ENDOGENOUS GROWTH

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MOTIVATION AND SETTING THE SCENE

How to sustain growth? Under standard assumptions (e.g., diminishing returns and the Inada-conditions), we need "A" to be increasing.

We would like to think about *mechanisms* which could generate this outcome

Observe that making A endogenous is not straight forward, if we'd like to maintain competitive markets. CRTS to K,L implies

$$Y = F(K, AL) = F_K K + F_L AL = rK + wL$$

Hence: No rents left to remunerate "A". As a result: We cannot ask the firm's to pay for it (directly)

MOTIVATION AND SETTING THE SCENE

Conceptually, there are 5 different approaches to making growth endogenous, and resolving the problem of "funding" technological change

1. Forget "A". Assume capital is sufficiently productive. The simplest approach. We start here, to figure out what we have to assume, *mechanically*, to genereate endogenous growth.

2. Nobody is paying; externalities. Technological progress is a byproduct of production. Learning by doing. (Next "story")

3. Households are paying directly. Human capital could sustain growth perpetually in theory

4. Government pays (households and/or firms, indirectly). Public funded R&D. Investments in infrastructure etc.

5. Deviate from perfect competition. Privately funded R&D.

Recall the law of motion for capital per worker under the Solow model (no tech. progress)

$$\frac{k_{t+1}}{k_t} - 1 = \frac{1}{1+n} \left[s \frac{f(k_t)}{k_t} - (\delta + n) \right]$$

We observe

$$\lim_{k \to \infty} \left(\frac{k_{t+1} - k_t}{k_t} \right) = \frac{s}{1+n} \lim_{k \to \infty} \frac{f(k_t)}{k_t} - \frac{\delta + n}{1+n}$$

By diminishing returns, f/k will be declining. We know more, however, since

$$\lim_{k \to \infty} \frac{f(k_t)}{k_t} = \lim_{k \to \infty} f'(k) \stackrel{\text{Inada}}{=} 0$$

This is why growth cannot be sustained.

Example 1: The "AK" model. $y_t = f(k_t, A) = Ak_t$.

Clearly f'(k; A) = A > 0 (so we violate $\lim_{k \to \infty} f'(k) = 0$).

The law of motion becomes

$$\frac{k_{t+1} - k_t}{k_t} = \frac{1}{1+n} \left[s \frac{f(k_t)}{k_t} - (\delta+n) \right] = \frac{1}{1+n} \left[sA - (\delta+n) \right].$$

Perpetual growth in k and thus y is *feasible*.

Definition. Endogenous growth is said to be present if perpetual (ever lasting) growth in GDP *per worker* is *feasible* without variables growing at exogenous rates.

E.g., if we shut off all exogenous sources of growth in the model above we get $\frac{k_{t+1}-k_t}{k_t} = sA - \delta$. Endogenous growth.

[Insert phasediagram for the AK model]. Some properties of the model:

1. Permanent changes in structural charactaristics (s, n etc) will permanently affect growth in GDP per worker

Long-lasting growth differences across countries easy. Points to policies as a central source of growth differences. Obviously, with permanent growth differences -> huge GDP per worker differences

2. No association between growth and initial levels. Not even conditional on structural charactaristics.

... a problem, but remember that we now are assuming f'(k)=constant>0 for all k! Stronger than needed.

Example 2: The asymptotic Ak model. $y_t = f(k, A)$ Properties: f'(k) > 0, f''(k) < 0 (diminishing returns). **But** $\lim_{k\to\infty} f'(k) = A$. (no Inada)

Law of motion as usual

$$\frac{k_{t+1} - k_t}{k_t} = \frac{1}{1+n} \left[s \frac{f(k_t)}{k_t} - (\delta + n) \right]$$

Growth in the long-run (using the law of motion for capital

$$\lim_{k \to \infty} \frac{k_{t+1} - k_t}{k_t} = \begin{cases} \frac{1}{1+n} \left[sA - (\delta + n) \right] & \text{if } sA > \delta + n \\ 0 & \text{otherwise} \end{cases}$$

[Insert phasediagram for the asymptotic AK model]

Properties of the asymptotic AK model:

Endogenous growth (recall: feasibility)

Inverse association between growth and initial levels, *conditional* on structural charactaristics

Interesting property: Regimes. Low $A \rightarrow$ changes in s has only level effects (as in Solow model). High "A" \rightarrow changes in s spurs growth.

