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Endogenous Time Preference with Investment and Development Traps

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1. Introduction

There is a large and persistent disparity in observed per capita output levels in different economies. E.g. Jones (1997), McGrattan and Schmitz (1998) and Parente and Prescott

(2000) document these facts, and discuss possible explanations for the phenomenon. A multitude of theoretical explanations have been put forward to explain, why these differences in per capita output levels persist. Azariadis (1996, 2001) surveys theoretical explanations for the development (poverty) traps. These traps are situations, where an economy can end up in a bad equilibrium even though better equilibria are available. Recently Graham and Temple (2001) have argued that empirically development traps are a significant factor in explaining the international income differentials and their persistence.

In this paper we present another mechanism to generate development traps. In the previous work surveyed by Azariadis the mechanism underlying the multiplicity of equilibria is the endogeneity of time preference due to the influence of consumption on the intertemporal marginal rate of substitution. The formal framework is thus based on recursive utility functions e.g. as explored in Becker and Boyd (1997). In this paper we generate the endogenous time preference by assuming that consumers can make investments that change it. In addition to investing in physical capital economic agents can invest in patience, i.e. they can spend resources to decrease rate of time preference (i.e. to increase their discount factor). We use this idea in a growth model to generate development traps. This mechanism has also been used in a model put forward by Becker and Mulligan (1994, 1997). The idea is closely related, but not identical, to the idea in recent "psychological economics" that people can make conscious (and costly) decisions to overcome the "weakness of will" created by time inconsistent preferences (e.g. Benabou and Tirole 2002). It has obvious implications for savings and growth (Harris and Laibson 2001).

We argue that the possibility for people to invest on farsightedness in addition to productive capital may explain why there exist development traps. It may also explain why economic growth is affected by initial conditions. By investment in farsightedness we mean such things as investment in personal health or better nutrition, which prolong life and increase the weight people assign to their future welfare. In a similar fashion, investment in activities that help to understand culture and social traditions (e.g. investment in understanding modern art) may improve perseverance.[1]

It is also interesting to note that already Fisher (1977) argued that the rate of time preference increases the wealthier consumers become, i.e. indifference curves become flatter. One way to represent this idea is to assume that discount factor can be increased (i.e. future is made more important) by spending resources on it.

At the face value, our attempt is perhaps futile: some of the existing empirical evidence seems to be against our theory. Mokyr (1990, p.155-156) rejects outright Boulding's hypothesis that increased life expectancy was a major driving force behind the growth of innovative activities leading to the First Industrial Revolution. He points out that there is no evidence for such a link between innovations and increased life expectancy. Furthermore, Braudel (1978) remarks that the life expectancy varied very much between socioeconomic groups.

While we agree that it is hard to see how increased farsightedness could have been a major source of bursts in innovative activities, we think, in particular, that it may be an important part of the growth process. Growth may induce investment in farsightedness, which again may increase accumulation of productive capital. And if growth has this cumulative element, it is easy to understand why the economy might end up in a development trap.

There is quite a bit of evidence supporting our view. Stark (1995, ch. 2) shows evidence that life expectancy and per capita income are internationally positively correlated. Correlation, of course, does not say anything about causation, and we, in fact, argue that they are interrelated. Deaton (2001) provides a survey of these issues confirming the role of income in improving individual health. Even more closely supporting our argument is the fact

[1] Becker and Mulligan (1997) give other similar examples.

that the income elasticity of public spending on health is higher in countries with high per capita income than in countries with low income (World Development Indicators, World Bank 1998, p 91).

A rough look at the data brings up similar related findings. We have taken the five richest (USA, Japan, Norway, Denmark, and UK) and the five poorest countries (Guinea-Bissau, Mongolia, Sierra Leone, Central African Republic, and Chad) in terms of per capita GDP (World Development Indicators 1998, Table 1.1), and looked at various indicators of health. In 1995 the average child immunization rate (DPT) was among the poorest countries 52 per cent while among the richest it was 90 per cent (World Development Indicators 1998, Table 2.14). Similarly, in 1995 93 per cent of population in the richest countries had access to sanitation while only 17 per cent in the poorest countries in 1995 (World Development Indicators 1998, Table 2.14, figures for Mongolia and Central African Republic are not available). Finally, the maternal mortality rate (per 100 000 live births) is 8.8 in the richest countries and 617 among the poorest countries.

