Monetary Policy with Sectoral Linkages and Durable Goods*

Ivan Petrella[†]
Birkbeck, University of London

Raffaele Rossi[‡]
Lancaster University

Emiliano Santoro[§]
Catholic University of Milan
University of Copenhagen

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Abstract

We study the normative implications of a New Keynesian model featuring intersectoral trade of intermediate goods between two sectors that produce durables and non-durables. The interplay between durability and sectoral production linkages fundamentally alters the intersectoral stabilization trade-off as it emerges in otherwise standard two-sector models. We compare the welfare properties of a timeless-perspective monetary policy with the performance of simple instrumental rules that adjust the policy rate in response to the output gap and alternative aggregate measures of final goods price inflation. Aggregating durable and non-durable inflation depending on the relative degrees of sectoral price stickiness may induce a severe bias. Input materials attenuate the response of sectoral inflations to movements in the real marginal costs, so that the effective slopes of the sectoral supply schedules are not properly accounted for by conventional measures of core inflation.

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[†]Department of Economics, Mathematics and Statistics, Birkbeck, University of London. *Address*: Malet St, London WC1E 7HX, UK. *E-mail*: i.petrella@bbk.ac.uk.

[‡]Department of Economics, Lancaster Management School, Lancaster University. *Address*: Lancaster LA1 4YX, UK. *E-mail*: r.rossi@lancaster.ac.uk.

[§]ITEMQ, Catholic University of Milan and Department of Economics, University of Copenhagen. Address: Via Necchi 5, 20123 Milan, Italy. *E-mail*: emiliano.santoro@unicatt.it.

1 Introduction

Along with major differences in the time span over which they yield utility, durable and non-durable consumption goods are characterized by deep peculiarities in their production and price-setting. These structural traits are paramount to the monetary transmission mechanism and need to be accounted for in the normative analysis of multi-sector economies.[1] This paper deals with monetary policy-making in a New Keynesian model with two sectors that produce durable and non-durable goods. The key departure from the existing literature is to allow for the presence of factor demand linkages between sectors: gross output in each sector serves either as a final consumption good, or as an intermediate input in both sectors.

Intersectoral trade of intermediate inputs is a prominent feature of modern industrialized economies.[2] As such, sectoral production linkages should be seen as essential building blocks of multi-sector business cycle models that aim at generating realistic degrees of sectoral output volatility and co-movement.[3] In fact, it is well known that sticky-price models incorporating sectoral heterogeneity in price stickiness – usually in the form of sticky non-durable goods prices and flexible prices of durables – cannot generate positive sectoral co-movement in the face of monetary policy innovations (Barsky et al., 2007). Despite sectoral production linkages have been proposed as a remedy to the lack of co-movement (Bouakez et al., 2012; Sudo, 2012; Di Pace, 2012), the normative literature has generally neglected their importance.

In the economy under examination the monetary authority cannot attain the Pareto optimal allocation consistent with the full stabilization of sectoral productions and in-

[1] Bouakez et al. (2009a, 2009b) have shown that heterogeneity in price rigidity is the most important factor to understand the cross sectional heterogeneity in sectoral inflation responses to monetary policy shocks, while the most relevant characteristic to explain sectoral output responses is whether the sector produces a durable good.

[2] The U.S. input-output accounts compiled by the Bureau of Labor Statistics (BLS) show that 70% of the material-input expenditures by the durables sector goes into goods produced by the non-durables sector. The converse proportion is around 10%, which is much smaller but still not negligible.

[3] Horvath (1998, 2000) and Carvalho (2009) show that cross-industry flows of input materials can reinforce the effect of sectoral shocks, generating aggregate fluctuations and co-movement between sectors, as originally hinted by Long and Plosser (1983). Kim and Kim (2006) show that a similar mechanism generates widespread co-movement of sectoral economic activity. See also Hornstein and Praschnik (1997).

flation rates, even when distortions in the labor market (imperfect labor mobility) and the goods market (monopolistic competition) are removed. Thus, we turn our attention to policy strategies capable of attaining second best outcomes. We start by exploring equilibrium dynamics under the assumption that the policy maker can credibly commit to a rule derived from the minimization of his objective function. To this end, we derive an appropriate welfare metric through a quadratic approximation to the utility function of the representative household (Woodford, 2003). The interplay between durability and factor demand linkages has major implications for intersectoral stabilization. Due to the near constancy of their shadow value (Barsky et al., 2007), durable goods imply that changes in their relative price are instantly passed onto the marginal utility from non-durable consumption. Therefore, a drop in the relative price of durables almost entirely reflects into a rise in non-durable consumption, for sufficiently low depreciation rates. Concurrently, sectoral production linkages are responsible for expanding the production of non-durables, while depressing the production of durables. Introducing asymmetric price stickiness – in the form of relatively more flexible durable goods prices – increases the relative price volatility and the (intrinsically higher) volatility of durables, thus posing an even tougher challenge to intersectoral stabilization.

We study the transmission of sectoral shocks to both technology and markup pricing under timeless-perspective commitment (Woodford, 1999, 2003). Factor demand linkages imply that the price of non-durables in terms of durables not only affects their marginal rate of substitution, but also exerts a direct impact on the sectoral real marginal costs. The capacity of this channel depends on the off-diagonal elements in the input-output matrix. Consequently, a shock to the technology of one sector also affects the other sector's potential output and consumption, even if preferences over different types of consumption goods are separable. This feature of the model with sectoral linkages has major implications for monetary policy-making. Consider a technology shock to non-durable production. In a model without input materials keeping the consumption of non-durables at potential requires a sharp and persistent fall in the real interest rate, while closing the durable consumption gap calls for a sharp rise in the policy rate. The latter incentive prevails in Erceg and Levin (2006). By contrast, in our variant economy with input materials the monetary authority relies on the endogenous stabilization operating through intersectoral trade of intermediate goods, so that it can accommodate the contraction in non-durable consumption. The intermediate input channel also modifies the transmission of (positive) sectoral cost-push shocks in two main respects: (i) first, the deflationary effect in the sector which is not hit by the shock is attenuated; (ii) second, the drop in the production of both sectors is amplified. Both features of the model imply marked differences in the policy response, as compared with models that neglect the presence of sectoral linkages.

