not inconsistent with limited pre take-off variation in GDP per worker. In a pre-take off environment changes in technology would be expected to manifest themselves in changes in population density, but not in sustained increases in living standards; see Ashraf and Galor (2011) for theory and evidence; see Comin el al. (2010) for evidence that early technological sophistication (pre 1500) seems correlated with contemporary income differences.

as the year in which employment in industry exceeds that in agriculture.[8] These data are described below.

The second issue is how to think about "g"; the underlying rate of technology growth. We will assume that once a take-off is achieved the country taps into the world technology frontier. Hence, g can be viewed as exogenously given, from the perspective of the individual country.[9] As a result, g is common across countries in the sample which means that we can think of it as a parameter to be estimated (though see footnote 13 below).

The third issue relates to the neoclassical controls: n in particular. Empirically, peak population growth rates, during the demographic transition, vary considerably across countries (e.g., Reher, 2004; Strulik, 2008). As a result, even conditional on the transition occurring, we expect differences in average growth rates of the labor force, *post* transition. We *proxy* post transition labor force growth by 1960-2000 average growth rates; similarly, s (also an 1960-2000 average) is to be thought of as the post transition investment rate in physical capital.

A final issue is how to deal with $\log(A_\tau)$. The simplest approach is to follow Mankiw et al. (1992) and assume it is subject to random variation. Indexing countries by i this would imply $\log(A_{i\tau}) = \log(\bar{A}_\tau) + \epsilon_i$. For our baseline regressions we also invoke an arguably slightly more reasonable approach by assuming $\log(A_\tau)$ is subject to systematic *regional* variation, but random variation within regions.[10] Denoting regions by R this would mean $\log(A_{Ri\tau}) = \log(A_{R\tau}) + \epsilon_i$. This can be handled in a cross-sectional context by adding regional fixed effects to the right hand side of the regression equation.[11] For the robustness checks we push the matter a little further by controlling for plausible correlates with A_τ (such as early levels of schooling), as well as (fundamental) determinants of A_τ: geography, culture and institutions (Acemoglu, 2009, Ch. 4).

In sum the regression equation to be estimated below is:

$$\log(y_i) = \beta_0 + \beta_1 \log(s_i) + \beta_2 \log(n_i + \delta + g) + \beta_3 \tau_i + \mathbf{X}_i' \gamma + \epsilon_i. \tag{6}$$

[8] See Strulik and Weisdorf (2008) for a unified growth theory that explicitly studies both the fertility transition and the process of structural change, thereby providing theoretical foundations for the use of either of the two adopted indicators as proxies for the timing of the take-off in GDP per worker.

[9] Globally, of course, g is undoubtedly endogenous. See e.g., Howitt (2000).

[10] The celebrated book by Diamond (1997) provides historical foundations for systematic differences in historical levels of technological sophistication across regions or continents.

[11] The regions included in our regressions are: East Asia and the Pacific; Europe and Central Asia; Latin America and the Caribbean; Middle East and North Africa; South Asia and Sub-Saharan Africa.

where the matrix $\mathbf{X}_i$ contains regional fixed effects and alternative confounders to be discussed below; γ is the associated vector of regression coefficients.[12]

The history augmented Solow model holds several strong predictions, which we test below.

- The predictions that $\beta_1 > 0$, $\beta_2 < 0$ and $\beta_3 < 0$.
- The restriction that the coefficients on $\log(s)$ and $\log(n + \delta + g)$ are numerically identical: $\beta_1 + \beta_2$=0.
- The prediction that $\beta_1 = -\beta_2$ is between 0.5 and 2/3; this mirrors that the capital share α is expected to be between 1/3 and 0.4.
- The prediction that β_3 should capture the frontier productivity growth rate in the sample; a reasonable prior for this rate is about 2 percent on account of the growth trajectory of the US economy (arguably the technological leader) during the 20th century.[13]

Hence, the model offers predictions regarding the signs of the parameters (1); the relative size of the parameters (2); as well as their numerical value (3), (4). In particular, β_3 is negative as each year of delay of the fertility transition lowers GDP per worker relative to countries that have taken off. The extent of the shortfall in GDP per worker depends on trend growth, g, which we expect is about two percent per year.

Before we turn to the empirical analysis it is worth considering what the prospects are for identification by way of ordinary least squares (OLS). In order for OLS to deliver unbiased estimates the right hand side variables need to be orthogonal to the residual in equation (6), which structurally should contain information about the *level* of technology at the time of take-off.

As discussed in Mankiw et al. (1992), the independence between s, n and the level of A is plausible. The fact that the potential covariance involves the level of technology lagged by up to a century would not appear to make the case any worse. Based on unified growth theory it is also plausible to assume that the onset of the fertility transition is largely orthogonal to the (pre take-off) *level* of A; rather, it is the (expected) growth rate that might trigger the fertility decline (Galor and Weil, 2000). In these respects the key orthogonality conditions are plausible.

[12] Comparing equation (6) with equation (5) it follows immediately that, structurally, $\beta_0 = A_\tau + g \cdot t$, and that ϵ_i therefore captures unobserved country variation in A_τ.

[13] Alternatively, one could assume that g varies across countries. For example, suppose individual countries are visualized as gradually converging towards their *technological* steady state, as in the model by Nelson and Phelps (1966). Then g would exhibit country specific variation in transition; in the long run, however, all countries would converge to the frontier rate. In this case the interpretation of β_3 changes from the unique "frontier rate" to the *average* growth rate in the country sample (see e.g., Zellner, 1969). Again one would think that 2 % might be a reasonable prior as the global distribution of income seems to have remained fairly stable during the second half of the 20th century, which suggests that the frontier and average expanded at roughly similar rates (Acemoglu and Ventura, 2003).

Nevertheless, identification is obviously not ensured. In theory, the (unobservable) A_τ impacts on GDP per worker. Moreover, A_τ might be determined by factors that are correlated with "contemporary" savings, labor force growth as well as τ_i. If so, identification is jeopardized. In Section 3.3 we try to gauge the likely severity of this concern by controlling as best as we can for plausible correlates with A_τ (such as early levels of schooling) as well as fundamental determinants of A_τ: geography, culture and institutions (Acemoglu, 2009, Ch. 4).

3. Regression Analysis

3.1. **Data.** We estimate the history augmented Solow model in two samples, following the approach of Mankiw et al. (1992). The first sample only excludes oil producing nations, whereas the second sample excludes both oil producers and "small nations" with less than two million inhabitants. Mankiw et al. defend these sample restrictions by arguing that labor productivity likely is poorly understood by the mechanisms emphasized in the Solow model in both oil nations (where resource extraction dominates) and small nations (where "idiosyncratic factors" might dominate).[14]

The Appendix provides details on our data sources, and Table 1 reports summary statistics for our main variables. Investment rates and growth in the labor force are calculated as averages for the period 1960-2000, as explained above.