In his survey Deaton (2001) concludes that the social environment is crucial for the individual health and for the overall health of nations. To give some further, somewhat more micro level evidence from economic history, on the possible importance of the mechanism we propose, consider the account of Thompson (1988), why the birth rates declined and the average age at marriage increased in the 19th century Britain. E.g. he argues that because the possibility to get their children positions in government offices improved, the middle classes began to exercise birth control. This was necessary, because otherwise they would not have been able to provide proper education for their children (especially because the cost of education was increasing) and take care of children's welfare.

One could argue that this is an example of a new investment opportunity opening, not of increasing weight put on children's welfare. But what was the value to parents of their children receiving better positions than they themselves did if they did not put additional weight on children's welfare? And if they cared before of their children equally much, why they did not restrict the number of children then? And when we look at the present developed world, what is the consequence of people spending increasing amounts of money on health in general and physical fitness in particular? And why has this spending increased? A natural answer is to claim that the time preference was changing. Becker and Mulligan (1997) argue that much of the observed time series evidence cannot be understood unless one allows the rate of time preference to respond to changes in economic environment in the manner assumed in the model we present.

Finally, one potentially interesting application of our model is to problems of transition, especially to those found in Russia. As is well known transition has everywhere included a period of falling real GDP. In Russia the decline has continued for a longer time and been deeper than in most of the other countries. At the same time the life expectancy at birth has declined in Russia. The adult mortality rate (both for men and women) has increased dramatically (World Development Indicators 1998, Table 2.17). Our theory is a way to see the continued decline of income and increase in the mortality as different aspects of the same phenomenon.

We want to argue that the exact mechanism creating development traps is the complementarity between investment in physical capital and investment in patience. Complementarity implies that income must be at a high enough level for investment in patience to take place, or that multiple equilibria, with one equilibrium being the development trap, exist.

2. The Model:investment in patience and saving decision

We consider an overlapping generations economy where agents live for two periods. There is no population growth. To focus on investment in patience alone we assume away all effects coming from changing wealth distributions. The agent born at the beginning of period t has the following lifetime utility function,

(1) $$v(c_1^t, c_2^t, x_t) = u(c_1^t) + \beta(x_t)u(c_2^t).$$

The periodic utility function, $u(c)$, is assumed to be strictly increasing and concave, and to fulfill the Inada conditions: $u'(c) > 0$, $u''(c) < 0$ and $\lim_{c \to 0} u'(c) = \infty$. c_1^t and c_2^t denote consumption in youth and in old age. Consumers supply inelastically one unit of labor in their youth. $\beta(x) = [(1+\rho(x)]^{-1}$, where ρ is the rate of time preference. It depends on the amount invested in making consumer less impatient.

To focus sharply on the patience investment we assume the discount factor function to have the following simple form:

$$\beta(0) = \beta_1 > 0 \text{ , } \beta(\bar{x}) = \beta_2 > 0 \text{ and } 1 > \beta_2 > \beta_1 .$$

The assumed form of discounting means that consumer must invest at least $\bar{x}$ to patience for the future to have an added importance.

Next we consider consumer's optimal decisions. His periodic budget constraints are

(2) $$c_1^t + s_t + x_t = w_t$$

(3) $$c_2^t = R_{t+1}s_t.$$ [2]

w_t is the wage rate, s_t denotes saving, and R_{t+1} is the interest factor from period t to period $t+1$. The lifetime budget constraint is then

(4) $$c_1^t + x_t + \frac{c_2^t}{R_{t+1}} = w_t .$$

The consumer's decision problem is

(PC) $$\frac{\max}{\{c_1^t, c_2^t, x_t\}} \{u(c_1^t) + \beta(x_t)u(c_2^t)\}$$

subject to (4) or to (2) and (3).