To conclude, we compare the welfare properties of simple monetary policy rules to the optimal policy under timeless-perspective commitment. While the welfare criterion derived from consumers' utility involves sector-specific variables, we consider interest rate rules that adjust the policy rate in response to aggregate measures of real activity and inflation. The response coefficients are computed so as to minimize the welfare metric consistent with households' utility. Aggregate inflation is obtained as a weighted average of sectoral inflations, with the weights depending on the relative size of each sector in the model economy. When dealing with asymmetric stickiness, we also consider the possibility of reacting to an index of core inflation that weighs the sectoral rates of inflation depending on both the relative size of each sector and the relative degree of nominal rigidity in price-setting. Notably, a rule that responds to aggregate inflation always outperforms core inflation targeting in a model with intermediate goods. The key to resolve this puzzle is that in the presence of input materials the impact of the real marginal cost on current inflation is attenuated – relative to model economies with no input materials – and more so when cross-industry flows of intermediate goods are in place. Therefore, even when a sector is characterized by a lower degree of nominal rigidity in price-setting, the effective slope of its supply schedule depends on the presence of input materials and the entries of the input-output matrix. When the price of durables is relatively flexible, the resulting index of core inflation attaches too much importance to non-durable inflation.

The studies available to date have shown that sectoral heterogeneity presents the monetary policy authority with a clear challenge: with only a single instrument, the Central Bank cannot replicate the equilibrium allocation under flexible prices (see, e.g., Aoki, 2001). According to our study monetary policy-making in multi-sector economies should necessarily account for sectoral heterogeneity along three dimensions that appear to be deeply integrated: durability, nominal rigidity in price-setting and factor demand linkages. Our key insight is that a realistic blend of these ingredients dramatically affects the policy maker's perspective on the intersectoral stabilization trade-off.

The remainder of the paper is laid out as follows: Section 2 introduces the theoretical setting; Section 3 reports the calibration of the model economy; Section 4 discusses the implementation of the optimal monetary policy under timeless-perspective commitment; Section 5 examines the stabilization properties of alternative instrumental policy rules. Section 6 concludes.

2 The Model

We develop a DSGE model with two sectors that produce durable and non-durable goods, respectively. The model economy is populated by a large number of infinitely-lived households. Each of them is endowed with one unit of time and derives utility from the consumption of durable goods, non-durable goods and leisure. The two sectors of production are connected through factor demand linkages.[4] Goods produced in each sector serve either as a final consumption good, or as an intermediate production input in both sectors. The net flow of intermediate goods between sectors depends on the input-output structure of the production side.

[4]Throughout the paper we will refer to 'factor demand linkages' as indicating cross-industry flows of input materials. Should a specific feature of the model economy be essentially determined by the use of intermediate goods in the production process (i.e., inter-sectoral relationships are not essential), we will explicitly refer to 'input materials'.

2.1 Consumers

Households derive income from supplying labor to the production sectors, investing in bonds, and from the stream of profits generated in the production sectors. Their consumption preferences are defined over H_t – a composite of non-durable goods (C_t^n) and an "effective" stock of durable goods ($\mathfrak{D}_t$) – as well as labor, L_t. They maximize the expected present discounted value of their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(H_t, L_t\right), \tag{1}$$

where $H_t = (C_t^n)^{\mu_n} \mathfrak{D}_t^{\mu_d}$, μ_n and $\mu_d (= 1 - \mu_n)$ denote the expenditure shares on non-durable and durable goods and β is the discount factor. We assume that the representative household's period utility function takes the form:

$$U\left(H_t, L_t\right) = \frac{H_t^{1-\sigma}}{1-\sigma} - \varrho \frac{L_t^{1+v}}{1+v}; \quad \varrho > 0 \tag{2}$$

where σ is the inverse of the intertemporal elasticity of substitution and v is the inverse of the Frisch elasticity of labor supply. Durable goods are accumulated according to the following law of motion:

$$D_t = C_t^d + (1-\delta) D_{t-1}, \tag{3}$$

where δ is the depreciation factor. The effective stock of durables scales the effect of a quadratic cost of adjustment (see, e.g., Bernanke, 1985):[5]

$$\mathfrak{D}_t = D_t - \frac{\Xi}{2} \frac{\left(D_t - D_{t-1}\right)^2}{D}, \quad \Xi \geq 0, \tag{4}$$

where D denotes the steady state stock of durable consumption goods.

[5] Including an adjustment cost of the stock of durables allows us to obtain results in line with the empirical evidence on the behavior of durable consumption over the business cycle. King and Thomas (2006) show how the partial adjustment mechanism helps at accounting for the aggregate effects of discrete and occasional changes in durables consumption at the microeconomic level. Adda and Cooper (2000) provide evidence on the discrete nature of durable expenditure at the individual level.