In measuring the year of the take-off, τ_i, we employ three variables. Our main variable records the year of the fertility decline (YFD), and is taken from Reher (2004) who determines the "date" of the decline as (p. 21): "...the first quinquennium after a peak, where fertility declines by at least 8% over two quinquennia and never increases again to levels approximating the original take-off point." The fertility measure underlying Reher's analysis is the crude birth rate, which is available for extended periods of time in most countries around the world. A minor issue with the resulting data is that Reher codes countries that, as of the time of his research, had *not* undergone the transition as having experienced it in "2000". Obviously, including these countries in our sample would introduce considerable measurement error, for which reason we limit the sample to countries that had undergone the transition before 2000.[15] With the YFD indicator we are able to test the

[14] We follow Mankiw et al. in the interest of comparability with earlier studies; nothing substantive changes if we use a "full" sample of countries that only is limited by data availability. The difference in sample size between an unrestricted sample, and our non-oil sample, is only eight observations. Specifically, the countries that drop out in the non-oil sample are Bahrain, Benin, Brunei, Iran, Kuwait, Oman, Qatar and Saudi Arabia.

[15] By focusing on countries in which the take-off has (in theory) occurred, we also limit the risk of misspecification error, as it implies that equation (6) can legitimately be said to characterize all countries in our sample as of the year 2000, where GDP per worker is measured. Before the take-off a different equation would likely be appropriate for GDP per worker, see Ashraf and Galor (2011), Dalgaard and Strulik (2013).

model on a sample involving up to 110 countries. In light of the discussion above, we expect to see YFD enter the regression with a negative sign, and with a coefficient in the vicinity of 0.02.

Table 1: Summary Statistics of Main Variables

variable	obs	mean	stdev	min	max
y	110	18136	18373	1052	67078
s	110	0.15	0.08	0.02	0.45
n	110	0.02	0.01	-0.01	0.05
YFD	110	1961	29	1865	1995
YFD2	91	1955	36	1800	1993
YIT	56	1956	37	1801	1998

The table reports summary statistics for GDP per worker in 2000 (y), the average investment rate 1960-2000 (s), the average rate of labor force growth (n), the year of the fertility transition dated by Reher (2004) (YFD) and Caldwell and Caldwell (2001) (YFD2) as well as the year of the industrial transition (YIT) from Bentzen et al. (2013).

Our second measure of the fertility transition, which we employ in our robustness checks, is taken from Caldwell and Caldwell (2001). These data speak to the year of the fertility transition, judged from total fertility rates rather than the crude birth rate, which could make a difference. This variable is labeled YFD2, and it allows for a test of the model on a sample of up to 91 countries.

Finally, our third measure of the year of the take-off captures structural change: "The year of the industrial transition" (YIT). This variable records the year in which employment in industry exceeds employment in agriculture, and is taken from the work of Bentzen et al. (2013). As with the year of the fertility decline from Reher (2004) we ignore countries that experienced the transition after 2000. With this sample limitation we can estimate the model on a sample involving up to 56 countries.

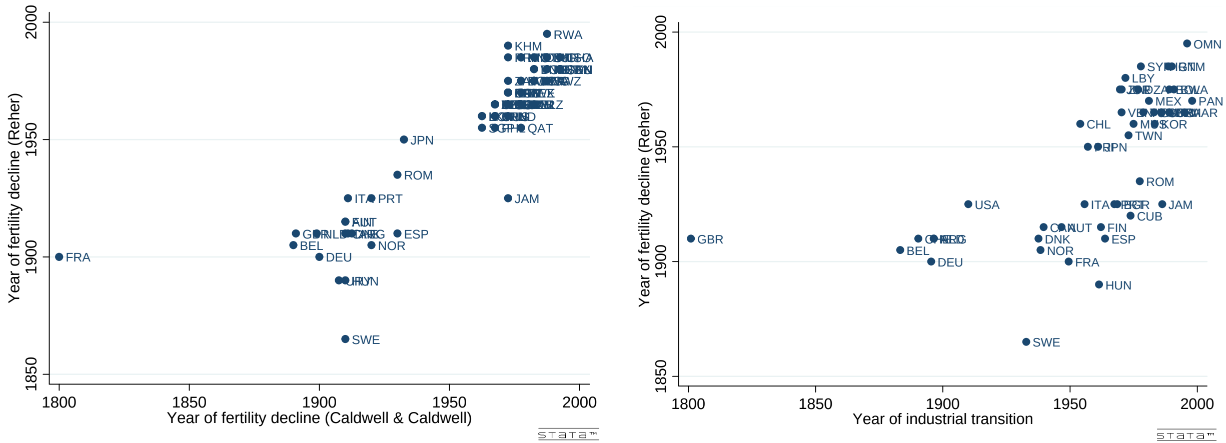
Descriptively it is interesting to note that the three measures of the "take-off" have fairly similar characteristics (albeit they are available for different samples of countries): On average the transition occurs around the same time (late 1950s), exhibits fairly similar cross-country variation (i.e., a standard deviation of roughly 35 years) and is observed within a broadly similar time frame (19th century onwards). Figure 2 provides a visual sense of the overall concordance between the three indicators.

3.2. **Baseline results.** Table 2 reports our baseline results. The first six columns refer to the "non oil" sample, whereas the last six columns concern the more limited sample where we also exclude countries with less than 2 million inhabitants.

Figure 2: Correlation Between Alternative Indicators for Year of take-off

A: Fertility Decline: Reher vs. Caldwell and Caldwell

B: Fertility Decline vs. Industrialization



The figure depicts the correlation between year of the fertility decline according to Reher (2004) and Caldwell and Caldwell (2001), respectively. The pure correlation is 0.88; 0.93 if France is omitted.

The figure depicts the correlation between year of the fertility decline (Reher version) vs. year of industrial transition (Bentzen et al, 2013). The pure correlation is 0.65; 0.72 if England is omitted.

In column 1 and 7 we test the unrestricted baseline Solow model in the "full sample". The sign of the included variables (average investment rate, s, and average growth in the labor force, n) are as expected, and the basic specification accounts for some 40% of the cross-country variation in GDP per worker in 2000. Moreover, the structural prediction of the coefficients being the same in absolute value cannot be rejected on neither the full sample nor the more limited sample (see bottom of column 1 and 7, respectively). Hence, in the remaining we employ the restricted model where the coefficients are required to be the same in absolute value.