This will yield the saving function $s_t = s(w_t, R_{t+1}; \beta_i)$ ($i = 1,2$), which for a given level of patience investment is obtained from the Euler equation

(5) $$u'(w_t - x_i - s_t) = R_{t+1}\beta_i u'(R_{t+1}s_t),$$

where x_i is either zero or $\bar{x}$.

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[2] In principle we could have set the price of x to differ from unity. We can interpret there to be a linear "technology", which transforms patience investment (x) into "output". Thus the price of x will be unity.

Consumer invests in patience either nothing or $\bar{x}$. He will invest $\bar{x}$, if that decision yields him higher utility. Thus in consumer's optimum $x = \bar{x}$, if total utility is at least as high as not investing anything in patience, i.e.

$$(6) \quad \Delta(w_t, R_{t+1}) \equiv u(w_t - \bar{x} - s_t^2) + \beta_2 u(R_{t+1} s_t^2) - u(w_t - s^1) - \beta_1 u(R_{t+1} s_t^1) \geq 0,$$

where s_t^i, ($i = 1,2$), is the appropriate optimal saving decision.[3] The "indirect" indifference curve, $\Delta(w, R) = 0$, gives all the wage and interest factor combinations such that consumer is indifferent in investing in patience or not.

We first note that if the wage income is very small, less than or equal to the required patience investment, $\bar{x}$, consumer will not invest anything in patience. On the other hand if that income is so large that the proportion of $\bar{x}$ to the wage is small, he will always invest in patience. This argument is formalized in

Proposition 1. There is a wage level, $\underline{w}$ ($> \bar{x}$), below which consumers do not invest anything in patience, i.e. $\Delta(w, R) < 0$ for all $w < \underline{w}$. Furthermore, $\lim_{w \to \infty} \Delta(w, R) > 0$.

Proof: For all $w \leq \bar{x}$, consumer cannot invest anything in patience, because he does not have enough income. Consider now a situation with a large enough wage level, i.e. where $w \approx w - \bar{x}$, and denote the optimal saving without patience investment as s^1. Since $\beta_2 > \beta_1$, we have $\Delta(w, R) = u(w - \bar{x} - s^1) + \beta_2 u(Rs^1) - u(w - s^1) - \beta_1 u(Rs^1)$ = $(\beta_2 - \beta_1) u(Rs^1) + u(w - \bar{x} - s^1) - u(w - s^1) > 0$. By continuity of the utility function and the respective saving functions there must be at least one wage level, say $\underline{w}$, which is greater than $\bar{x}$ such that $\Delta(\underline{w}, R) = 0$. Q.E.D.

Next we will find out those combinations of the wage rate and the interest factor such that consumer is indifferent of being of each type, i.e. we want to find out the slope of the "indirect" indifference curve, $\Delta(w, R) = 0$. We drop subscripts in (6), take into account the appropriate Euler conditions, and totally differentiate to obtain

[3] If the utilities are equal we assume that he invests in patience.

(7) $\quad \{u'(c_1^{\beta 2}) - u'(c_1^{\beta 1})\}dw = \{\beta_1 s^1(w,R)u'(c_2^{\beta 1}) - \beta_2 s^2(w,R)u'(c_2^{\beta 2})\}dR$.

Superscripts βi ($i = 1,2$) refer to consumer's type. It is not clear from (7), what the slope of the indifference curve is with a general utility function. Obviously we can get technical conditions e.g. for the downward sloping indifference curve by inspecting equation (7), but those conditions do not seem to have a nice economic interpretation.