We assume that labor can be either supplied to sector n or d according to a CES aggregator:

$$L_t = \left[\phi^{-\frac{1}{\lambda}} \left(L_t^n\right)^{\frac{1+\lambda}{\lambda}} + (1-\phi)^{-\frac{1}{\lambda}} \left(L_t^d\right)^{\frac{1+\lambda}{\lambda}}\right]^{\frac{\lambda}{1+\lambda}}, \tag{5}$$

where λ denotes the elasticity of substitution in labor supply, and ϕ is the steady state ratio of labor supply in the non-durable goods sector over total labor supply (i.e., $\phi = L^n/L$). This functional form conveniently allows us to account for different degrees of labor mobility between sectors, depending on λ.[6] For $\lambda = 0$ labor is prevented from moving across sectors. For $\lambda \to \infty$ workers devote all time to the sector paying the highest wage. Hence, at the margin, all sectors pay the same hourly wage and perfect labor mobility is attained. For $\lambda < \infty$ hours worked are not perfect substitutes. An interpretation of this is that workers have a preference for diversity of labor and would prefer working closer to an equal number of hours in each sector even in the presence of wage differences across sectors.[7] An important difference between (5) and the CES aggregator used by Horvath (2000) is that the former allows us to neutralize the impact of labor market frictions in the steady state.

The following sequence of (nominal) budget constraints applies:

$$\sum_{i=\{n,d\}} P_t^i C_t^i + B_t = R_{t-1} B_{t-1} + \sum_{i=\{n,d\}} W_t^i L_t^i + \sum_{i=\{n,d\}} \Psi_t^i - T_t, \tag{6}$$

where B_t denotes a one-period risk-free nominal bond remunerated at the gross risk-free rate R_t, W_t^i denotes the nominal wage rate in sector $i = \{n, d\}$ and T_t denotes a lump-sum tax paid to the government. The term $\Psi_t^n + \Psi_t^d$ captures the nominal flow of dividends from both sectors of production.[8]

[6] The available evidence suggests that labor and capital are not perfectly mobile across sectors. Davis and Haltiwanger (2001) find limited labor mobility across sectors in response to monetary and oil shocks. Bouakez, Cardia, and Ruge-Murcia (2008) report evidence suggesting that perfect labor mobility across sectors, with its implication that sectoral nominal wages are the same (at the margin), is an imperfect characterization of the data.

[7] Horvath (2000) motivates a similar specification based on the desire to capture some degree of sector-specificity to labor while not deviating from the representative consumer/worker assumption. In a similar vein, we conveniently employ this mechanism to allow for imperfect labor mobility between sectors.

[8] The first order conditions from consumers' optimization are available in Appendix A.

2.2 Producers

The production side of the economy consists of two distinct sectors producing durable (sector d) and non-durable goods (sector n). Each sector is composed of a continuum of firms producing differentiated products. Let Y_t^n (Y_t^d) denote gross output of the non-durable (durable) goods sector:

$$Y_t^i = \left[\int_0^1 \left(Y_{ft}^i \right)^{\frac{\varepsilon_t^i - 1}{\varepsilon_t^i}} df \right]^{\frac{\varepsilon_t^i}{\varepsilon_t^i - 1}}, \qquad i = \{n, d\} \tag{7}$$

where ε_t^i denotes the time-varying elasticity of substitution between differentiated goods in the production composite of sector $i = \{n, d\}$. Each production composite is produced in the "aggregator" sector operating under perfect competition. The fth firm in sector i faces the following demand schedule:

$$Y_{ft}^i = \left(\frac{P_{ft}^i}{P_t^i} \right)^{-\varepsilon_t^i} Y_t^i, \qquad i = \{n, d\} \tag{8}$$

where P_t^i is the price of the composite good in the ith sector. From (7) and (8) the relationship between the firm-specific and the sector-specific price is:

$$P_t^i = \left[\int_0^1 \left(P_{ft}^i \right)^{1-\varepsilon_t^i} df \right]^{\frac{1}{1-\varepsilon_t^i}}, \qquad i = \{n, d\}. \tag{9}$$

Sectors are related by factor demand linkages. Part of the output of each sector serves as an intermediate input in both sectors. The allocation of output produced in the ith sector is such that:

$$Y_t^i = C_t^i + M_t^{in} + M_t^{id}, \qquad i = \{n, d\} \tag{10}$$

where C_t^i denotes the amount of consumption goods produced by sector i, while M_t^{in} (M_t^{id}) is the amount of goods produced in sector i and used as input materials in sector n (d).

The production technology of a generic firm f in sector i is:

$$Y_{ft}^{i} = Z_{t}^{i}\left[\frac{\left(M_{ft}^{ni}\right)^{\gamma_{ni}}\left(M_{ft}^{di}\right)^{\gamma_{di}}}{\gamma_{ni}^{\gamma_{ni}}\gamma_{di}^{\gamma_{di}}}\right]^{\alpha_i}\left(L_{ft}^{i}\right)^{1-\alpha_i}, \qquad i=\{n,d\} \tag{11}$$

where Z_t^i ($i=\{n,d\}$) is a sector-specific productivity shock, L_{ft}^i denotes the number of hours worked in the fth firm of sector i, M_{ft}^{ji} ($j=\{n,d\}$) denotes material inputs produced in sector j and supplied to firm f in sector i. Moreover, γ_{ij} ($i,j=\{n,d\}$) denotes the generic element of the (2×2) input-output matrix, Γ, and corresponds to the steady state share of total intermediate goods used in the production of sector j and supplied by sector i. The input-output matrix is normalized, so that the elements of each column sum up to one: $\sum_{j=\{n,d\}}\gamma_{jn}=1$ (and $\sum_{j=\{n,d\}}\gamma_{jd}=1$).