The restricted model is estimated in column 2 and 8, from which we can elicit information about the size of the structural parameter, α. Theoretically α should reflect the capital share in national accounts and thus fall somewhere in an interval from 1/3 and 0.4. However, as is familiar from the Mankiw et al. (1992) study, the parameter is estimated to be much higher at about 0.6.

In columns 3 and 9 we include the year of the fertility transition; in column 4 and 10 we further include regional fixed effects (without substantive implications beyond a slightly higher R^2), and in columns 5-6 and 11-12 we limit attention to countries that went through the fertility transition before 1970. There are two reasons why the latter sample restriction is of interest. First, our "Solow controls" are measured as averages from 1960–2000, and are thought to proxy for post transition developments. By restricting the sample to "early transition countries" this approximation should improve. Second, when we derived the equation for estimation we assumed countries are reasonably

Table 2: The History Augmented Solow Model

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	log GDP per worker, 2000											
$\log(s)$	1.215***						1.239***					
	(0.148)						(0.144)					
$\log(n+d+g)$	-1.707***						-1.701***					
	(0.435)						(0.443)					
$\log(s) - \log(n+d+g)$		1.255***	0.595***	0.594***	0.564***	0.662***		1.278***	0.583***	0.523***	0.662***	0.729***
		(0.135)	(0.162)	(0.153)	(0.198)	(0.207)		(0.133)	(0.157)	(0.141)	(0.190)	(0.200)
YFD			-0.0222***	-0.0186***	-0.0111***	-0.0128***			-0.0227***	-0.0177***	-0.0119***	-0.0128***
			(0.0034)	(0.0038)	(0.0030)	(0.0037)			(0.0033)	(0.0035)	(0.0031)	(0.0038)
regional FE	no	no	no	yes	no	yes	no	no	no	yes	no	yes
$\beta_1 + \beta_2 = 0$ (p-value)	0.31						0.34					
implied α		0.56	0.37	0.37	0.36	0.40		0.56	0.37	0.34	0.40	0.42
$\alpha = 1/3$ (p-value)		0.00	0.53	0.51	0.73	0.38		0.00	0.58	0.87	0.34	0.19
observations	110	110	110	110	58	58	99	99	99	99	52	52
R^2	0.448	0.445	0.634	0.735	0.462	0.589	0.481	0.478	0.681	0.792	0.546	0.643

i) $\log(s)$ refers to the 1960-2000 average investment rate, $\log(n + delta + g)$ is the sum of 1960-2000 labor force growth and 0.05, YFD is the year of the fertility decline, all models contain a constant term; six regional fixed effects. (ii) Robust standard errors in parentheses; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. (iii) Cols. 1-4 represents the "full" sample where only oil countries are omitted; cols. 5 and 6 further removes non-oil countries with YFD in 1970 or later; cols. 7-10 considers non-oil and non-small countries, and cols. 11-12 further drop non-small and non-oil countries with YFD in 1970 or later.

close to their steady state path by 2000, which seems more plausible in the case of early transition countries. As is clear from Table 2, however, we obtain very similar results in either of the three samples; i.e., whether we examine non-oil countries, non-oil and non-small and non-oil (and non-small) that went through the fertility transition before 1970.[16]

Several results from columns 3-6 and 9-12 are worth noting. First, the year of the fertility transition is highly significant in statistical terms and contributes significantly to the overall R^2, which increases to between 0.6 and 0.7 (column 3, 9). If regional fixed effect are added, the model can account for up to 80% of the variation in GDP per worker in 2000 (column 10).

Second, the parameter estimate is reasonable on *a priori* grounds; we obtain estimates in the vicinity of -0.02. Hence, each year the transition is delayed comes (on average) at a cost of roughly 2% lower GDP per worker in 2000. While this number matches expectations on structural grounds it is worth spelling out its implications. From Table 1 we know that there is a lot of variation in the timing of the fertility decline; the mean is 1961, but the standard deviation is roughly a generation (i.e., about 30 years). If each year "costs" 2 percent lost output per worker, a one standard deviation's worth of delay lowers GDP per worker in 2000 by nearly 50%.

The third noteworthy result is that the "anomaly" detected originally by Mankiw et al. (1992), and that is re-confirmed in column 2 and 8, is eliminated: the estimated share of capital is around 1/3 (see bottom of column 3-6 and 9-12 for a formal test). It is worth pausing to reflect on why this result emerges. Naturally, the mechanical explanation is that the year of the fertility decline is negatively correlated with savings (at -0.45 in the non-oil sample), and positively correlated with the growth rate of the labor force (at 0.63 in the non-oil sample). As a result, once the fertility decline is included in the regression, the parameter estimate for the savings rate and for the term involving n will shrink in absolute value. But what is a plausible economic interpretation?

The positive correlation between the year of the fertility decline and observed growth in the labor force 1960–2000 is relatively easy to rationalize. In the aftermath of the fertility decline, growth in the labor force gradually declines. Ceteris paribus, one would therefore expect to see faster growth in the labor force 1960-2000 in countries for which the transition occurred relatively recently.[17]

[16] More formally, on statistical grounds one cannot reject the hypothesis of equality between the point estimates for YFD in columns 4 and 6, and columns 10 and 12, respectively.

[17] Moreover, late transition countries tend to attain higher peak-fertility rates compared with early transition countries (Reher, 2004; Strulik, 2008), which will serve to reinforce the positive correlation between YFD and average (post-transition) labor force growth.

The observed negative correlation between the year of the take-off and the average savings rate might also have a relatively straightforward explanation. It has long been known that the average propensity to save tends to increase in the process of industrialization. Indeed, Lewis (1954, p. 155) viewed this fact as the central problem of development economics to explain. In more recent work Laitner (2000) provides a theory, according to which gradual structural change during the take-off shifts savings from largely consisting of capital gains on land holdings to savings in the form of physical capital; during development (NIPA) savings therefore increase. Since the fertility transition is the critical watershed beyond which the growth process takes off, according to unified growth theory, one would expect the savings rate to be higher in places farther away from the transition (i.e., where the YFD is a "low" number), ceteris paribus.[18] This mechanism can therefore account for a negative correlation between the year of the fertility decline, and the average savings rate 1960-2000.[19] Taken together, the year of the fertility transition should reduce (in absolute value) both the coefficient on $\log(s)$ and $\log(n + \delta + g)$, leading to a lower estimate for α.