Bottom line. Central assumption needed to generate endogenous growth is

$$\lim_{k \to \infty} f'(k) > 0.$$
 (CEG)

Endogenous growth requires the marginal product of capital is bounded away from zero.

An equivalent way of stating the condition CEG

Endogenous growth requires (asymptotically) constant returns to scale in reproducible factors of production.

Example 1: y = Ak. 1 reprocible factor -> condition is $\frac{\partial y}{\partial k}\frac{k}{y} = 1$. Check:

$$\partial y/\partial k = A, \ k/y = 1/A.$$
 Hence $\frac{\partial y}{\partial k} \frac{k}{y} = 1$ for all k .

Example 2:y = f(k; A). 1 reprocible factor. Check:

$$\partial y/\partial k = f'(k), k/y = \frac{k}{f(k)}.$$
 We get $\frac{\partial y k}{\partial k y} = \frac{f'(k)k}{f(k)} \neq 1.$ But
$$\lim_{k \to \infty} \frac{f'(k) k}{f(k)} = A \lim_{k \to \infty} \frac{k}{f(k)} = A \lim_{k \to \infty} \frac{1}{f'(k)} = 1.$$

With one reproduciple factor of production we therefore have to equivalent conditions for endogenous growth

Marginal production of capital bounded away from zero

$$\lim_{k \to \infty} f'(k) > 0.$$
 (CEG)

or constant returns to scale in reproduciple inputs:

$$\lim_{k \to \infty} \frac{\partial y k}{\partial k y} = 1$$
 (CEG')

Suppose we have competitive factor markets. Notice anything troubling with CEG' (and therefore with the condition for endogenous growth) from an empirical standpoint? We need to resolve this.

A classic case study: The Liberty Ship. Same ship produced throughout WWII.

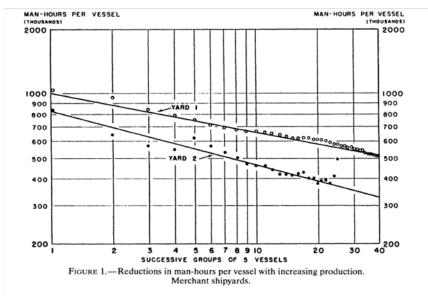


Figure 1: From Lucas (1993).

Classic study by Searle (1945); re-examined by Rapping (1965, Re-Stat) and recently by Thompson (2001, JPE).

Basic idea studied by Rapping

$$Y_t = A_t K_t^{\phi} L_t^{\delta},$$

where

$$A_t = T_t \tilde{Y}_t^\beta$$

and $T_t = T_0 (1+g)^t \tau_i$ (exogenous tech. change; τ_i a productivity shock), whereas \tilde{Y} is *cumulated output* at time t

Conditional on capital, a time trend, and labor input, Rapping found $\beta \approx 0.3$. Thompson finds the effect is smaller: around 0.15. Note: Internal learning (at a given shipyard)

External learning; Past production in firm i matters for productivity in firm j.

Irwin and Klenow (1994, JPE): Semi-conductors. Confirm this effect. You have impacts across industries and *across* countries!

A simple way to capture this effect is to assume that technological knowledge, A, is an increasing function of the *aggregate* level of production, or capital stock ("learning-by-investing")

For example, to make things particularly simple, we could assume (inspired by Arrow, 1962, RES)

$$A_t = \bar{A}K_t^{\phi}, \ \phi > 0.$$

So the production function of our *representative* firm is

$$Y_t = K_t^{\alpha} \left(A_t L_t \right)^{1-\alpha},$$

where $A_t = \bar{A}K_t^{\phi}$. When optimizing the firm *does not* take A_t into account. Argument: External learning, and small firms.

Hence the first order conditons from firm profit max

$$r_t + \delta = \frac{\partial Y_t}{\partial K_t} = \alpha \frac{Y_t}{K_t}$$
$$w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) \frac{Y_t}{L_t}$$

The *reduced form* production function

$$Y_t = K_t^{\alpha} \left(\bar{A} K_t^{\phi} L_t \right)^{1-\alpha} = K_t^{\alpha + (1-\alpha)\phi} \left(\bar{A} L_t \right)^{1-\alpha}$$

Observe: Increasing returns to K and L in reduced form.