Next we study (7) by assuming that the lifetime utility function has constant intertemporal elasticity of substitution ($1/\sigma$), i.e. $u(c) = c^{1-\sigma}/(1-\sigma)$. We also assume that $1/\sigma > 1$ so as to make saving an increasing function of the interest factor.[4]

Solving for saving and consumption we get

(8) $$s^1 = \frac{1}{1 + \beta_1^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}} w, \quad c_1^{\beta 1} = \frac{\beta_1^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}}{1 + \beta_1^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}} w$$

(9) $$s^1 = \frac{1}{1 + \beta_2^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}} (w - \bar{x}), \qquad c_1^{\beta 2} = \frac{\beta_2^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}}{1 + \beta_2^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}} (w - \bar{x}).$$

We will next study equation (7) for these functional forms. We present two lemmas, where we make inferences about the signs of the left-hand and right-hand sides of (7).

Lemma 1. For a utility function with constant intertemporal elasticity, $c_1^{\beta 1} > c_1^{\beta 2}$.

Proof: We make a counter hypothesis and assume $c_1^{\beta 1} < c_1^{\beta 2}$. Cancelling out some terms, this can be rewritten as $\frac{1}{1 + \beta_1^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} w < \frac{1}{1 + \beta_2^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} (w - \bar{x})$, from where it follows that $1 < \frac{w}{w - \bar{x}} < \frac{1 + \beta_1^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}{1 + \beta_2^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}$. Given the fact that $\beta_1 < \beta_2$, the right-hand-side of the last expression is less than one. This is a contradiction, and thus we must have $c_1^{\beta 1} > c_1^{\beta 2}$. Q.E.D.

[4] Our emphasis in this paper is on the dynamics created by investment in patience, and thus we want to keep the rest of the model as standard as possible.

This lemma, in particular, means that $u'(c_1^{\beta 2}) > u'(c_1^{\beta 1})$. As a corollary to Proposition 1 and Lemma 1 we get

Lemma 2. For a utility function with constant intertemporal elasticity, $\Delta_w(w,R) > 0$.

Proof: Taking into account the Euler conditions we differentiate $\Delta(w,R)$ to get $\Delta_w(w,R) = u'(c_1^{\beta 2}) - u'(c_1^{\beta 1})$. Since the utility function is strictly concave, it follows from Lemma 2 that this partial derivative is positive. Q.E.D.

This lemma also gives conditions for $\underline{w}$ in Proposition 1 to be unique.

Next we compute the arguments on the right-hand side of (7) to obtain

$$\text{(10)} \qquad \beta_1 s^1 u'(c_1^{\beta 1}) = \frac{\beta_1 w^{1-\sigma}}{R^{\sigma}\left[1+\beta_1^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}\right]^{1-\sigma}}, \; \beta_2 s^2 u'(c_1^{\beta 2}) = \frac{\beta_2 (w-\bar{x})^{1-\sigma}}{R^{\sigma}\left[1+\beta_2^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}\right]^{1-\sigma}}.$$

We get the following lemma.

Lemma 3. For a utility function with constant intertemporal elasticity, $\beta_1 s^1(w,R)u'(c_2^{\beta 1}) - \beta_2 s^2(w,R)u'(c_2^{\beta 2})$ is negative at least for large wage levels, and it is negative everywhere, if the utility functions is logarithmic.

Proof: Assume that $\beta_1 s^1 u'(c_1^{\beta 1}) > \beta_2 s^2 u'(c_1^{\beta 2})$. This inequality can be rewritten as $\frac{\beta_1 w^{1-\sigma}}{\left[1+\beta_1^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}\right]^{1-\sigma}} > \frac{\beta_2 (w-\bar{x})^{1-\sigma}}{\left[1+\beta_2^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}\right]^{1-\sigma}}$, and again $\frac{\dfrac{\beta_1}{\left[1+\beta_1^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}\right]^{1-\sigma}}}{\dfrac{\beta_2}{\left[1+\beta_2^{-\frac{1}{\sigma}} R^{1-\frac{1}{\sigma}}\right]^{1-\sigma}}} > \left(\frac{w-\bar{x}}{w}\right)^{1-\sigma}$. We know that the left-hand side of the previous expression is clearly less than unity since $\beta_1 < \beta_2$, but the right-hand side gets closer to unity the larger the wage rate is. Thus for large wage rates we must have $\beta_1 s^1 u'(c_1^{\beta 1}) < \beta_2 s^2 u'(c_1^{\beta 2})$. If the utility function is logarithmic we get that $\beta_1 s^1 u'(c_1^{\beta 1}) - \beta_2 s^2 u'(c_1^{\beta 2}) = (\beta_1 - \beta_2)/R$, which is clearly negative. Q.E.D.