The baseline results leave some clue as to the relative importance of the timing of the take-off and the forces emphasized by neoclassical growth theory (cf. the discussion in Section 2), in accounting for cross-country inequality in labor productivity. In order to compare the estimate for YFD and the "Solow term" on an equal footing one can calculate the standardized regressions coefficients, which then reflect the impact from the two controls on GDP per worker in 2000, if either variable is changed by one standard deviation. Accordingly, conditional on regional fixed effects the standardized regression coefficient associated with the "neoclassical term" is 0.31, whereas it is 0.46 for YFD in the non-oil sample; the corresponding numbers for the sample that excludes small countries as well are 0.28 and 0.45. Taken at face value these findings suggests that the forces emphasized by unified growth theory are somewhat more powerful than those traditionally emphasized by neoclassical growth theory in explaining GDP per worker in 2000.

3.3. **The History Augmented Solow Model: Robustness of Baseline Results.**

3.3.1. *Alternative Measures of the "Take-off".* Table 3 explores whether our results are sensitive to the choice of indicator for the "year of the take-off". The first check consists in employing an

[18] In the theory developed by Strulik and Weisdorf (2008) it is demonstrated that the fertility decline is associated with rapid growth in relative employment in industry during the take-of.

[19] Complementary explanations include the lowered relative importance of subsistence consumption in the aftermath of the take-off (Strulik, 2010) and that the development process instigates rising patience, which elevates savings (Strulik, 2012).

alternative indicator for the year of the onset of the fertility decline due to Caldwell and Caldwell (2001) (C&C). As seen from Figure 2 there are instances where the two measures give rise to widely different transition years. The extreme example is France for which C&C suggest the year 1800, whereas Reher estimates the onset of the decline to take place around 1900. But overall the difference between the two measures is actually not all that large: On average the fertility transition occurs only 3.5 years (median: 2.5 years) later according to C&C's estimates compared to those from Reher.

In this light it is perhaps unsurprising that our baseline results are robust to the change in the indicator for the year of the fertility decline (see Columns 1-4 of Table 3). The only change is that the implied estimate for α edges slightly higher, and in some cases rises significantly above 1/3. Still, it is never significantly different from 0.4 (see bottom of Table 3 for formal tests).

The second check consists of using an altogether different indicator for the year of the take-off, namely the "year of the industrial transition" (YIT). In most respects the results remain unaffected. The year of the take-off is highly significant (along with the "Solow controls"), and we also obtain similar results with respect to α, which is estimated around 0.4. However, the point estimate for the take-off declines, when the YIT indicator is used.

A possible explanation is that the regressions involve considerably fewer observations when YIT is used, compared with those involving YFD ad YFD2. Moreover, most of the countries that drop out are found among the currently poorest countries in the world thus limiting the variation in the data appreciably. To check this account we re-ran the regressions using YIT and YFD2 on the same sample.[20] Conditional on regional fixed effects and the "Solow term", we find -0.0059 for YIT (p-value of 0.007), which is insignificantly different from the result in Table 3 column 6, and -0.0086 (p-value of 0.047) for YFD2, which *is* significantly different (at a 10% level of significance) from the result for YFD2 reported in Table 3, column 2. This suggests that the difference in results, when moving from YFD2 to YIT, probably is driven by the change in sample.

The overall impression is that our baseline results are reasonably robust to the use of alternative indicators. In particular, the amount of the variation explained by the history augmented Solow model, the strong statistical significance of the year of the take-off, and the lowered estimate for the capital share all carry over with little or no qualifications to the use of YFD2 or YIT.

[20] We use YFD2 because it allows for the maximum common sample, N= 45. Naturally, this involves the "non-oil" sample.

Table 3: The History Augmented Solow Model: Alternative Indicators for Take-off

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	log GDP per worker, 2000							
$\log(s) - \log(n + d + g)$	0.721***	0.728***	0.691***	0.663***	0.742***	0.693***	0.751***	0.682***
	(0.180)	(0.152)	(0.187)	(0.163)	(0.131)	(0.156)	(0.134)	(0.169)
YFD2	-0.0121***	-0.0119***	-0.0134***	-0.0129***				
	(0.0026)	(0.0031)	(0.0030)	(0.0039)				
YIT					-0.0068***	-0.0055***	-0.0069***	-0.0056***
					(0.0017)	(0.0016)	(0.0017)	(0.0016)
regional fe	No	Yes	No	Yes	No	Yes	No	Yes
implied α	0.42	0.42	0.41	0.40	0.43	0.41	0.43	0.41
$\alpha = 1/3$ (p-value)	0.16	0.08	0.25	0.27	0.03	0.16	0.03	0.23
$\alpha = 0.4$ (p-value)	0.75	0.67	0.89	0.98	0.55	0.86	0.51	0.93
observations	91	91	81	81	56	56	50	50
R^2	0.615	0.684	0.650	0.745	0.617	0.696	0.633	0.713

(i) All regressions contain a constant. (ii) Robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$. (iii) YFD2 is the year of the fertility transition according to Caldwell and Caldwell (2001), whereas YIT is the year of the industrial transition from Bentzen et al. (2013). (iv) Cols. 1-2, 5-6 estimate the model in the non-oil sample, cols. 3-4, 7-8 also exclude small countries.

3.3.2. *Confounders: Additional Controls for A_τ.* A lingering concern is that omitted variables might be affecting the significance of the year of the fertility transition in the regressions above. In theory our regression model involves the level of technological sophistication at the time of the take-off, A_τ, which we cannot measure directly. Some of this unobserved variation is likely picked up by our regional fixed effects, but this naturally leaves out unobserved variation in A_τ within regions.

In brief the concern is then the following. If we denote by $\tilde{A}_\tau$ unobserved variation (within regions) in the pre-take off level of technology our OLS estimates for our parameter of main interest, $\hat{\beta}_1$ can be written as

$$\hat{\beta}_1 = \beta_1 + \left(\frac{cov(YFD, \tilde{A}_\tau)}{var(YFD)} \right). \tag{7}$$

As discussed in Section 2, the key orthogonality condition, $\left(\frac{cov(YFD, \tilde{A}_\tau)}{var(YFD)} \right) = 0$, is theoretically plausible: in Galor and Weil (2000) the onset of the fertility transition is unleashed when the rate of technological change attains a critical level; it is not triggered by a sufficiently high *level* of A. Similar arguments can be made regarding covariances pertaining to $\log(s)$ and $\log(n + \delta + g)$ (see Mankiw et al (1992), or Section 2 above). Nevertheless, at this stage the orthogonality conditions amounts to an *assumption*, which seems worth testing to the greatest extent possible.