What do we have to require for endogenous growth? CRTS in $K \Leftrightarrow$ the marginal product is bounded away from zero.

Hence

$$\alpha + (1 - \alpha)\phi = 1 \Leftrightarrow \phi = 1.$$

On empirical grounds, this is a tall order if motivated solely by *learning-by-doing* (supports perhaps $\phi \approx 0.15$). Other "stories" could increase it.. textbook discuss *reduced form* evidence which suggests $\phi \approx 0.45$ could be appropriate. Assume $\phi = 1$ anyway, for now.

If $\phi = 1$

$$Y_t = \left(\bar{A}L_t\right)^{1-\alpha} K_t \Rightarrow y_t = \left(\bar{A}L_t\right)^{1-\alpha} k_t$$

Otherwise: "Solow" dynamics:

$$\frac{k_{t+1}}{k_t} - 1 = \frac{1}{1+n} \left[s \left(\bar{A} L_t \right)^{1-\alpha} - (\delta+n) \right].$$

Observe something worrisome: The presence of L_t .

The model features a *scale effect*.

Why? No diminishing returns but we do have complementarity between K and $L (\partial^2 Y / \partial K \partial L > 0)$. Since $\partial Y / \partial K$ is constant, Y/K is constant, any so change in L becomes permanent.

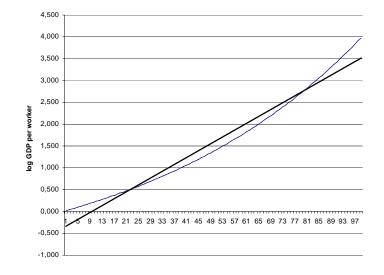


Figure 2: Simulation of the model with scale effects. Log GDP per worker vs "time". 100 years. Note: The straight line is a (linear) trend.

With expanding population the model implies that the growth rate should be accelerating. Growth increases from 2 to 7 percent per annum. Clearly counterfactual. Cross-country? Larger countries should be growing faster than smaller countries? No support here either.

How to eliminate scale effects?

So far we have assume all firms can benefit from all other firms production (knowledge). This is an extreme assumption

Possibly when the economy grows larger, firms become more specialized, and the potential to learn from each other may be reduced. Increasing "technological distance"

If we proxy the size of the economy by L_t , these considerations would call for something like

$$A_t = \bar{A}K_t^{\phi}L_t^{-\beta}$$

where β parameterizes the extent to which specialization reduces knowledge spillovers.

The modified dynamics:

$$\frac{k_{t+1}}{k_t} - 1 = \frac{1}{1+n} \left[s \left(\bar{A} L_t^{1-\beta} \right)^{1-\alpha} - (\delta+n) \right]$$

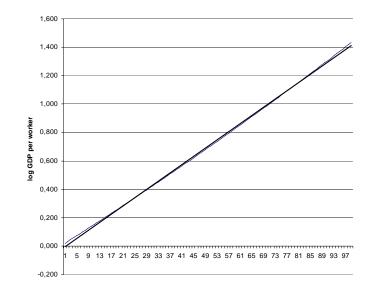


Figure 3: Simulation of the model with scale effects and increasing technological distance. Log GDP per worker vs "time". 100 years. $\beta = 0.9$. Note: The straight line is a (linear) trend.

A large literature has pondered the issue of how scale effects might "evaporate". By now, many theories which suggest $\beta \approx 1$ might be reasonable. If so, we have

$$\frac{k_{t+1}}{k_t} - 1 = \frac{1}{1+n} \left[s\bar{A}^{1-\alpha} - (\delta+n) \right] = \frac{y_{t+1}}{y_t} - 1.$$

The model has the properties of the simple "AK" model. But there is one key difference.

The share of capital in national accounts

$$\frac{(r+\delta) K}{Y} \stackrel{\text{firm}}{=} \frac{\Theta \frac{Y_t}{K_t} K_t}{Y_t} = \alpha$$

which is *consistent* with Kaldor's facts. But didn't we need CRTS in K?