We have given conditions for describing the behavior of the consumer in a situation, where he must decide, whether to invest in patience or not. Figure 2 describes an indifference curve. It follows from Proposition 1 and Lemma 2 that the area above the curve is the one, where consumer wants to invest in patience.

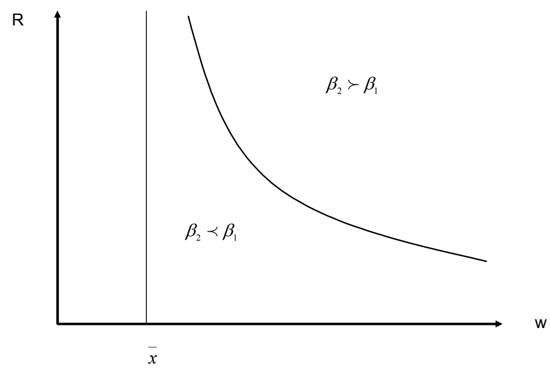

Figure 1.

Next we describe the production side of our model. The firms have a constant returns to scale technology, $F(K_t, L_t)$, to transform capital (K_t) and labor (L_t) into output. There is no capital depreciation. This technology can be expressed in factor intensive form to give $F(K_t, L_t)/L_t = f(k_t)$, where k_t (= K_t/L_t) is the per capita level of capital. The per capita production function has the standard properties: $f' > 0$ and $f'' < 0$. Furthermore, we assume $\lim_{k \to 0} f'(k_t) = \infty$ and $\lim_{k \to \infty} f'(k_t) = 0$.

3. Equilibrium and development trap

Next we define the competitive equilibrium.

Definition. A sequence of a price system and a feasible allocation,

$\left\{R_t, w_t, c_1^t, c_2^{t-1}, k_t\right\}_{t=1}^{\infty}$ is a competitive equilibrium, if

(i) given the price system, consumers and firms solve their decision problems and

(ii) markets clear for all t = 1,2,...,T,...

Market clearing conditions are

(11a) $\quad c_1^t + c_2^{t-1} + x_t = f(k_t)$

(11b) $s_t = k_{t+1}$

(11c) $f'(k_t) = r_t$

(11d) $f(k_t) - k_t f'(k_t) = w_t$

(11a) is the resource constraint for all t, and (11b) is the asset market clearing condition, where today's savings of the young are used as capital in the next period. Equations (11c) and (11d) in turn are the marginal productivity conditions, and they determine the evolution of factor prices, r_t and w_t.

Using the market clearing conditions (11b), (11c) and (11d), and the saving function we can express the fundamental dynamical equation as follows

(12) $$k_{t+1} = s\left[1 + f'(k_{t+1}), f(k_t) - k_t f'(k_t); \beta_i\right].$$

This means that equilibria can also be described as a sequence of capital stocks, $\{k_s\}_{s=0}^{\infty}$, such that (12) holds. When the discount factor is constant, and there is no investment in patience, the properties of (12) are well known since Diamond (1965).

To isolate the effects of patience investment in the most transparent way, we assume that the dynamics of (12) without patience investment is such that there is one non-trivial stable stationary equilibrium.[5] E.g. Cobb-Douglas technology with constant returns to scale and logarithmic preferences will result in such an equilibrium. We also assume that dynamics with patience investment is "standard". Here we should, however, note that there are two nontrivial steady states. Both dynamics are described in Figures 2 and 3.