Since we cannot measure A_τ directly we have to take an indirect route, which involves a two pronged strategy. First, we control for outcomes that might be correlated with A_τ: primary school enrollment rates in 1900, and GDP per capita in 1900. The motivation for the former control is simple: If education is important for technological change (either via adoption or innovation), early levels of education might be a marker for the initial level of technology.[21] The main problem with this control is that unified growth theory predicts that the fertility transition unleashes a process of human capital accumulation (the quantity-quality trade-off is triggered), which stimulates growth and technological change. If we control for post-take off human capital then the year of the fertility transition *should* turn insignificant, which makes the proposed robustness check hard to interpret. We return to this issue in the context of the results below, and at greater length in Section 3.4. The

[21] In early states of development primary schooling might be highly important for technology diffusion along the intensive margin. That is, technology usage lags might be shorter (Comin et al, 2008) in societies where the citizen's have at least basic education. In addition, primary school enrolment rates might well be highly correlated with secondary and tertiary enrolment rates. Observe also that Murtin (2012) documents a clear link between fertility and schooling over the period 1870-2000; in accordance with unified growth theory, Murtin finds that changes in schooling are negatively correlated with changes in fertility during this period.

motivation for the use of GDP per capita in 1900 is that technology enters the production function for which reason GDP per capita in 1900 could act as a stand-in for early technology.[22]

The second strategy is to control for fundamental determinants of productivity: institutions, culture and geography (Acemoglu, 2009, Ch. 4). If these factors influence productivity, they might well have helped shape A_τ making them relevant indirect controls for A_τ. Moreover, they might also have worked to influence the timing of the fertility transition, as argued in several recent papers (Andersen et al, 2010; Basso and Cuberes, 2011), for which reason these checks are non-trivial on a priori grounds.

In controlling for institutions we employ direct measurement of executive constraints in 1900, as well as possible historical controls for the development of institutions: population density in 1500, fraction of population speaking a European language as well as a full set of legal origins dummies (Acemoglu et al., 2001, 2002; Hall and Jones, 1999; Glaeser and Shleifer, 2002).[23] In controlling for "culture" we deploy three sets of determinants: religious affiliations (see e.g. Barro and McCleary, 2003), ethno-linguistic fractionalization (e.g., Devleeschauwer et al, 2003; Michalopoulos, 2012), and finally the genetic distance between country citizens and the population in the UK anno 1500. The latter control is inspired by Spolaore and Wacziag (2009) and might be interpreted as capturing cultural distance between individual countries and the population that saw the first industrial revolution. Finally, our geography controls are distance to the equator (absolute latitude), tropical area and distance to coast or river, in addition to a full set of regional fixed effects (e.g., Hall and Jones, 1999; Gallup and Sachs, 2000; Rappaport and Sachs, 2003). Data sources are given in the Appendix.

Table 4 reports our results. In column 1 and 3 we check for the impact on the point estimate of interest from including early education and income alongside the "standard" controls. As seen, this does seem to leave an imprint in the point estimate for YFD, which could be a sign of omitted variable bias. Another possibility, however, is that enrollment and GDP per capita in 1900 represent an outcome from the fertility transition, as discussed above.

[22] But it might not; in a Malthusian steady state environment A and GDP per capita are uncorrelated (e.g., Ashraf and Galor, 2011; Dalgaard and Strulik, 2013).

[23] Why not use measures of contemporary institutions? The reason is that institutional quality likely is determined by human capital (at least in part), which in theory is determined by the timing of the fertility decline as noted above (see for instance, Glaeser and Saks (2006) who document that human capital accumulation has worked to lower corruption in the US and, more generally, Glaeser et al. (2004)). Consequently, adding measures of *contemporary* institutions would not allow for a discriminatory test since these (in part) are an outcome from a process which may have begun with the fertility transition.

Table 4: The History Augmented Solow model and Potential Confounders

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	log GDP per worker, 2000									
$\log(s) - \log(n+d+g)$	1.106**	0.835	0.497**	0.477**	0.354**	0.537***	0.485***	0.550***	0.580***	0.525***
	(0.420)	(0.488)	(0.207)	(0.210)	(0.162)	(0.173)	(0.155)	(0.159)	(0.156)	(0.133)
YFD	-0.007*	-0.016**	-0.007**	-0.011*	-0.011***	-0.011**	-0.015***	-0.0169***	-0.0189***	-0.0162***
	(0.004)	(0.006)	(0.003)	(0.006)	(0.003)	(0.005)	(0.003)	(0.004)	(0.004)	(0.004)
log GDP per capita 1900	-0.059	-0.238								
	(0.173)	(0.212)								
log prim enr. rate 1900			0.054	0.030						
			(0.114)	(0.110)						
Executive constraints (1900)					0.140***					
					(0.0349)					
log Population density 1500						-0.053				
						(0.068)				
Fraction speaking european language						0.461**				
						(0.184)				
Legal origin: British							0.192			
							(0.245)			
Legal origin: French							0.133			
							(0.201)			
Legal origin: Socialist							-0.574**			
							(0.243)			
Legal origin: German							0.589*			
							(0.307)			
Fraction Catholic (1980)								0.004		
								(0.003)		
Fraction Muslim (1980)								-3.84e-05		
								(0.003)		
Fraction Protestant (1980)								0.005		
								(0.004)		
Ethnic linguistic fractional (1985)								-0.001		
								(0.003)		
Genetic distance to UK, 1500									0.080	
									(0.245)	
Absolute latitude										-0.001
										(0.011)
Tropical area										-0.536*
										(0.313)
Distance to coast or river										-0.305*
										(0.164)
Regional FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
p-value (added controls, joint)	.	.	.	.	.	0.04	0.00	0.39	.	0.11
observations	30	28	58	55	43	94	109	107	110	104
R^2	0.777	0.794	0.739	0.751	0.897	0.768	0.768	0.739	0.735	0.772

(i) Robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. (ii) All models contain a constant term. (iii) The reported p-value refers to the joint exclusion of added controls (if more than one). (iv) In cols. 2 and 4 countries with transition before 1900 are excluded. (iv) See Data appendix for sources of controls.

