GROWTH AND HUMAN CAPITAL ACCUMULATION

- The Augmented Solow model

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MOTIVATION FOR AUGMENTING THE MODEL

The Solow model leaves us with two problems in need of being fixed

1. Under plausible assumptions we *cannot* account for the magnitude of observed productivity differences

2. When we estimate the model the data "tells us" capital's share (α) is about 0.6, which is too large to be attributable to physical capital (national accounts: 1/3 - 0.4).

We begin by thinking about (2)

MOTIVATION FOR AUGMENTING THE MODEL

If we are confident that α *should* be 1/3-0.4, it follows that our OLS results somehow are biased

Recall, if we estimate $y_i = a + bx_i + \varepsilon_i$, we can write the OLS estimate for b

$$\hat{b} = b + \frac{cov\left(e_i, x_i\right)}{var\left(x_i\right)}$$

In the case of the Solow model: x is savings and population growth.

Perhaps the covariance isn't zero after all? If $cov\left(e_i, s_i\right) > 0$, $\hat{b} > b$.

Note: If we do not control for all relevant variables (are misspecifying the empirical model), these will be left in e. And they could be correlated with s and n. Ideally, something which is *positively* correlated with s, and *negatively* correlated with n.

MOTIVATION FOR AUGMENTING THE MODEL

A candidate: Skills, or *human capital*.

Human capital?Analytical skills, facts and figures (schooling). It could be more informal knowledge (learning by doing; return to that later)

From microdata we know that more schooling tends to increase wages; it seems productive. Fairly confindent in introducing it as an "input" in the production function

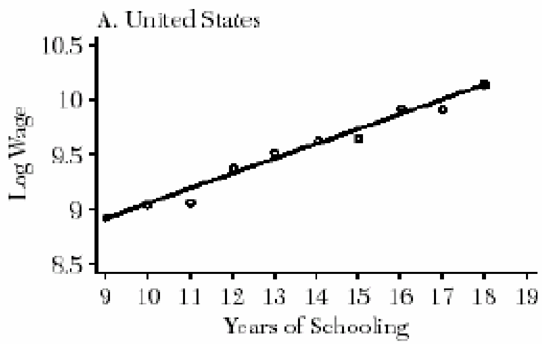

Figure 1: Taken from Krueger and Lindahl, 2001.

MOTIVATION FOR AUGMENTING THE MODEL

A couple of other useful observations:

- Schooling tends to rise when fertility declines (recall, we need $cov\,(n, hc) < 0$ to help us empirically)

- Societies with high investment rates in K tend also to invest in schooling (measured by enrollment rates, for example). This suggests $cov\,(n, hc) > 0$ could be plausible

There is a third reason why we might want to include human capital ....

MOTIVATION FOR AUGMENTING THE MODEL

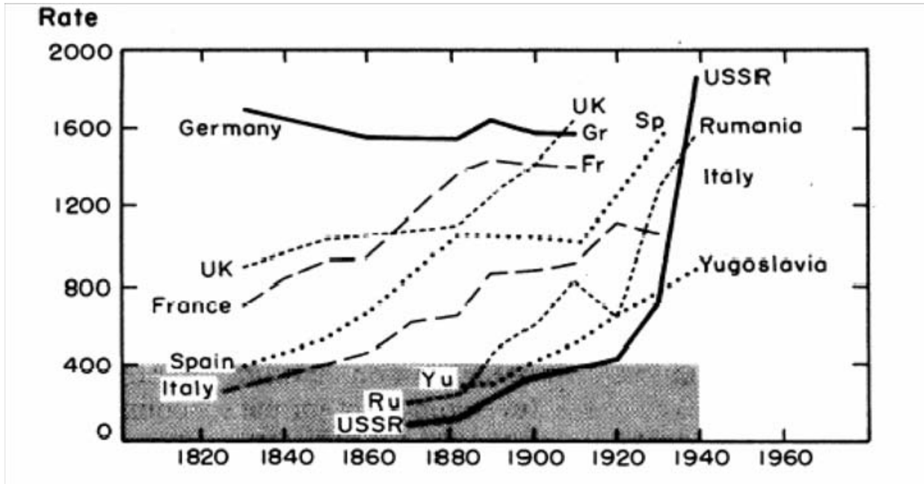

Figure 2: Source: Easterlin, 1981. Primary school enrolment per 10.000 inhabitants

Mass education is a recent phenomena (as growth is); HC came in increasing supply around the time of the take-off to sustained growth

Hence, a tentative conclusion: Perhaps the Solow model is an *incomplete* (stylized) description of the growth process? Augmenting the model by including HC seems worthwhile, and might “fix” our problems.