PROPERTIES OF THE MODEL

The model can generate long-run growth differences; permanent changes in s leads to higher growth

The model offers a theory of TFP growth. Note

 $Y = K_t^{\alpha} \left(TFP_t \cdot L_t \right)^{1-\alpha}$

where $TFP_t = A_t = \overline{A}k_t, \ k_t \equiv K_t/L_t$.

It seems consistent with Kaldor's facts

It is not consistent with empirically detected negative association between growth and initial level, conditional on structural charactaristics. But that can be fixed: **NOT** a problem for endogenous growth per se (only asymptotically do we need, say, learning to ensure a positive marginal product)

PROPERTIES OF THE MODEL AND EMPIRICS

Scale is a problem. Still, plausible resolutions to this issue exist as well. The required magnitude of ϕ (=1) is a problem *on empirical grounds*. Learning is unlikely to be the only source of perpetual growth.... and there are some bigger problems up ahead ...

Chad Jones (1995, JPE) launched an empirical critique of these sorts of models. Central piece of evidence: Growth in GDP per worker (capita) in OECD has been stable over time (growth is stationary). BUT investment shares (s) are NOT.

Under the AK model: $s \uparrow \Rightarrow$ growth \uparrow . It hasn't.

EMPIRICS ON THE AK MODEL

Cross country evidence. Across the globe you tend to find

Table 3								
Inv	vestment Rate	s Are Mo	re Persist	tent than Gro	wth Rate	s		
	<u>1980-20</u>	000 vs. 1960	-1980	Decade to Decade				
	<i>Y/L</i> Growth	s_I	S _H	<i>Y/L</i> Growth	S_I	S_H		
World	.34 (.13)	.56 (.07)	1.02 (.04)	.20 (.07)	.77 (.04)	1.00 (.02)		

Figure 4: Klenow and Rodriguez-Clare (2006) Handbook of Economic Growth. The cofficients reported derive from simple bivariate OLS regressions. Standard deviations in paranthesis.

Since s is the key structural charactaristics we would expect growth to "inherit" its properties. It does not. Moreover ...

EMPIRICS

		Та	able 4				
Investment I	Rates Correl	ate More	with Level	s than with	n Growth	Rates	
	Independ	lent Varial	ble = S_I	Independent Variable = S_H			
	Dependen	Dependent Variable			Dependent Variable		
	Y/L Growth Rates	Y/L Log Levels	# of countries	<i>Y/L</i> Growth Rates	<i>Y/L</i> Log Levels	# of countries	
All countries	.111 (.017) R ² = .32	1.25 (0.13) $R^2 = .48$	96	.210 (.060) $R^2 = .15$.313 (.026) $R^2 = .67$	74	

Figure 5: Klenow and Rodriguez-Clare (2006) Handbook of Economic Growth. The cofficients reported derive from simple bivariate OLS regressions. Standard deviations in paranthesis.

Hence, when it comes to investment rates, it seems the "truth" is closer to the modelling approach of the Solow model....

As for TFP growth. Across countries, TFP growth varies *much* more than growth in K/L. Hence, this too is not a "home run".

CONCLUSION ON AK MODEL VIA LEARNING BY DO-ING

The problems with the theory:

 $\phi = 1$ not realistic if Learning-by-Doing; $\phi < 1$ is fine though

Growth in TFP and capital accumulation does not have the same properties. Suggest A = K is not right

s and growth do not have similar time series properties. Evidence more in favour of $s \rightarrow$ levels, rather than growth rates.

Other worries: Scale (albeit theory to resolve it), Lack of "conditional convergence" (albeit theory to resolve it)

An approach suggested by Chad Jones offers a possible resolution of these problems: *Semi*-endogenous growth.

SEMI-ENDOGENOUS GROWTH

Formally, this model assumes $\phi < 1$ (and $\beta = 0$)

In per worker terms the law of motion for capital is $(Y_t = y_t L_t = L_t k_t^{\alpha} A_t^{1-\alpha}, A_t = \overline{A} K_t^{\phi}, k_t \equiv K_t/L_t)$:

$$\gamma_{t+1} \equiv \frac{k_{t+1}}{k_t} - 1 = \frac{1}{1+n} \left[sk_t^{\alpha-1} K_t^{\phi(1-\alpha)} - (\delta+n) \right]$$
$$= \frac{1}{1+n} \left[sk_t^{\alpha+\phi(1-\alpha)-1} L_t^{\phi(1-\alpha)} - (\delta+n) \right]$$