[5] For details of the Diamond model, and in particular the conditions for a unique non-trivial steady state equilibrium, see de la Croix and Michel (2002), ch. 1.

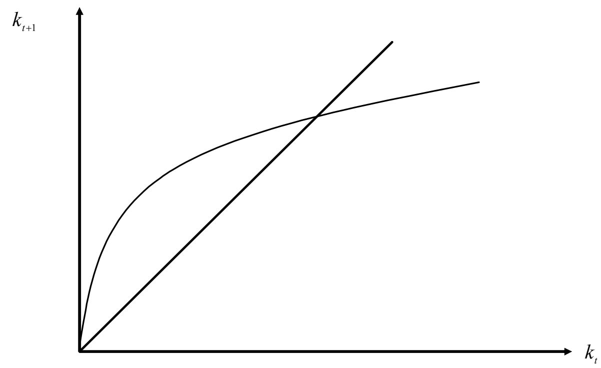

Figure 2.

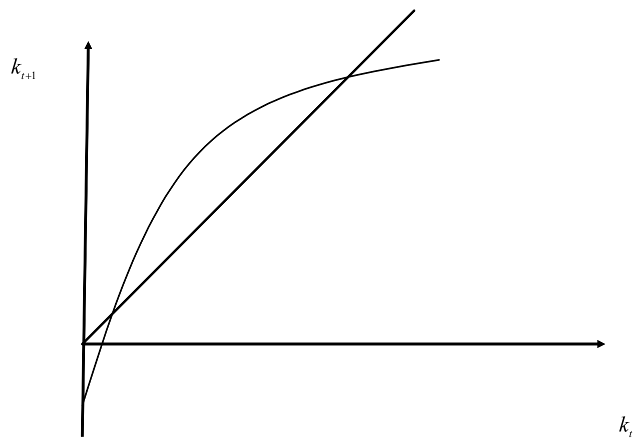

Figure 3.

Equations (11c) and (11d) define the factor price frontier, which, as is well known, is downward sloping. This means that we can describe the indifference relation, $\Delta(w_t, R_{t+1}) = 0$, also as a relation between the capital stocks, k_t and k_{t+1}. It follows from the factor price frontier that this relation is also downward sloping. We denote this relation by $\Gamma(k_t, k_{t+1}) = 0$. For a given level of k_{t+1}, k_t is bigger, and thus is the wage rate. According to Proposition 1 and Lemma 2 consumer prefers to invest in patience above the curve.

Azariadis (1996, p.450-451) defines poverty traps as "…nonergodic equilibrium growth paths that contain several attractors, e.g. steady states, balanced growth paths, or asymptotic distributions of world income. We call the lowest of these attractors a "poverty trap" or a "low-level development trap"." In our model the simplest case of a poverty trap is a situation, where there are two stable steady states, e.g. as those described in Figure 4. $\underline{k}$ is the level of capital, which corresponds to the wage level $\underline{w}$. Both stationary equilibria, k_1^s and k_2^s, are stable. If $k_0 < \underline{k}$, the economy ends up in the lower equilibrium (development trap), and if $k_0 \geq \underline{k}$ the economy converges toward an equilibrium with a higher level of capital stock, k_2^s.

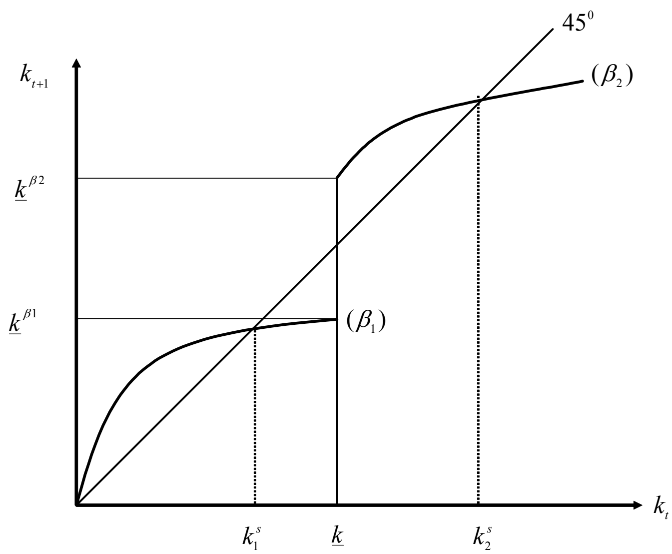

Figure 4.