Material inputs are combined according to a CES aggregator:

$$M_{ft}^{ji} = \left[\int_0^1\left(M_{kf,t}^{ji}\right)^{\left(\varepsilon_t^j-1\right)/\varepsilon_t^j}dk\right]^{\varepsilon_t^j/\left(\varepsilon_t^j-1\right)}, \tag{12}$$

where $\left\{M_{kf,t}^{ji}\right\}_{k\in[0,1]}$ is a sequence of intermediate inputs produced in sector j by firm k, which are employed in the production process of firm f in sector i.

Firms in both sectors set prices given the demand functions reported in (8). They are also assumed to be able to adjust their price with probability $1-\theta_i$ in each period. When they are able to do so, they set the price that maximizes expected profits:

$$\max_{P_{ft}^i} E_t\sum_{s=0}^{\infty}(\beta\theta_i)^s\Omega_{t+s}\left[P_{ft+s}^i(1+\tau_i)-MC_{ft+s}^i\right]Y_{ft+s}^i, \qquad i=\{n,d\} \tag{13}$$

where Ω_{t+s} is the stochastic discount factor consistent with households' maximizing behavior, τ_i is a subsidy to producers in sector i, while MC_{fs}^i denotes the marginal cost of production of firm f in sector i. The optimal pricing choice, given the sequence $\left\{P_t^n, P_t^d, Y_t^n, Y_t^d\right\}$, reads as:

$$\overline{P}_{ft}^i = \frac{\varepsilon_t^i}{\left(\varepsilon_t^i-1\right)(1+\tau_i)}\frac{E_t\sum_{s=0}^{\infty}(\beta\theta_i)^s\Omega_{t+s}MC_{ft+s}^iY_{ft+s}^i}{E_t\sum_{s=0}^{\infty}(\beta\theta_i)^s\Omega_{t+s}Y_{ft+s}^i}, \qquad i=\{n,d\}. \tag{14}$$

Note that assuming time-varying elasticities of substitution translates into sectoral cost-push shocks that allow us to account for sector-specific shift parameters in the supply schedules.

In every period each firm solves a cost minimization problem to meet demand at its stated price. The first order conditions from this problem result in the following relationships:

$$MC_{ft}^{i}Y_{ft}^{i}=\frac{W_{t}^{i}L_{ft}^{i}}{1-\alpha_{i}}=\frac{P_{t}^{n}M_{ft}^{ni}}{\alpha_{i}\gamma_{ni}}=\frac{P_{t}^{d}M_{ft}^{di}}{\alpha_{i}\gamma_{di}},\qquad i=\{n,d\}\,. \tag{15}$$

It is useful to express the sectoral real marginal cost as a function of the relative price and the real wage prevailing in each sector $i,j=\{n,d\}$, $i\neq j$:

$$\frac{MC_{t}^{i}}{P_{t}^{i}}=\frac{\overline{\phi}^{i}\left(Q_{t}^{i}\right)^{\gamma_{ji}\alpha_{i}}\left(RW_{t}^{i}\right)^{1-\alpha_{i}}}{Z_{t}^{i}}, \tag{16}$$

where $\overline{\phi}^{i}$ is a convolution of the production parameters, $RW_{t}^{i}=W_{t}^{i}/P_{t}^{i}$ is the real wage in sector i and Q_{t}^{i} denotes the price of sector i relative to that of sector j. Since $Q_{t}^{n}=\left(Q_{t}^{d}\right)^{-1}$, in what follows we normalize so as to have a single relative price $Q_{t}=P_{t}^{n}/P_{t}^{d}$.

Equation (16) makes it clear that the relative price exerts a direct effect on the real marginal cost of each sector, whose magnitude depends on the size of the cross-industry flows of input materials. Specifically, for the ith sector the absolute impact of Q_{t} on MC_{t}^{i}/P_{t}^{i} is related to the "importance" of the other sector as input supplier, i.e. on the magnitude of the off-diagonal elements in the input-output matrix (γ_{nd} and γ_{dn}). This is a distinctive feature of the framework we deal with. By contrast, in traditional multi-sector models without factor demand linkages (e.g., Erceg and Levin, 2006), the relative price only affects the real marginal cost indirectly, through the marginal rate of substitution between different consumption goods.

2.3 Market Clearing

The allocation of output produced by each sector requires that sectoral gross output is partly sold on the markets for consumption goods, while a proportion is sold on the markets for input materials. Therefore, (10) must be met in each sector.

2.4 The Government and the Monetary Authority

The government serves two purposes in the economy. First, it delegates monetary policy to an independent Central Bank. In this respect, we initially assume that the short-term nominal interest rate is used as the instrument of monetary policy and the policy maker is able to pre-commit to a time-invariant rule. We then explore the welfare properties of interest rate rules whose reaction coefficients to output and inflation are computed so as to minimize a quadratic welfare function consistent with consumers' utility. The second task of the government consists of taxing households and providing subsidies to firms to eliminate distortions arising from monopolistic competition in the markets for both classes of consumption goods. This task is pursued via lump-sum taxes that maintain a balanced fiscal budget.