To see the main conclusion viz growth it is easiest to look at changes in the growth rate

$$\gamma_{t+1} - \gamma_t = \left[\frac{sk_t^{\alpha - 1}K_t^{\phi(1 - \alpha)}}{1 + n}\right] \left[(1 + \gamma_t)^{(\phi - 1)(1 - \alpha)} (1 + n)^{\phi(1 - \alpha)} - 1 \right]$$

The steady state is where $\gamma_{t+1} = \gamma_t = \gamma^* > 0$.

SEMI-ENDOGENOUS GROWTH

In this model, then, perpetual growth in k_t is *feasible*. The long-run growth rate is

$$1 + \gamma^* = (1+n)^{\frac{\phi}{1-\phi}}$$

Notice: growth is *not* feasible if n = 0; we do not have "endogenous growth" in the sense of our definition. Yet, growth does not require *technology* to be growing at an exogenous rate. It's "sort of" endogenous growth, or

Definition. "Semi-endogenous growth" is said to be present if perpetual (ever lasting) growth in GDP *per worker* is *feasible* without *technology* growing at an exogenous rate.

You can show that the model is globally stable, gradual convergence prevail etc. (textbook p. 224-26; note typo p. 225; exercises).

We now know that $1+\gamma^* = (k_{t+1}/k_t) = \left(\frac{K_{t+1}}{K_t}\right)^* \left(\frac{L_t}{L_{t+1}}\right) = (1+n)^{\frac{\phi}{1-\phi}}$. Hence the rate of productivity growth

$$\left(\frac{A_{t+1}}{A_t}\right)^* = \left[\left(\frac{K_{t+1}}{K_t}\right)^*\right]^\phi = \left[(1+n)^{\frac{\phi}{1-\phi}+1}\right]^\phi = (1+n)^{\frac{\phi}{1-\phi}}$$

And, so GDP per worker growth

$$\left(\frac{y_{t+1}}{y_t}\right)^* = \left(\left(\frac{A_{t+1}}{A_t}\right)^*\right)^{1-\alpha} \left(\left(\frac{k_{t+1}}{k_t}\right)^*\right)^\alpha = (1+n)^{\frac{\phi}{1-\phi}},$$

as well. A few interesting aspects of the model:

The reason why perpetual growth is feasible is that we assume increasing returns to scale in K,L.

$$Y = (\lambda K)^{\alpha + \phi(1 - \alpha)} (\lambda L)^{1 - \alpha} = \lambda^{1 + \phi(1 - \alpha)} K^{\alpha + \phi(1 - \alpha)} L^{1 - \alpha}$$

Doubling K,L we get more than twice the output. We get $2^{1+\phi(1-\alpha)}$. This is motivated by learning - the external effect from increasing K.

1. Growth is feasible in the long run when $\phi < 1$. Hence, we require *less* increasing returns in the semi-endogenous growth setting, than under endogenous growth ($\phi = 1$). A virtue of the model, since $\phi < 1$ is *empirically realistic*. To sustain growth we need population to be growing (consequence of less increasing returns)

2. The long-run growth rate, $(1+n)^{\frac{\phi}{1-\phi}}$, is independent of policies (insofar as these chiefly affect things like s). This too is a virtue, since the empirical association between s and "g" is questionable.

Yet. s does affect the *level* of GDP per worker (data seems to agree). Indeed you can show that in the steady state

$$y_t^* = \left(\frac{s}{g_A^* + \delta}\right)^{\frac{\alpha + \frac{\phi}{1 - \phi}}{1 - \alpha}} L_0^{\frac{\phi}{1 - \phi}} \left(1 + g_A^*\right)^t$$

If $\phi \approx 0.15$, $\alpha = 1/3$, then $\frac{\alpha + \frac{\phi}{1-\phi}}{1-\alpha} \approx 3/4$. An improvement. 1/2 in the Solow model, and (effectively speaking) 2/3 in the augmented Solow model. Again, thanks to the external effects.