NEW ELEMENTS OF THE MODEL

Fundamentally we include a new input into the production function

$$Y_t = K_t^{\alpha} H_t^{\phi} L_t^{1-\alpha-\phi}$$

Note: Constant returns is *maintained*. Human capital, H, is a rival input of production.

We have a pretty good idea about the size of α. But what about ϕ? And how do we think about labor's share? National accounts still says: Wage income + Capital income = GDP

Realize: Wages compensate for L ("brawn") as well as H ("brains"). But how much "goes to each"?

NEW ELEMENTS OF THE MODEL

To come up with a reasonable guess, we proceed in a few steps

step1. Show that

$$\frac{Y_t}{L_t} = \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} h_t^{\frac{\phi}{1-\alpha}}, \; h_t = H/L$$

or the human capital per worker.

step2. With competitive markets, wages equal the marginal product of labor

$$w_t = \frac{\partial Y}{\partial L} = (1 - \alpha - \phi) \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} h_t^{\frac{\phi}{1-\alpha}}$$

step3. Parameterize ϕ, using wage data. Consider two individuals; a skilled (s) and a unskilled (u). Their human capital levels: h^s and h^u

NEW ELEMENTS OF THE MODEL

If they work with same capital equipment, their relative wage

$$\frac{w_t^u}{w_t^s} = \left(\frac{h^u}{h^s}\right)^{\frac{\phi}{1-\alpha}} \Leftrightarrow \log\left(\frac{w_t^u}{w_t^s}\right) = \frac{\phi}{1-\alpha}\log\left(\frac{h^u}{h^s}\right)$$

or (since $\frac{w^s - w^u}{w^s} \approx \log\left(\frac{w_t^u}{w_t^s}\right)$ if $\frac{w_t^u}{w_t^s}$ is small):

$$\frac{w^s - w^u}{w^s} = \frac{\phi}{1-\alpha}\left(\frac{h^s - h^u}{h^s}\right) \approx \frac{\phi}{1-\alpha},$$

for $\frac{h^u}{h^s}$ small as well.

NEW ELEMENTS OF THE MODEL

step4. Think of w^u as the minimum wage ("unskilled"), and w^s as the average wage in the economy.

Then

$$\left(1 - \frac{w^{\min}}{w^{mean}}\right)(1-\alpha) \approx \phi$$

In the US $\frac{w^{\min}}{w^{mean}}$ is roughly 0.5. Capital's share (α) is about $1/3$. It follows that

$$\phi \approx \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}.$$

NOTE: Labor's share is still $1 - \alpha = 2/3$. But this calculation suggests that *half* the wage is compensation for brains (human capital), half is remuneration for brawn ("raw labor", L). We now have a fully parameterized production function.

NEW ELEMENTS OF THE MODEL

Second new element is the law of motion for human capital. We assume

$$H_{t+1} = s_H Y_t + (1-\delta) H_t$$

where $s_H Y_t$ is *investment* in human capital. Can human capital accumulation go on forever? Bounded human capacity? Quality.

Can either be taken literately, or metaforically - a statement about production. Inserting for production

$$s_H Y_t = (s_H K_t)^{\alpha} (s_H H_t)^{\phi} (s_H L_t)^{1-\alpha-\phi}$$

Så s_H: share of inputs used to produce human capital, or, share of income used to pay for human capital (tuition etc)

What is δ? obsolete knowledge? Mortality? The same rate as capital?

SOLVING THE MODEL

Per worker growth

$$\frac{h_{t+1} - h_t}{h_t} = \frac{1}{1+n}\left[s_H \frac{y_t}{h_t} - (\delta + n)\right] \equiv \Gamma\left(k_t, h_t\right)$$

In addition we have

$$\frac{k_{t+1} - k_t}{k_t} = \frac{1}{1+n}\left[s_K \frac{y_t}{k_t} - (\delta + n)\right] \equiv \Psi\left(k_t, h_t\right)$$

as

$$y_t \equiv Y_t / L_t = k_t^{\alpha} h_t^{\phi}.$$

Definition The steady state of the model is a $k_{t+1} = k_t = k^*$ and $h_{t+1} = h_t = h^*$, such that $k^* = \Psi\left(k^*, h^*\right)$ and $h^* = \Gamma\left(k^*, h^*\right)$.