3 Solution and Calibration

To solve the model, we log-linearize behavioral equations and resource constraints around the non-stochastic steady state and take the percentage deviation from their counterparts under flexible prices. The difference between log-variables under sticky prices and their linearized steady state is denoted by the symbol "$\hat{}$", while we use "$*$" to denote percent deviations of variables in the efficient equilibrium (i.e., flexible prices and constant elasticities of substitution) from the corresponding steady state value. Finally, we use "$\tilde{}$" to denote the difference between linearized variables under sticky prices and their counterparts in the efficient equilibrium.[9]

The model is calibrated at a quarterly frequency. We set $\beta = 0.993$ and $\sigma = 2$. The expenditure share on non-durable goods, μ_n, is set to 0.682. The inverse of the Frisch elasticity of labor supply, v, is set to 3, while $\lambda = 1$, which implies limited labor mobility. As to the parameters characterizing the production technologies of the two sectors, we rely

[9] The steady state conditions are reported in Appendix B. We omit the time subscript to denote variables in the steady state. Appendix C presents the economy under flexible prices.

on Bouakez, Cardia, and Ruge-Murcia (2009b) and set $\alpha_n = 0.746$ and $\alpha_d = 0.643$. The entries of the input-output matrix are set in accordance with the input-use table of the US economy: $\gamma_{nn} = 0.899$ and $\gamma_{nd} = 0.688$.[10] These values imply a positive net flow of input materials from the non-durable goods sector to the durable goods sector. The depreciation rate of the stock of durables is assumed to be 2.5%, while $\Xi = 600$, as in Erceg and Levin (2006). We assume that sectoral elasticities of substitution have a steady state value equal to 11. At different stages of the analysis we allow for both symmetric and asymmetric degrees of nominal rigidity across sectors. In the symmetric case we set $\theta_n = \theta_d = 0.75$ (i.e., an average duration of four quarters). In the case of asymmetric price stickiness we set $\theta_n = 0.75$ and $\theta_d = 0.25$ (i.e., an average duration of 1.33 quarters). These values imply that durable prices are relatively more flexible, as suggested by Bils and Klenow (2004). This view is also supported by Bouakez, Cardia, and Ruge-Murcia (2009b), who construct and estimate a six-sector DSGE model of the US economy. While the null hypothesis of price flexibility can be rejected for non-durable manufacturing and services, it cannot be rejected for agriculture, mining, construction and durable manufacturing. Also Barsky, House, and Kimball (2007) suggest the possibility that some long-lived durables have relatively flexible prices. They show that inflation in the median (and average) price of new houses displays negative serial correlation, suggesting that these prices jump and indeed tend to overshoot. This is inconsistent with incomplete (partial) nominal adjustment, which implies that prices should undershoot.

As discussed above, the system features two sector-specific technology shocks, z_t^n and z_t^d. The cost-push shocks, η_t^n and η_t^d, are reduced-form expressions for the time-varying cost-shift parameters in the sectoral New Keynesian Phillips curves. Exogenous variables are assumed to follow a first-order stationary VAR with *iid* innovations and, unless we state otherwise, diagonal covariance matrix. We set the parameters capturing the persistence and variance of the productivity growth stochastic processes so that $\rho^{z^n} = \rho^{z^d} = 0.95$ and $\sigma^{z^n} = \sigma^{z^d} = 0.02$, respectively. These values are consistent with

[10] These shares have been computed using the table 'The Use of Commodities by Industries' for 1992 produced by the BLS. Sudo (2012) shows that the matrix is fairly stable over time. A key implication of taking these values is that the marginal impact of changes in the relative price on the real sectoral marginal cost is (in absolute value) greater for the durable goods sector, as $\alpha_n \gg \alpha_d$ and $\gamma_{nd} \gg \gamma_{dn}$.

the empirical evidence showing that technology shocks are generally small, but highly persistent (see Cooley and Prescott, 1995; Huang and Liu, 2005). As to the cost-push shocks, we follow Jensen (2002), Walsh (2003) and Strum (2009), assuming that these are purely transitory, with $\sigma^{\eta^n} = \sigma^{\eta^d} = 0.02$.

4 Monetary Policy

In the present context the Central Bank cannot attain the Pareto optimal allocation consistent with the full stabilization of inflation and the output gap in both sectors, even after distortions in the labor market (i.e., imperfect labor mobility) and the goods market (i.e., monopolistic competition) have been removed. Thus, we turn our attention to policy strategies capable of attaining second best outcomes. We first explore equilibrium dynamics under the assumption that the policy maker can credibly commit to a rule derived from the minimization of a utility-based welfare loss function. The optimal policy consists of maximizing the conditional expectation of intertemporal household utility subject to private sector's behavioral equations and resource constraints. A 'timeless perspective' approach is pursued (Woodford, 1999, 2003). This involves ignoring the conditions that prevail at the regime's inception, thus imagining that the commitment to apply the rules deriving from the optimization problem had been made in the distant past.[11] We then consider interest rate rules whose reaction coefficients to aggregate activity and alternative measures of overall price inflation are computed so as to minimize the loss of social welfare.

[11] Dennis (2010) shows that discretionary policy-making can be superior to timeless perspective policy-making when the supply schedule is relatively flat. Given that intermediate goods exert a marked influence on the slope of the supply schedule, Petrella, Rossi, and Santoro (2012) explore the relative performance of timeless perspective policy-making and discretion, showing that input materials enlarge the set of possible equilibrium outcomes in which discretion prevails.

4.1 The Welfare Criterion

To evaluate social welfare we take a second-order Taylor approximation to the representative household's lifetime utility.[12] Our procedure follows the standard analysis of Woodford (2003), adapted to account for the presence of factor demand linkages. The resulting intertemporal social loss function reads as:

$$
\begin{aligned}
\mathcal{SW}_0 \approx\ & -\frac{U_H(H)\,H}{2}\Theta E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma-1}{\Theta}\left(\mu_n \widetilde{c}_t^n + \mu_d \widetilde{d}_t\right)^2 \right. \\
& + \varsigma\left[\varpi\left(\pi_t^n\right)^2 + (1-\varpi)\left(\pi_t^d\right)^2\right] + (1+v)\left[\omega \widetilde{c}_t^n + (1-\omega)\widetilde{c}_t^d\right]^2 \\
& \left. + S\left(\widetilde{d}_t - \widetilde{d}_{t-1}\right)^2 \right\} + \text{t.i.p.} + O\left(\|\xi\|^3\right), \qquad (17)
\end{aligned}
$$