A simple way to check which of the two possibilities is the viable one consists of focusing on countries in which the fertility transition had yet to occur as of 1900. As it turns out, in the sample underlying column 1 only two countries experienced the transition before 1900, implying GDP per capita in 1900 might have been affected by the fertility transition: Sweden and Uruguay. In Column 2 we therefore omit these two countries, after which the point estimate rises (in absolute value) to a level reasonably comparable to that reported in Table 2 (see Table 4, column 2). Similarly, in the (slightly larger) sample involving primary school enrolment rates, the list of pre-1900 take-off countries is Hungary, Sweden and Uruguay. Column 4 omits them, which also produces a numerically greater point estimate for YFD. In general, these results seem to provide some credence to the orthogonality condition discussed above.[24]

The remaining columns of Table 4 report the results from the "fundamental determinants" check. In some cases the added controls appear significant (individually or collectively), yet they seem to have little influence on the coefficient for YFD. This suggests that while (say) early institutions might have impacted on the level of technological sophistication, the orthogonality between the level of technology and the timing of the transition ensures consistent OLS estimates.[25] In other cases the controls turn out insignificant. That should not be taken as evidence of "lack of importance". Rather, it suggests that the variables in question likely have impacted on contemporary economic development through one or more proximate sources of income: the year of the take-off, or post take-off investment and fertility.[26]

3.4. **The Fertility Transition and Current Labor Productivity: Exploring the Channels of Influence.** In the regressions above we did not control for *current* human capital. The reason is that human capital accumulation, in theory, is the key mechanism by which the fertility decline influences contemporary income. If true, it follows that once contemporary human capital levels are included in the regression, the impact from the fertility decline should evaporate.

To see the connection more clearly, suppose the evolution of average years of schooling in the labor force is described by the following linear first order differential equation (Aghion and Howitt, 2008,

[24] If we employ YFD2, the list of pre-1900 transition countries grows: Belgium, France, United Kingdom, Iceland, Netherlands, New Zealand and Ireland. Nevertheless, all results in Table 4 carry over to this alternative indicator. The point estimate for YFD2 is around -0.01 in all specifications, with all countries included. If we then drop the pre-demographic transition countries we once again see an increase in economic significance for YFD2. In the regression involving GDP per capita 1900 we obtain -0.012,$t = 2.32$; in the human capital setting the point estimate is -.013, $t = 2.46$. These results are available upon request. We have also explored an alternative data source for human capital (average years of schooling), which recently has been compiled by Morrison and Murtin (2009). The basic results are very similar. A difference is, however, that when we use the YFD indicator both human capital and YFD turn insignificant; this is true whether we omit countries that went through the fertility decline before 1900 or not. When we use YFD2 as take-off indicator along side Morrison and Murtin (2009)'s data for human capital, only YFD2 remains significant. This is true whether or not we exclude countries that observed a fertility decline before 1900, albeit the economic significance of the transition year is increased if we exclude countries that experienced the fertility transition in the 19th century.

[25] The significance of institutions in the regression is consistent with the analysis in Eicher et al. (2006), which establishes empirically a causal impact of institutions on productivity, conditional on the Mankiw et al. set of controls, that is, institutions seem to raise productivity chiefly via A.

[26] A case in point would be genetic distance to the UK. Basso and Cuberes (2011) document that a greater distance to the US leads to a delayed fertility decline, which (as shown above) leads to lower GDP per worker today.

Ch. 13):

$$\dot{s}(t) = u - \mu s(t)$$

where $\dot{s}$ is change in years of schooling, u is the fraction of time spend per year accumulating skills, and $\mu s(t)$ is the loss of schooling years originating from people exiting the labor force. For simplicity u and μ are treated as constants. The solution to this differential equation, assuming $s(\tau) = 0$, is

$$s(t) = \frac{u}{\mu}\left[1 - e^{-\mu(t-\tau)}\right].$$

Accordingly, the level of schooling at some point in time t (say, the year 2000) depends on how much time is invested on net per period (u/μ), but also on the calendar date at which investments commenced, τ.

The influence from the latter is intuitive: even at a 100% enrollment rate, average years of schooling will only gradually rise when well educated cohorts replace less well educated ones. For investments given, average years of schooling must be – in part – determined by the time at which such investments started being undertaken. Now, according to the bulk of the literature on growth in the long run, the year of the fertility decline should be a good marker for the time at which human capital investments start being made in earnest; a good proxy for τ in other words (e.g., Galor and Weil, 2000; Lucas, 2002). As a result, the fertility transition should matter to current labor productivity *via* its impact on current human capital.

Of course, the influence from human capital on growth may involve multiple channels. The influence could be direct, as in Lucas (1988). Alternatively, or in addition, it could influence growth via faster technology adoption, as in e.g. Nelson and Phelps (1966). Moreover, one may hypothesize that a higher level of schooling in the population at large gradually works to improve institutional quality (e.g., Glaeser et al, 2004). Hence, for any of these reasons one would expect that current human capital levels should appear as a determinant of contemporary labor productivity. If the fertility transition is influencing productivity via human capital, its significance should diminish once current human capital is controlled for in the regression.[27]

In order to assess whether this is the case or not, we invoke three measures of outcomes from (past and present) "quality investments": Average years of schooling in the population (+15) in

[27] As noted in the introduction, the fertility decline also influences post-transition growth via reduced capital dilution; with (gradually) slower population growth the capital-labor ratio is increased. Notice, however, that the regressions above already controls for this channel, as the specification includes a control for the rate of labor force growth – post transition, proxied by labor force growth 1960-2000. The significance of the timing of the fertility transition above cannot therefore be attributed to reduced capital dilution, leaving only the human capital channel.

2000; the average IQ score in the population, and life expectancy at birth in 2000. The results from re-estimating our baseline model – human capital augmented – are reported in Table 5.

All three "child quality variables" are highly significant correlates with GDP per worker, as expected. However, whereas average years of schooling and IQ scores eliminate the influence from the fertility transition (column 2 and 4) the same is not true for life expectancy (column 1). As can be seen from column 3 and 5, years of schooling and IQ render the fertility transition insignificant mainly due to a marked reduction in the point estimate on the latter; the coefficient associated with the fertility decline is reduced by 2/3 or more when either the schooling variable or IQ is introduced. These results are consistent with the main prediction of unified growth theory: the differential timing of the fertility transition is a strong determinant of current income differences due to its influence on the path of human capital accumulation, which in turns stimulates growth in multiple ways.

At the same time, some of the results in Columns 2-4 appear problematic. Specifically, as seen from Column 2, the estimated return to schooling is large: about 20%. This return estimate is much too large to be attributed to the direct impact of schooling on individual-level productivity; in practise, micro estimates are typically "only" about 10% (e.g., Caselli, 2005). An interpretation of this result is that the estimates from Table 5 convolute the direct effect of human capital on productivity (which is what micro studies estimate) and the *indirect* effects, via innovation. Such indirect effects may either result from the production of new ideas, as emphasized in the R&D growth literature (e.g., Romer, 1990; Aghion and Howitt, 1992), or be the consequence of more rapid technology adoption (e.g., Nelson and Phelps, 1966; Benhabib and Spiegel, 2005).

In order to check the viability of this explanation for the elevated return estimate, we introduce the R&D investment rate. If indeed the large return on skills (recovered above) is attributable to the *indirect* effects of human capital on technology innovation or adoption, we expect to see it decline if the R&D investment rate is introduced, as the latter should pick up the innovation channel.