3. But is this an improvement viz understanding *TFP* growth? Stricktly speaking, we still have the prediction that $A \propto K$, and that's a problem (growth in A more variable). But it can be fixed, by assuming instead that $A \propto Y$ (cf. exercises)

-> It is therefore more reasonable to look at the *prediction* that growth in A should be related to n

Q1: Are the two correlated?

Q2: Do they have similar properties in the time series dimension (persistency?)

A1: They are certainly correlated ... just not in the right way

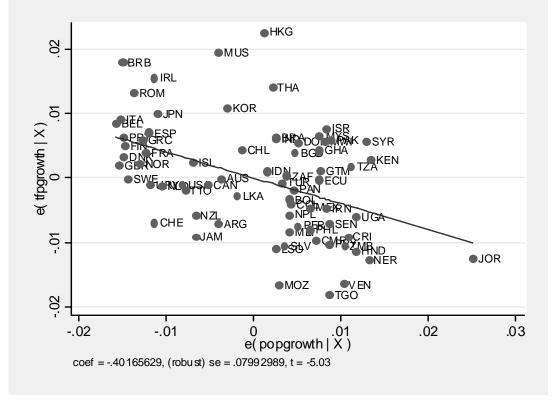


Figure 6: Semi-endogenous growth? The figure shows the partial correlation between TFP growth and population growth 1960-2000, conditional on a constant. The line is fitted by OLS. TFP data and population data from Klenow and Rodriguez-Clare (2006).

As mentioned: TFP growth is not particularly persistent:

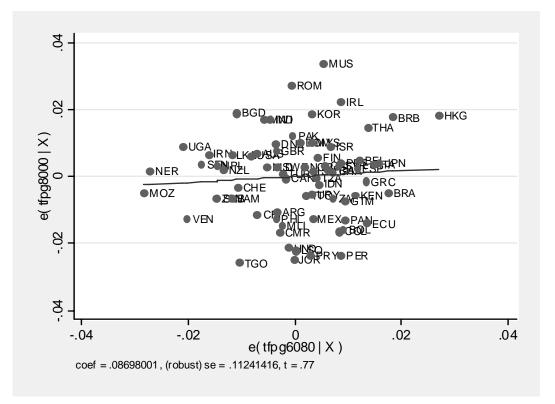


Figure 7: The figure shows the partial correlation between TFP growth 1960-1980 vs. TFP growth 1980-2000.

... But population growth is

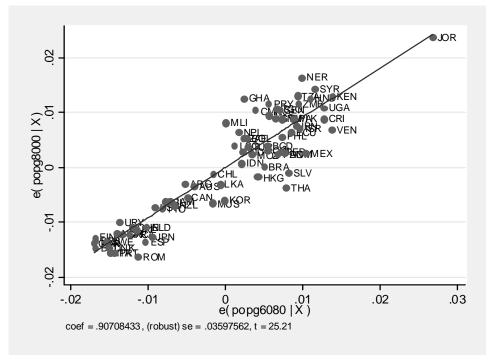


Figure 8: The figure shows the partial correlation between TFP growth 1960-1980 vs. TFP growth 1980-2000.

A2: They do not match in terms of persistency.

4. Scale effects? The dramatic scale effect on *growth* has been removed. But scale still matters. Recall,

$$y_t^* = \left(\frac{s}{g_A^* + \delta}\right)^{\frac{\alpha + \frac{\phi}{1 - \phi}}{1 - \alpha}} L_0^{\frac{\phi}{1 - \phi}} \left(1 + g_A^*\right)^t$$

Hence, a large population should enable higher *levels* of productivity, conditional on investment shares.

Any tendency for this to be true?

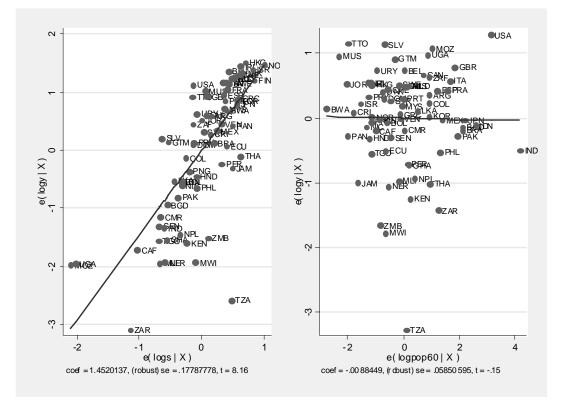


Figure 9: Regression results: Estimating GDP per worker (log), using average investment shares (s), and level of population as explanatory variables. Data: PWT 6.1. and Klenow and Rodriguez-Clare (2006)

No.