where:

$$S = \mu_d \Theta^{-1} \Xi + (1-\delta)(1-\omega)\delta^{-2}, \qquad (18)$$

$$\Theta = \frac{\mu_n\left[1-\beta(1-\delta)\right] + \mu_d \delta}{1-\beta(1-\delta)}, \qquad (19)$$

$$\varpi = \phi \varepsilon^n \left(\kappa_n \varsigma\right)^{-1}, \qquad (20)$$

$$\omega = \frac{\mu_n\left[1-\beta(1-\delta)\right]}{\mu_n\left[1-\beta(1-\delta)\right] + \mu_d \delta}, \qquad (21)$$

$$\varsigma = \phi \varepsilon^n \left(\kappa_n\right)^{-1} + (1-\phi)\varepsilon^d \left(\kappa_d\right)^{-1}, \qquad (22)$$

$$\kappa_i = \frac{(1-\beta\theta_i)(1-\theta_i)}{\theta_i}, \; i = \{n, d\}, \qquad (23)$$

and t.i.p. collects the terms independent of policy stabilization, whereas $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher.

The welfare criterion (17) balances, along with fluctuations in aggregate consumption

[12] We assume that the shocks that hit the economy are not big enough to lead to paths of the endogenous variables distant from their steady state levels. This means that shocks do not drive the economy too far from its approximation point and, therefore, a linear quadratic approximation to the policy problem leads to reasonably accurate solutions. Appendix F reports the derivation of the quadratic welfare function.

(or, equivalently, value added), sectoral inflation variability and a term that reflects a preference to smooth the accumulation of the stock of durable goods.[13] The weights of the time-varying terms in (17) can be interpreted as follows: (i) ς indexes the total degree of nominal stickiness in the economy and is inversely related to both κ_d and κ_n; (ii) ϖ accounts for the relative degree of price stickiness in the non-durable goods sector; (iii) ω is the relative weight of non-durable consumption over total consumption when durable goods are reported as a flow. This is an inverse function of Θ. In turn, the latter depends on the degree of durability of goods produced in sector d. For $\delta = 0$ it reduces to μ_n, whereas for $\delta = 1$ it equals one. Therefore, as the degree of durability increases, the weight attached to the non-durable consumption gap increases with respect to that attached to the durable term. Notice also that the relative importance of sector-specific inflation variability depends on the steady state ratio of labor supplied to the non-durable goods sector to the total labor force (ϕ).[14]

Note that for specific assumptions about the parameters, the welfare criterion nests popular specifications in the New Keynesian literature. When input materials are not employed in the production process (i.e., $\alpha_n = \alpha_d = 0$) the loss function reduces to that obtained in traditional two-sector models where consumption and gross output are equalized (e.g., Erceg and Levin, 2006). If we also set $\delta = 1$ and $\Xi = 0$ we end up in the case considered by Woodford (2003, pp. 435-443).[15]

4.2 Durability, Sectoral Linkages and the Intersectoral Stabilization Trade-off

This section aims at uncovering the nature of the trade-off the Central Bank faces in the stabilization of sectoral output gaps and inflation rates. Figure 1 reports the loss of welfare

[13] Further details on the linear approximation of this term are available in the technical appendix. Assuming durables accumulation smoothing as a stabilization objective should help at counteracting the amplification effect of changes in the stock demand of durables on the flow demand of newly produced durable goods. However, as discussed by Erceg and Levin (2006) this term makes a relatively minor contribution to the overall loss. To see this, consider that $S\Delta\widetilde{d}_t^2 \simeq \left[\mu_d \Theta^{-1} \Xi \delta^2 + (1-\delta)(1-\omega)\right] \left(\widetilde{c}_t^d\right)^2$.

[14] For $\alpha_n = \alpha_d$ we obtain $\phi = \frac{L^n}{L} = \frac{Y^n}{Y^n + Y^d}$.

[15] Needless to say that assuming perfectly correlated shocks would make the two-sector model observationally equivalent to the standard (single sector) New Keynesian model.

under alternative benchmark models as a function of the degree of labor mobility.[16] In a similar setting Petrella and Santoro (2011) show that even under perfect labor mobility (i.e., $\lambda \to \infty$) the Central Bank may not attain the Pareto optimal allocation consistent with the full stabilization of both sectors, unless the technology shock buffeting one sector equals the other one, scaled by the ratio between the sectoral income shares of input materials. In the present context the condition necessary to attain full stabilization is:

$$\Delta z_t^n = \frac{1-\alpha_n}{1-\alpha_d}\Delta z_t^d. \tag{24}$$

The left-hand panel of Figure 1 considers a situation with perfectly correlated sectoral shocks. As as predicted by (24) a symmetric production structure (i.e., $\alpha_n = \alpha_d$) always ensures full stabilization under perfect labor mobility, even in the presence of durability. Moreover, full stabilization is attainable even at low values of λ in the presence of no durables (i.e., $\delta = 1$). Therefore, as implied by Erceg and Levin (2006), durability inevitably amplifies the loss of welfare in the presence of limited labor mobility, even if the relative price remains at its steady state level by virtue of $z_t^n = z_t^d$. Note also that the loss of welfare is attenuated when $\lambda < \infty$ and input materials are employed by symmetric production technologies, as compared with the case of $\alpha_n = \alpha_d = 0$. The mitigation induced by intermediate goods on the intersectoral trade-off is more evident at low values of λ: the underlying intuition is that in the model with input materials firms have the chance to adjust the mix of their production inputs even if labor cannot move across sectors. This option is *a priori* ruled out when input materials are not employed in the production or they feature asymmetric income shares, in which cases a higher loss of social welfare is produced.