As seen from column 6, when R&D investment (as a fraction of GDP) is added to the model the estimated return on schooling shrinks to about 10%. To isolate the pure effect of introducing R&D investments we also provide the results from estimating the model without R&D, but in the same sample (column 7); in this setting the estimated return is considerably higher at 15%, as expected. Interestingly, however, the estimate for IQ does not change much; when the average IQ score increases by 1 it is associated with an increase in labor productivity of about 4% whether we control for R&D or not.

Table 5. Channeling the Effect of the Fertility Decline

	1	2	3	4	5	6	7	8	9
	log(y)	log(y)	log(y)	log(y)	log(y)	log(y)	log(y)	log(y)	log(y)
$\log(s) - \log(n+d+g)$	0.464***	0.3	0.573***	0.08	0.471*	0.642***	0.800***	0.31	0.596**
	(0.14)	(0.19)	(0.19)	(0.28)	(0.28)	(0.21)	(0.21)	(0.26)	(0.22)
YFD	-0.0123***	-0.0052	-0.0149***	-0.0022	-0.00927**				
	0.0038	0.0042	0.0045	0.0036	0.0045				
life expectancy (at birth, 2000)	0.0549***								
	(0.01)								
avg. years of schooling (above 15, 2000)		0.211***				0.103*	0.143***		
		(0.05)				(0.06)	(0.04)		
IQ score				0.0548***				0.0482**	0.0495***
				(0.01)				(0.02)	(0.02)
$\log(s_{R\&D})$						0.197*		0.277**	
						(0.11)		(0.12)	
regions	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
constant	30.04***	18.27**	38.98***	9.94	28.39***	8.988***	8.390***	5.657***	5.415***
	(8.09)	(8.29)	(8.85)	(7.23)	(8.73)	(0.65)	(0.38)	(1.75)	(1.55)
observations	110	80	80	49	49	54	54	39	39
R^2	0.78	0.78	0.72	0.78	0.73	0.67	0.63	0.76	0.72

(i) Robust standard errors in parentheses. (ii) ***: $p < 0.01$, **: $p < 0.05$, *: $p < 0.1$. (iii) The sample excludes oil producers. Column 3 re-estimates the model on the same sample as in Column 2 to assess the importance of the sample to the insignificance of YFD; the same exercise is conducted in Column 4 in the context of the alternative "child quality indicator": IQ. Columns 6-9 probe the consequences of adding R&D investment rates for the return on skills and IQ.

Overall, the results from Table 5 paint a coherent picture. Early transition countries managed to start accumulating human capital sooner than late take-off countries. The resulting human capital divergence instigated divergence in livings standards. Hence, while the timing of the fertility decline is a determinant of contemporary comparative development, its effect is operating through human capital and ideas; the more proximate sources of growth.

Finally, one may observe that the above results are fully consistent with the various augmentations of the Solow model that have been proposed (Mankiw et al, 1992; Ram, 2007; Knowles and Owen, 1995; Nunnemann and Vanhout, 1996). Yet it adds one important additional piece of information. Namely, that the origin of observed differences in these additional "capital stocks" (be it schooling, IQ, life expectancy or technological "ideas") is found in the differential timing of the take-off to sustained growth, which is connected to the onset of the fertility transition.

4. Concluding Remarks

The present paper has augmented the empirical Solow model by taking into account the timing of the take-off to sustained growth. According to unified growth theory, the timing of the fertility decline should be a good proxy for the timing of the take-off to growth. We have shown theoretically how one can combine key insights from unified growth theory with the neoclassical model in a simple way.

Our empirical results support the notion that contemporary income differences are highly influenced by both the year of the take-off and post take-off influences captured by the neoclassical growth model. We find that adding the onset of the fertility transition to the neoclassical growth model improves the empirical fit considerably. Taken together the onset of the fertility decline and (post take-off) investment and labor force growth can account for about 70% of the variation in GDP per worker; nearly 80% if one on top adds regional fixed effects. When we examine the standardized regression coefficients we find that the influence from the fertility transition is slightly greater than the influence from the "neoclassical determinants" of productivity, as of the year 2000.

Another key finding is that the obtained parameters square well with priors. The share of capital is estimated to be between 1/3 and 0.4, and the point estimate for the fertility decline suggests that each year the transition is delayed lowers GDP per worker by about two percent. The latter is consistent with the view that once the take-off has occurred individual countries (on average) manage to tap into the "world technology frontier", thus experiencing a rate of underlying productivity growth similar to "leader nations" like the US, conditional on physical capital investments etc.

The influence from the fertility transition is robust to the inclusion of controls for institutions, culture, geography, as well as correlates with early levels of technological sophistication, like early education. But the partial correlation between the year of the fertility decline and current GDP per worker is eliminated if we control for the channels through which the fertility transition is supposed to affect contemporary comparative development: reduced post take-off capital dilution and increased human capital accumulation.

Admittedly, our regressions do not necessarily establish causality, although the obtained point estimates square well with priors. Hence, an important topic for future work is to obtain a clearer empirical understanding of the determinants of the fertility transition. This will eventually pave the way for a study where the causal impact from the fertility transition on global cross-country inequality in labor productivity can be assessed.

Appendix A. Derivation of equation (5)

The Solow mode consists of the aggregate production function $Y = K^{\alpha}(AL)^{1-\alpha}$, the equation of motion of capital $\dot{K} = I/K - \delta K$ and the investment equation $I = sY$. Production in efficiency units is defined as $\hat{y} = Y/(AL)$ and capital stock in efficiency units is defined as $\hat{k} \equiv K/(AL)$, implying $\hat{y} = \hat{k}^{\alpha}$. Log-differentiating $\hat{k}$ we obtain

$$\frac{\dot{\hat{k}}}{\hat{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L} = \frac{I}{K} - \delta - g - n, \tag{A.1}$$

in which the last equality makes use of the equation of motion for K and the definitions of productivity growth $\dot{A}/A \equiv g$ and population growth $\dot{L}/L \equiv n$. Rewriting the investment rate we get

$$\frac{I}{K} = s\frac{Y}{K} = s\frac{\hat{y}}{\hat{k}} = s\hat{k}^{\alpha-1}.$$

Inserting this information into (A.1) we get the fundamental equation of the Solow growth model in efficiency units (A.2).

$$\frac{\dot{\hat{k}}}{\hat{k}} = s\hat{k}^{\alpha-1} - \delta - g - n. \tag{A.2}$$