Next we elaborate conditions under which we can get the situation described in Figure 4. It is obvious that the indifference curve, $\Gamma(k_t, k_{t+1}) = 0$, must lie between the steady states described in Figure 4. E.g. if we have a situation, where indifference curve cuts the curve describing dynamics under patience investment (denoted by (β_2) above the steady state, k_2^s, we cannot have a poverty trap, since the only steady state in the economy will then be k_1^s.

We describe a poverty trap in Figure 5. Denote by $\hat{k}$ a level of capital stock such that $\Gamma(\hat{k}, \hat{k}) = 0$. From Figure 5 we see that to get a poverty trap we need to have $k_1^s < \hat{k} < k_2^s$. By inspecting the Figure we note that $\Gamma(k_t^{''}, k_{t+1}^{''\beta 1}) = \Gamma(k_t^{'}, k_{t+1}^{'\beta 2}) = 0$. If the initial capital stock, k_0, fulfills $k_0 < k_t^{'}$, consumer does not want to invest anything in patience. If $k_0 > k_t^{''}$, consumer wants to invest in patience. What happens, if $k_t^{'} < k_0 < k_t^{''}$? From Figure 5 we conclude that $\Gamma(k_t^{''}, k_{t+1}^{''\beta 2}) > 0$ and $\Gamma(k_t^{'}, k_{t+1}^{'\beta 1}) < 0$. This, in particular, means that by continuity of the utility function there must be a three tuple, $(\underline{k}, \underline{k}^{\beta 1}, \underline{k}^{\beta 2})$, such that $k_t^{'} < \underline{k} < k_t^{''}$ and

$\Gamma(\underline{k},\underline{k}^{\beta 1}) = \Gamma(\underline{k},\underline{k}^{\beta 2})$. The allocations, which are indifferent, are perhaps described more clearly in Figure 4 above.

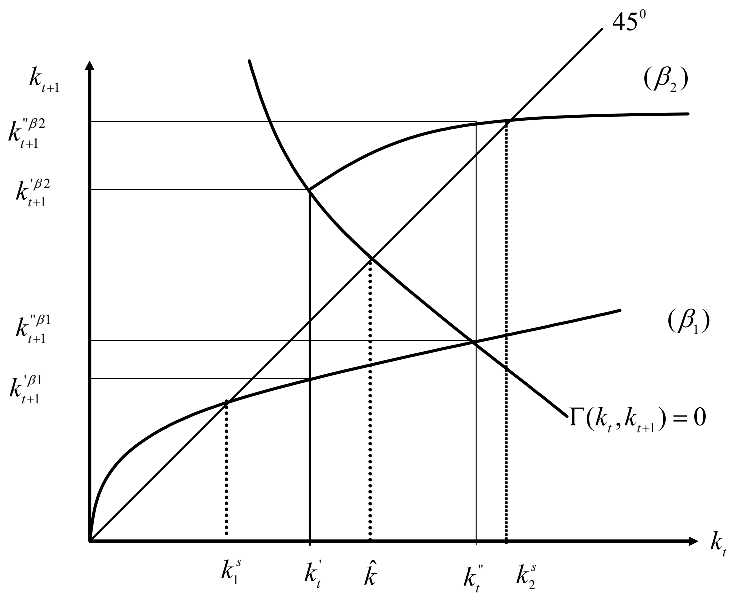

Figure 5.