Insert Figure 1 here

Let us now shift our focus on a situation with uncorrelated technology disturbances, which is portrayed in the right-hand panel of Figure 1. As we know from (24) full sta-

[16] The monetary authority is assuemed to implement a timeless-perspective commitment policy. Moreover, we temporarily rule out sectoral cost-push shocks.

bilization can never be attained in this case, even if the two sectors feature the same production structure and labor can move between sectors so as to offset discrepancies between nominal wage rates. Note that perfect labor mobility tends to offset welfare discrepancies among model economies that employ intermediate goods in a symmetric fashion. Otherwise, in the baseline calibration with $\alpha_n \neq \alpha_d$ the loss of welfare is substantially higher that the alternative scenarios, even for $\lambda \to \infty$. Nevertheless, we should note that durability tends to attenuate the loss of welfare, no matter whether input materials are employed in the same proportion across different sectors or they are excluded from the set of production inputs. To provide some intuition on this result we assume, without loss of generality, $\Xi = 0$ and re-write the Euler equation for durable consumption in a more compact form by applying repeated forward substitution:

$$\frac{U_{C_t^n}}{Q_t} = \sum_{j=0}^{\infty} (1-\delta)^j \beta^j E_t \left[U_{D_{t+j}} \right], \tag{25}$$

where $U_{C_t^n}$ (U_{D_t}) is the marginal utility with respect to non-durable (durable) consumption. Barsky, House, and Kimball (2007) note that in the case of durables with low depreciation rates, the right-hand side of (25) is heavily influenced by the marginal utilities of durable service flows in the distant future. When shocks hitting the economy are temporary, the forward-looking terms do not deviate from their steady-state values, and so even significant variation in the first few terms only have a small impact on the present value. This means that the present value is close-to-invariant, even in the face of substantial temporary movements in U_{D_t}.[17] Given that the right hand-side of (25) remains fairly constant, any variation in the relative price instantly impacts on the marginal utility of non-durable consumption: this is exactly what happens under $\delta = 0.025$. In this case, uncorrelated technology disturbances induce the relative price Q_t to deviate from its steady state level, so that $U_{C_t^n}$ fluctuates accordingly. Thus, from a policy-making standpoint stabilizing the relative price is equivalent to stabilizing non-durable expenditure and vice

[17]This approximation is equivalent to saying that the demand for durable goods displays an almost infinite elasticity of intertemporal substitution. Even a small drop in the relative price of durables today relative to tomorrow would cause people to delay their purchases.

versa. Otherwise, for $\delta \to 1$ the stock-flow ratio for durables increases and both $U_{C_t^n}$ and U_{D_t} vary in the face of movements in Q_t, so that it is impossible for the policy maker to jointly stabilize the marginal utilities of different consumption goods and their relative price.[18]

The last insight into the model with uncorrelated disturbances is that input materials amplify the loss welfare, regardless of the degree of durability of goods produced by sector "d". This is because uncorrelated disturbances induce fluctuations in Q_t that make the sectoral real marginal costs move in opposite directions, thus exacerbating the intersectoral stabilization trade-off. This effect is even more evident in the case of asymmetric production technologies (i.e., $\alpha_n \neq \alpha_d$).[19] Note that assuming relatively more flexible prices in the durable goods sector would induce even stronger volatility in the relative price, an element that may pose a further challenge to the policy maker. Overall, the interplay between durability and sectoral production linkages deeply alters the nature of the conventional trade-off between stabilizing sectoral inflation rates and production gaps that has been studied by Aoki (2001) and Woodford (2003, pp. 435-443) among others.

4.3 Impulse-response Analysis

To isolate the contribution of factor demand linkages to the transmission of shocks under the optimal policy, we compare our baseline setting with a model that neglects the presence of input materials. As such, the benchmark scenario is akin to that considered by Erceg and Levin (2006). Figure 2 reports equilibrium dynamics following a one-standard-deviation technology shock in the non-durable goods sector, under different assumptions about the production structure.[20] All variables but the interest rate are reported in percentage deviation from their frictionless level. Symmetric nominal rigidity is assumed,

[18] Recall that the relative price does not fluctuate in the presence of perfectly correlated sectoral shocks. Thus, for $\delta = 1$ the marginal utilities accruing from each type of consumption good are always equalized and full stabilization can always be attained, even under limited labor mobility.

[19] In fact, asymmetric production technologies always imply the highest loss of welfare, regardless of the correlation structure between sectoral shocks.

[20] The responses to sectoral innovations in the durable goods sector are reported in Appendix H.

with $\theta_n = \theta_d = 0.75$.[21]

Insert Figure 2 here

A technology shock in the non-durable goods sector causes production of these goods to become relatively cheaper, thus increasing their production and consumption. However, their price is prevented from reaching the level consistent with flexible prices. This determines a negative consumption gap of non-durables. As to the response of the Central Bank, the two models return opposite prescriptions: while in the economy without input materials the nominal rate of interest increases so as to counteract the sharp rise in non-durable expenditure, in the model with factor demand linkages the policy response aims at accommodating the contraction in non-durable consumption. As discussed by Erceg and Levin (2006), keeping the consumption of non-durables at potential requires a "sharp and persistent fall" in the real interest rate. By contrast, a sharp rise in the policy instrument is required to close the consumption gap of durable goods. Although the latter incentive prevails in both model economies, in the model with sectoral linkages the real interest rate response is attenuated. This result is intimately related to the existence of factor demand linkages, which amplify the response of non-durable consumption under flexible prices, thus inducing a greater drop in the consumption gap. In addition, cross-industry flows of input materials amplify the rise of durable consumption under flexible prices, as compared with the benchmark model without input materials. The latter represents an endogenous mechanism of adjustment that helps at closing the consumption gap of durables.[22]

It is worth recalling that, in the presence of input-output interactions between sectors, the relative price does not only have a direct effect on the marginal rate of substitution between durable and non-durable consumption goods. As shown by equation (16), Q_t

[21] As in Strum (2009) we opt for this choice to prevent the Central Bank from focusing exclusively on the stickier sector in the formulation of its optimal policy, as predicted by Aoki (2001). In the next section we draw some policy implications from the model under asymmetric degrees of nominal rigidity between sectors.