SOLVING THE MODEL

The two isoclines along which k and h are constant:

$$h_{t+1} = h_t \Rightarrow s_H \frac{k^\alpha h^\phi}{h} = \delta + n \Leftrightarrow h = \left(\frac{s_H}{n+d}\right)^{\frac{1}{1-\phi}} k^{\frac{\alpha}{1-\phi}}$$

$$k_{t+1} = k_t \Rightarrow s_K \frac{k^\alpha h^\phi}{k} = \delta + n \Leftrightarrow h = \left(\frac{\delta + n}{s_K}\right)^{\frac{1}{\phi}} k^{\frac{1-\alpha}{\phi}}$$

Note: If $\phi < 1 - \alpha$ the slope of the constant-human-capital isoline is smaller than 1, whereas the constant-physical-capital isocline is larger than 1.

If our calibration makes sense then this is the realistic scenario: $\phi = 1/3$, and capital's share $\alpha = 1/3$.

[Insert phase diagram]

STEADY STATE PROPERTIES

Unique (non-trivial) steady state

Stable

k^* and h^* determined by structural charactaristics. Steady state GDP per worker

$$y^* = \left(\frac{k^*}{y^*}\right)^{\frac{\alpha}{1-\alpha-\phi}} \left(\frac{h^*}{y^*}\right)^{\frac{\phi}{1-\alpha-\phi}}$$

Directly from $h_{t+1} = h_t$ and $k_{t+1} = k_t$ it follows

$$y^* = \left(\frac{s_K}{n+\delta}\right)^{\frac{\alpha}{1-\alpha-\phi}} \left(\frac{s_H}{n+\delta}\right)^{\frac{\phi}{1-\alpha-\phi}}$$

Hence, more investment increases k, h and y in the long-run. Now two types of investment. n lowers long-run productivity.

ACCOUNTING FOR GDP PER WORKER DIFFERENCES

Consider two countries; country A and B, where A has 4 times the investment rate in physical capital

$$\frac{y_A^*}{y_B^*} = 4^{\frac{\alpha}{1-\alpha-\phi}} = 4, \text{ if } \phi = 1/3 = \alpha$$

Why are investment in physical capital a more powerful determinant of labor productivity than in the Solow model?

Suppose next that A also has lower fertility (0.01 vs 0.03)

$$\frac{y_A^*}{y_B^*} = \left(\frac{0.01 + 0.05}{0.03 + 0.05}\right)^{-\frac{\alpha+\phi}{1-\alpha-\phi}} = 1.8, \text{ if } \phi = 1/3 = \alpha$$

So far we have motivated differences of a factor of 7. Much better than in the Solow model. We need 35 though.

ACCOUNTING FOR GDP PER WORKER DIFFERENCES

To be "home free", we need differences in hc investment of the amount

$$5 = \left(\frac{s_{H,A}}{s_{H,B}}\right)^{\frac{\phi}{1-\alpha-\phi}} \Leftrightarrow \frac{s_{H,A}}{s_{H,B}} \approx 5, \text{ if } \phi = 1/3 = \alpha$$

That is, if the richest countries invest about 5 times as big a fraction of domestic resources in human capital compared with poorer places, we can motivate GDP per worker differences of 1:35, without mentioning technology

The key reason why we get new results is that we are *increasing the factor share of accumulated factors*

In a Solow model: 1 factor which can be accumulated; share $\alpha = 1/3$. Here: 2 factors, with *combined* share $\phi + \alpha = 2/3$. *Positive* argument in favor of factor share increase.

GROWTH DIFFERENCES

Deriving the rate of convergence in this model is slightly more painful (2 difference equations). See textbook for derivations (p. 177-178)

In the end we find

$$\lambda = (1 - \alpha - \phi)\frac{n + \delta}{1 + n}$$

whereas in Solow model

$$\lambda_{\text{Solow}} = (1 - \alpha)\frac{n + \delta}{1 + n}$$

The upshot: Transitions are prolonged even further; the rate of convergence is lower with human capital

The augmentation therefore "buys us" a better ability to account for income differences, *and*, a larger scope for transitional dynamics to explain persistent growth differences.

TESTS

The steady state of the model can be expressed as follows

$$\log(y_i) = \log A + \frac{\alpha}{1-\alpha-\phi}\log(s_{Ki}) + \frac{\phi}{1-\alpha-\phi}\log(s_{Hi}) - \frac{\alpha+\phi}{1-\alpha-\phi}\log(n_i+\delta) + \epsilon_i$$

where A (a constant level of productivity) has been added to the production function. As before, $\log(A_i) = \log(A) + \epsilon_i$.