4. Multiple equilibria and development trap: An example

To investigate equilibria more closely we now turn to the following example: $u(c) = \ln(c)$, and $f(k) = Ak^{\alpha}$, where $1 > \beta_2 > \beta_1 > 0$, $1 > \alpha > 0$ and $A > 0$. The saving functions are

(13) $$s(R_{t+1}, w_t; \beta_1) = \frac{\beta_1}{1+\beta_1} w_t \text{, when } \beta = \beta_1$$

(14) $$s(R_{t+1}, w_t; \beta_2) = \frac{\beta_2}{1+\beta_2}(w_t - \bar{x}) \text{, when } \beta = \beta_2 .$$

Note that when discount factors increase, saving will go up.

They lead to the following equilibrium dynamics for the capital stock

(15) $$k_{t+1} = \frac{\beta_1}{1+\beta_1}(1-\alpha)Ak_t^{\alpha} \text{, when } \beta = \beta_1$$

(16) $$k_{t+1} = \frac{\beta_2}{1+\beta_2}(1-\alpha)Ak_t^{\alpha} - \frac{\beta_2}{1+\beta_2}\bar{x} \text{, when } \beta = \beta_2 .$$

Parts of equations (15) and (16) are described in Figure 4 above. Given the equilibrium sequence of capital stocks, $\{k_t\}_{t=1}^{\infty}$ with a given level of k_0, we can calculate the resulting utility levels under a particular equilibrium sequence. We will be looking for a particular combination, $\{k_t, k_{t+1}\}$, of capital stocks (i.e. also a particular combination of the wage level, w_t, and the interest factor, R_{t+1}) such that consumer is indifferent between investing nothing in patience and investing the amount $\bar{x}$.

We take into account the periodic budget constraints: $c_1^t + s_t + x_t = w_t$, $c_2^t = R_{t+1}s_t$, the equilibrium relation, $s_t = k_{t+1}$, and the pricing relations: $Af'(k_t) = r_t$ and $Af(k_t) - Ak_t f'(k_t) = w_t$. Given the functional forms the pricing relations yield: $\alpha A k_t^{\alpha-1} = r_t$ and $(1-\alpha)Ak_t^{\alpha} = w_t$. If combinations $\{\underline{k}, \underline{k}^{\beta 1}\}$ and $\{\underline{k}, \underline{k}^{\beta 2}\}$ (consult Figure 4 above) are such that the consumer is indifferent, then those points are the solutions for the following three equations:

(17)

$$\ln\left[(1-\alpha)A\underline{k}^{\alpha} - \underline{k}^{\beta 1}\right] + \beta_1 \ln\left[\underline{k}^{\beta 1} + \alpha A(\underline{k}^{\beta 1})^{\alpha}\right] = \ln\left[(1-\alpha)A\underline{k}^{\alpha} - \underline{k}^{\beta 2} - \bar{x}\right] + \beta_2 \ln\left[\underline{k}^{\beta 2} + \alpha A(\underline{k}^{\beta 2})^{\alpha}\right]$$

(18) $$\underline{k}^{\beta 2} = \frac{\beta_2}{1+\beta_2}(1-\alpha)A\underline{k}^{\alpha} - \frac{\beta_2}{1+\beta_2}\bar{x}$$

(19) $$\underline{k}^{\beta 1} = \frac{\beta_1}{1+\beta_1}(1-\alpha)A\underline{k}^{\alpha}.$$

Equation (17) gives the indifference of investing and not investing in patience. (18) and (19) indicate that we are picking the capital points from the appropriate curves (see Figure 4).

We use the following numbers to perform the numerical calculation: $\beta_1 = 1/2$, $\beta_2 = 3/4$, $\alpha = 1/3$, $\bar{x} = 0.01$ and $A = 2.7$. We get the following capital stocks as solutions: $\underline{k}^{\beta 1} = .495445$, $\underline{k}^{\beta 2} = .632715$, and $\underline{k} = .56303$. The steady states calculated for both cases are: when $\beta = 1/2$, $k_1^s = .464758$, and when $\beta = 3/4$, $k_2^s = .671115$ (only the larger steady state is reported).

5. Conclusions

To be added.

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