SEMI-ENDOGENOUS GROWTH: SOME CONCLUDING REMARKS

Virtues:

(i) better in accord with evidence on ϕ than the AK model; (ii) We have theory of growth differences which relegates the impact of s to "levels", and away from the growth rate; (iii) Essentially the theory no longer stipulates a tight link between A and K (g_A and g_K), which is sensible as well

Vices:

The theory puts A and L (g_A and n) to be related, but: (A) n much more persistent than g_A ; (B) g_A and n are *negatively* correlated; (C) No clear relationship between prosperity (y) and population, conditional on s.

WHERE TO GO FROM HERE?

These days most researchers believe *interdependence* is a keyword. Hence, the theory of adoption discussed earlier has found a revival. That is, structures such as

$$A_{t+1}^{w} = (1+g) A_{t}^{w}$$
$$T_{t+1} - T_{t} = \omega \cdot (A_{t}^{w} - T_{t}), \ \omega < 1.$$

Hence, to most countries the long run growth rate g is exogenous. That is, Denmark cannot does not affect g. But we can affect ω , and therefore temporarily affect g_T .

Endogenous growth theory (/semi) is thought to have bearing on g; the evolution of the frontier. For example, in terms of the semi-endogenous growth model the prediction would be that *world* population growth (or in "leader" countries) matters for g. Maybe more reasonable.

WHERE TO GO FROM HERE?

These sort of models would suggest an inverse association between TFP growth and initial TFP. This appears to be in the data

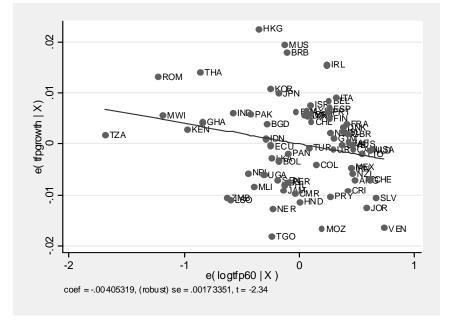


Figure 10: TFP growth 1960-2000 vs (log) TFP in 1960. OLS.

...the quest is to figure out what goes into ω though; R&D maybe?

WHERE TO GO FROM HERE?

The *pervasiveness* of the productivity slowdown

Table 2

Output Growth Declined Sharply Worldwide

	Average Y/L Growth				Average S _I			Average S _H		
	1960-75	1975-00	# of countries	1960-75	1975-00	# of countries	1960-75	1975-00	# of countries	
World	2.7%	1.1%	96	15.8%	15.5%	96	7.1%	9.7%	74	
OECD	3.4	1.8	23	23.2	22.9	23	11.4	14.3	21	
Non-OECD	2.5	0.9	73	13.5	13.2	73	5.4	8.0	53	
Africa Asia Europe North America South America	2.0 3.2 3.8 2.8 2.3	0.5 2.8 1.9 0.4 -0.1	38 17 18 13 10	12.3 14.5 24.9 14.3 17.3	10.5 19.9 23.1 14.5 15.0	38 17 18 13 10	3.9 6.9 10.7 7.5 7.1	6.0 9.9 13.7 10.2 9.8	19 16 16 13 10	
1 ^{at} quartile (poorest) 2 nd quartile 3 rd quartile 4 th quartile (richest)	1.6 2.6 3.5 3.0	0.5 1.4 1.1 1.5	24 24 24 24	9.6 14.8 15.4 23.6	9.9 14.2 16.3 21.9	24 24 24 24	3.1 5.7 7.5 12.3	5.0 8.9 10.3 15.1	19 19 18 18	

Notes: Y/L is GDP per worker. S_I is the physical capital investment rate, and S_H years of schooling attainment (for the 25+ population) divided by 60 years (working life). Data Sources: Barro and Lee (2000) and Heston, Summers, and Aten (2002).

Figure 11: Klenow and Rodriguez-Clare (2006)

This also suggest cross-country interdependence is important, and supports the "adoption" view; endogenous "world growth".