At the steady-state $\dot{\hat{k}}/\hat{k} = 0$ and thus solving for $\hat{k}$

$$\hat{k}^* = \left(\frac{s}{\delta + g + n}\right)^{\frac{1}{1-\alpha}} \qquad \Rightarrow \qquad \hat{y}^* = \left(\frac{s}{\delta + g + n}\right)^{\frac{\alpha}{1-\alpha}}, \tag{A.3}$$

where a star $(*)$ denotes a steady-state value. Taking the first order Taylor approximation of (A.2) evaluated at the steady state we obtain

$$\frac{\dot{\hat{k}}}{\hat{k}} \approx s(\alpha - 1)(\hat{k}^*)^{\alpha-2} \cdot (\hat{k} - \hat{k}^*) = -\underbrace{(1-\alpha)(\delta + n + g)}_{=\lambda} \cdot \left(\frac{\hat{k} - \hat{k}^*}{\hat{k}^*}\right),$$

where the right hand side follows after inserting $\hat{k}^*$ from (A.3). The parameter λ denotes the speed of convergence. To see this clearly consider another Taylor approximation at $\hat{k}^*$:

$$\log\left(\frac{\hat{k}}{\hat{k}^*}\right) \approx \log\left(\frac{\hat{k}^*}{\hat{k}^*}\right) + \frac{1}{\hat{k}^*}(\hat{k} - \hat{k}^*) = \frac{\hat{k} - \hat{k}^*}{\hat{k}^*}$$

Use this information in

$$\frac{\partial \log(\hat{k}/\hat{k}^*)}{\partial t} = \frac{\dot{\hat{k}}}{\hat{k}} = -\lambda\left(\frac{\hat{k} - \hat{k}^*}{\hat{k}^*}\right) = -\lambda \log\left(\frac{\hat{k}}{\hat{k}^*}\right)$$

and observe that this equation constitutes a linear ordinary differential equation for the variable $\log(\hat{k}/\hat{k}^*)$. The solution is given by $\log(\hat{k}(t)/\hat{k}^*) = \log(\hat{k}(\tau)/\hat{k}^*) \cdot \mathrm{e}^{-\lambda\cdot(t-\tau)}$ that is

$$\log(\hat{k}(t)) \approx \log(\hat{k}^*) \cdot \left(1 - \mathrm{e}^{-\lambda(t-\tau)}\right) + \mathrm{e}^{-\lambda(t-\tau)}\log(\hat{k}(\tau)). \tag{A.4}$$

Recall that $\hat{y}(t) = \hat{k}(t)^{\alpha}$ and thus $\log(\hat{y}(t)) = \alpha \log(\hat{k}(t)$. Inserting $\log(\hat{k}(t))$ from (A.4) provides (A.5).

$$\log(\hat{y}(t)) \approx \log(\hat{y}^*) \cdot \left(1 - \mathrm{e}^{-\lambda(t-\tau)}\right) + \mathrm{e}^{-\lambda(t-\tau)}\log(\hat{y}(\tau)). \tag{A.5}$$

Now consider income per worker defined as $y \equiv Y/L = A\hat{y}$. Let g_y denote the average growth rate of y between time t and τ and $g_{\hat{y}}$ the respective average growth rate of $\hat{y}$. Then from log-differentiating y

$$g_y = g + g_{\hat{y}} = g + \frac{\log(\hat{y}(t)) - \log(\hat{y}(\tau))}{t - \tau}$$

Inserting $\log(\hat{y}(t))$ from (A.5) this can be written as

$$g_y \approx g + \frac{1 - \mathrm{e}^{-\lambda(t-\tau)}}{t - \tau} \left(\log(\hat{y}^*) - \log(\hat{y}(\tau))\right). \tag{A.6}$$

From this follows for sufficiently large $t - \tau$ the approximation (3) in the main text.

Finally note that (A.6) can equivalently be written as $g_y(t-\tau) = g(t-\tau) = \log\left[\hat{y}^*/\hat{y}(\tau)\right]$ that is $\mathrm{e}^{(g_y - g)(t-\tau)} = \hat{y}^*/\hat{y}(\tau)$. This information can be used to rewrite the growth equation for income per capita:

$$y(t) = y(\tau)\mathrm{e}^{g_y(t-\tau)} = A(\tau)\hat{y}(\tau)\mathrm{e}^{g_y(t-\tau)} = A(\tau)\hat{y}^*\mathrm{e}^{g(t-\tau)}.$$

Inserting $\hat{y}^*$ from (A.3) provides (4) in the main text and taking logs provides (5) in the main text.

Appendix B. Data Sources

Absolute latitude:
Source: Gallup et al. (1999).
Distance to coast of river:
Source: Gallup et al. (1999).
Executive constraints in 1900:
Source: Polity IV., `http://www.systemicpeace.org/polity/polity4.htm`.
Fraction speaking a major European language:
Source: Hall and Jones (1999).
Fraction of area in tropics:
Source: Gallup et al. (1999).
GDP per worker in 2000:
Source: Penn World Tables, 6.3. (Heston et al., 2009),
GDP per capita in 1900:
Source: Barro–Ursúa macroeconomic data (2010), downloaded from `http://rbarro.com/data-sets/`.
Genetic distance to England in 1500:
Source: Spolaore and Wacziarg (2009).
Growth in the Labor Force:
Source: Penn World Tables 6.3. (Heston et al., 2009).
Investment rates 1960-2000:
Source: Penn World Tabels 6.3 (Heston et al., 2009).
IQ score:
Source: Jones and Schneider (2006).
Legal origin dummy's:
Source: Treisman (2007).
Life expectancy at birth in 2000:

Source. World Development Indicators, available at: `http://data.worldbank.org/`.

Population density in 1500:

Source: McEvedy and Jones (1978).

Primary school enrollment rates in 1900:

Source: Benavot and Riddle (1988).

R& D expenditures as a fraction of GDP, 2000:

Source: World Development Indicators, available at `http://data.worldbank.org/`

Religion: Measures of fraction of population muslim, catholic and protestant, respectively. All measured in 1980.

Source: Treisman (2007)

Schooling in 2000 (average years):

Source: Barro and Lee (2001).

Year of fertility decline, YFD:

Source: Reher (2004).

Year of fertility decline, YFD2:

Source: Caldwell and Caldwell (2001). In cases where Caldwell and Caldwell report an interval (say, 1990–1995) we date the decline to the middle of the interval. All observations without precise indications of the decline (reported in Caldwell and Caldwell as " transition before year X") are coded as missing observation.

Year of Industrialization, YIT, which measures the first year that industry accounts for a greater proportion of total employment than agriculture.

Source: Bentzen et al. (2013).

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