We now have to assume $cov(\epsilon_i, s_K) = 0 = cov(s_H, \epsilon) = cov(n, \epsilon)$ to estimate by OLS.

Measurement of s_H: average percentage of working aged population in secondary schooling. Proxies lost output; the alternative cost of education. The range is large: e.g. 2.5 (Zambia), 10.7 (Denmark). 1960-85 av.

TESTS

Formulated as a regression model

$$\log(y_i) = \beta_0 + \beta_1 \log(s_{Ki}) + \beta_2 \log(s_{Hi}) - \beta_3 \log(n_i + \delta) + \epsilon_i$$

with $\beta_1 = \frac{\alpha}{1-\alpha-\phi}$, $\beta_2 = \frac{\phi}{1-\alpha-\phi}$, $\beta_3 = \frac{\alpha+\phi}{1-\alpha-\phi}$.

Expectations:

(i) $\beta_1 > 0, \beta_2 > 0$ and $\beta_3 < 0$ (broad prediction)

(ii) $\alpha \approx 1/3$, $\phi \approx 1/3$ (parameter size)

(iii) $|\beta_3| = 2 \cdot \beta_1 = 2\beta_2$ (structure).

(iii) can be tested by examining the relative explanatory power of the restricted model

$$\log(y_i) = \beta_0 + \beta_1 [\log(s_{Ki}) - \log(n_i + \delta)] + \beta_2 [\log(s_{Hi}) - \log(n_i + \delta)] + \epsilon_i$$

TESTS

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\overline{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\overline{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Figure 3: Source: Mankiw et al. (1992)

TESTS

Correct signs for $\beta_1 - \beta_3$

High explanatory power: About 80% of variation can be motivated

The structure of the model is supported: $\beta_1 = \beta_2 = \frac{1}{2} \cdot |\beta_3|$

The implied parameter values conform with priors.

A remarkable success; almost too good to be true ....

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

The augmented Solow model offers an attractive framework for thinking about the issues we are concerned with

We can generate substantial GDP per worker differences with reasonable parametervalues; *without* appealing to technology

We can generate *growth* differences across countries in the long-run. Transitional dynamics + lengthy transitions. Growth "miracles" (e.g., Korea, Japan etc.) started far below steady state -> benefitted from catch-up growth

But at closer inspection "weirdness" shows up

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

A maintained assumption in the neoclassical paradigm is constant g (2 percent) and random A's (at least in expected terms)

In the Augmented Solow model we can write growth (in the vicinity of steady state - see p. 178)

$$\log(y_{t+1}) - \log(y_t) = g + \lambda\left[\log(\tilde{y}^*) - \log(\tilde{y}_t)\right] = g + \lambda\left[\log(y_t^*) - \log(y_t)\right]$$

So: Faster growth (above 2 percent) -> country is *below* steady state. Conversely, slower growth (below 2 percent) -> country is *above* its future steady state. Hence, by construction: Growth *disasters* must be *above* their steady state.

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

Some numbers are illuminating. Note that

$$\log(y_{t+1}) - \log(y_t) \equiv g^{\text{obs}} = g + \lambda \log\left(\frac{y_t^*}{y_t}\right)$$

or labor productivity initially relative to steady state:

$$e^{\frac{g^{\text{obs}}-g}{\lambda}} = \frac{y_t^*}{y_t}$$

Take Burundi. 1960-2000 growth: roughly 0 percent per year. Thus

$$e^1 \approx 2.7.$$

with $\lambda = 0.02$ also (see textbook). y_{1960} in Burundi= 1190 US\$. Steady state y_{1960}^* is c. 400 US\$... conventional: Subsistence minimum is 1-2 PPP dollar per day; about 548 PPP US\$ per year. Makes you a little suspicious

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

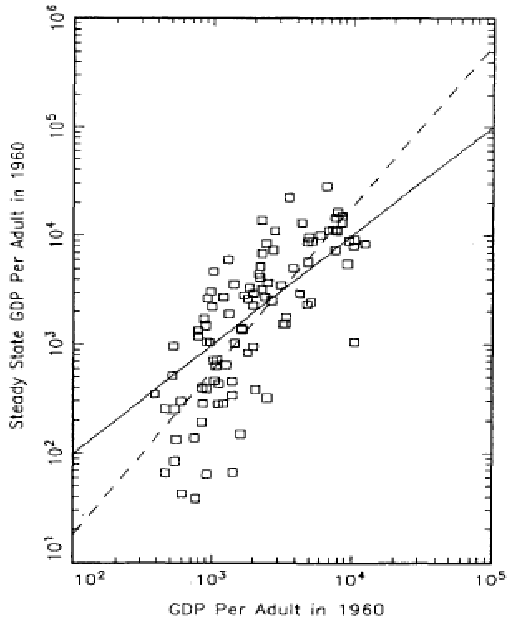

Fig. 1. Current vs. steady-state positions, 98 countries. Data from Mankiw et al. (1992).

Figure 4: Source: Cho and Graham, 1996. Note; The bold faced line is a 45 degree line.

By-and-large: poor countries in 1960 were "overdeveloped". A little more suspicious.

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

Another worrisome pience of evidence is this

Table 4 THE ROLES OF A AND X IN 1960–1985 GROWTH

	cov[Δ ln(Y/L), Δ ln (Z)]/*var* Δ ln(Y/L)			
Source[a]	$Z = \left(\frac{K_Y}{Y}\right)^{\frac{\alpha}{1-\alpha-\beta}}$	$Z = \left(\frac{H_Y}{Y}\right)^{\frac{\beta}{1-\alpha-\beta}}$	$Z = X$	$Z = A$
BK2	.03	12	.15	.85
BK3	03	12	.14	.86
BK4	.03	.06	.09	91

Figure 5: Source: Klenow and Rodriguez-Clare (1997).

Performing *growth accounting* on many countries you may ask how big a fraction of the *differences* in growth rates can be accounted for by factors, and "g". The answer: g! Nearly 90 percent!

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

Both "convergence from above", and growth accounting suggests that "g" likely differ. Do we have to give up "constant g" in the long-run?

Not nessesarily. Observe that g differences could be temporary in principle. Consider the following situation. Imagine there is a world frontier of technology (A^w) which expands at a constant rate

$$A_{t+1}^w = (1+g)\, A_t^w$$

In each country, people *adopts* from the frontier (at the rate ω). Specifically, in a given country actual technology

$$T_{t+1} - T_t = \omega \cdot \left(A_t^w - T_t\right), \ \omega < 1.$$

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

Define $x_t \equiv T_t / A_t^w$

$$x_{t+1} \equiv \frac{T_{t+1}}{A_{t+1}^w} = \frac{\omega}{1+g} + \frac{1-\omega}{1+g} x_t$$

When $x_{t+1} = x_t = x^*$

$$x^* = \frac{\omega}{\omega + g}.$$

with $\partial x^* / \partial \omega > 0$. Also, in steady state

$$\left(\frac{T_{t+1}}{T_t} \right)^* = (1+g).$$

[Insert phasediagram]. But note, now differences in *levels* of technology are systematic! Possibly ω and s are correlated! Our regressions are misspecified and the results are suspect.

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

So far our discussion suggests that at least levels of technology differ (casual observation suggests the same)

Whether technology differs is a testable hypothesis; requires panel data, however (time and countries). We need

$$\log(y_i) = \boldsymbol{\beta}_{i0} + \beta_1 \log(s_{Ki}) + \beta_2 \log(s_{Hi}) - \beta_3 \log(n_i + \delta) + \epsilon_i$$

and test whether $\beta_{i0} = \beta_{j0} = \beta$ for $i \neq j$. So is this assumption true?

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

No. You do see differences, and they are systematically related to y.but more than that ...

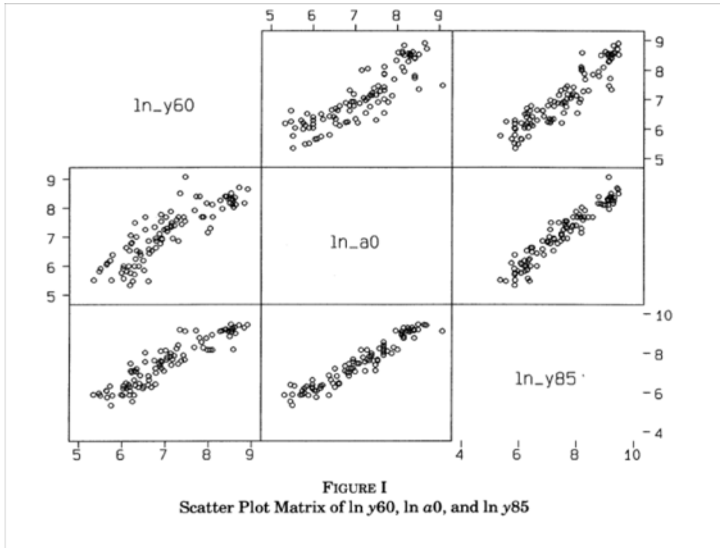

Figure 6: Source: Islam (1995)

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

The are systematically related to human capital ...

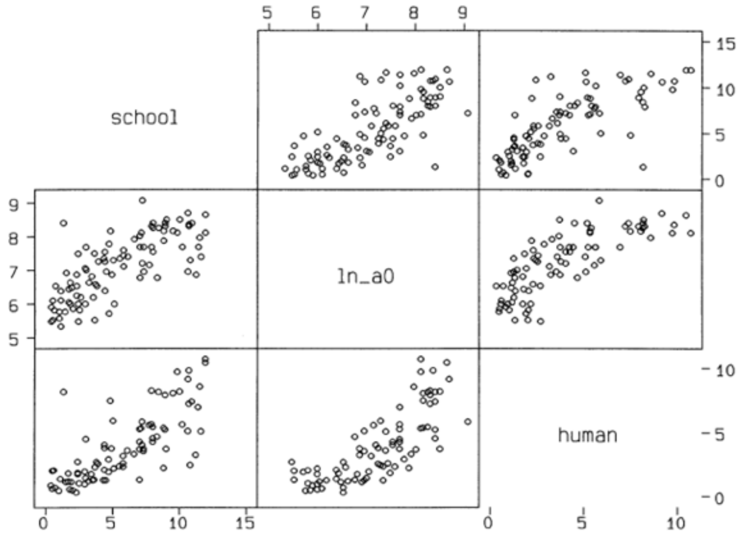

FIGURE III
Scatter Plot Matrix of SCHOOL, ln $a0$, and HUMAN

Figure 7: Source: Islam (1995).

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

But it seemed to work like a charm!? Human capital can account for differences without technology!

Most likely overestimating the impact of human capital. Compare the MRW model's predictions regarding income differences between US (School = 12) and Mali (School = 1). Predicted income difference

$$\log\left(\frac{y_{US}}{y_{MALI}}\right) = \frac{\phi}{1-\alpha-\phi}\left[\log\left(s_H^{US}\right) - \log\left(s_H^{MLI}\right)\right] \Rightarrow \frac{y_{US}}{y_{MALI}} \approx 12.$$

The labor literature also examines the impact of schooling (p. 4 above). Estimating equations of the form

$$\log(w) = \beta_0 + \rho u + \text{other controls},$$

where u is years of schooling. $\rho \approx 10\%$ about reasonable.

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

Incoorporating this effect. Suppose we have this production function

$$Y = K^{\alpha} (hL)^{1-\alpha}$$

where h is skills.

Wages

$$w = (1-\alpha) \underbrace{\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} h}_{Y/L} \Rightarrow \log(w) = \beta_0 + \log(h).$$

If $h = e^{\rho u}$, we get the "micro specification".

THE AUGMENTED SOLOW MODEL: A CRITICAL REAPPRAISAL

Re-calibrating income levels with microfoundations

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} e^{\rho u}$$

so

$$\frac{(Y/L)_{US}}{(Y/L)_{MALI}} = e^{\rho(u_{US}-u_{MLI})} = e^{0.1\cdot(12-0.876)} \approx 3.$$

which is to be compared with a factor of 12 under MRW!

Why the big difference? Overestimating "ϕ":

$$cov\,(\text{technology, human capital}) > 0.$$

Bottom line: Ultimately we have to think about *why* technology differs!

CONCLUSIONS

Augmenting the Solow model by Human capital is (a) reasonable and (b) improves its explanatory power *viz* income differences.

the MRW estimations were a remarkable triumf, at first sight. Technology essentially not needed!

Further work has raised doubts, however.

Circumstancial "evidence":

- Poor countries are *systematically* converging from above. Reason: Poor countries have grown slower than 2%. By the logic of the model: they start above. It's possible, but more plausible that the data is telling us g differs albeit perhaps temporarily (adoption)

- Growth accounting suggests g differs acorss countries.

CONCLUSIONS

- The impact of human capital, according to MRW's results, is too big to be consistent with labor literature

"Direct" evidence:

Panel data estimation tells us A differ. Systematically. This implies the MRW results are suspect, and potentially explain why they find too big of an effect from human capital.

We're not done yet. Why does A differ?