[22] This is an endogenous channel of adjustment that only operates in the model with factor demand linkages, no matter the value of the coefficient of relative risk aversion. To see this, consider a model with $\alpha_n = \alpha_d = 0$ and $\sigma = 1$: in this case durable consumption under flexible prices would not fluctuate following a shock to non-durables technology, due to the separability of households' preferences over durable and non-durable consumption.

also exerts a positive (negative) effect on the real marginal cost in the durable (non-durable) goods sector. A technology shock in the non-durable goods sector determines a positive relative price gap, which implies a substitution away from non-durable to durable consumption goods. Concurrently, the intermediate input channel is responsible for attenuating deflationary pressures in the non-durable goods sector, while inducing higher durable inflation, compared with the model without input materials. The contraction in non-durable gross output is partially offset by the increase in the demand of non-durable intermediate goods from firms in the durable goods sector, which eventually results in lower deflationary pressures on the price of non-durables. Similarly, stronger inflationary pressures in the durable goods sector are induced by a production gap which is greater than that obtained by setting $\alpha_n = \alpha_d = 0$. Moreover, the positive relative price gap reinforces this effect on durables inflation through its influence on the real marginal cost. These effects, combined with the expansionary policy pursued by the Central Bank, determine rising inflationary pressures at the aggregate level.

Insert Figure 3 here

Figure 3 reports equilibrium dynamics following a cost-push shock in the non-durables sector. A distinctive feature of the model with factor demand linkages is that the effect of the positive relative price gap on the marginal cost of firms producing durables partially counteracts the deflationary effect that operates through the conventional demand channel. Concurrently, the overall contractionary effect in consumption and production is magnified by the presence of factor demand linkages. This allows the Central Bank to pursue a weakly contractionary policy, initially accompanied by a negative real rate of interest. This policy reaction is also justified by the fact that changes in the relative price are channeled through the sectoral marginal costs and act as an endogenous attenuator of deflationary pressures in the sector which is not hit by the shock.

It is worth drawing attention to a subtle difference in the transmission of technology and cost-push shocks within this class of models. Sectoral technology shocks cause the consumption gaps in each sector to co-move negatively. The drop in the consumption gap of the sector that experiences the positive technology shock is compensated by a rise in the demand gap of intermediate goods from the other sector. Thus, each sector experiences opposite demand effects on the markets for the consumption and intermediate goods. By contrast, a sectoral cost-push shock determines a contraction of final goods consumption in both sectors. In turn, the contraction in the demand of both consumption goods causes a drop in the consumption of intermediate goods by both sectors, thus resulting in an even grater slump in the gross output.[23] The stark difference in the response of the output and consumption gaps to sectoral cost-push shocks has non-negligible implications for the implementation of the optimal policy and the choice of alternative policy regimes in the presence of a trade-off between inflation and output/consumption stabilization.

5 Monetary Policy with Simple Interest Rate Rules

Having examined monetary policy under a timeless perspective, we turn our attention to a family of simple monetary policy rules akin to those examined by Taylor (1993), Giannoni and Woodford (2003), Schmitt-Grohe and Uribe (2007) and Leith, Moldovan, and Rossi (2012). A number of empirical contributions (see, among others, Taylor, 1993, Clarida et al., 2000, Lubik and Schorfeide, 2004) have shown that the type of rules we propose capture, *prima facie*, the policy behavior of various Central Banks in the OECD countries. The aim of the next exercise is to evaluate how simple and implementable policy rules can mimic the optimal policy under timeless-perspective commitment, while abstracting from its stringent informational requirements.[24]

We first assume symmetric price stickiness and examine the welfare properties of the following rule:

$$i_t = i_t^* + \phi_\pi \pi_t + \phi_y \widetilde{y}_t, \tag{26}$$

[23]Importantly, imperfect labor mobility exacerbates this effect, increasing the wedge between consumption and production. When aggregate demand increases, as labor cannot flow across sectors without frictions, firms need to increase intermediate inputs by more than they would do under the assumption of perfect labor mobility to meet increasing demand. Consequently, fluctuations in production and consumption are wider in the presence of imperfect labor mobility.

[24]Erceg and Levin (2006) follow an analogous line of reasoning, studying the stabilization properties of targeting rules that, despite the fact the welfare criterion involves sector-specific variables, do not consider sector-specific output gaps and inflation rates.

where i_t^* is the nominal rate of interest under flexible prices and $\pi_t = \phi\pi_t^n + (1-\phi)\,\pi_t^d$ is an index of aggregate inflation. Recall that under symmetric price stickiness the parameter accounting for the relative size of the non-durable goods sector equals the weight attached to the squared rate of non-durable inflation in (17), i.e. $\phi = \varpi$. Under these circumstances, no distinction can be made between a measure of overall inflation that aggregates sectoral rates depending on the relative size of the two sectors and one that also accounts for the relative degree of nominal rigidity in price-setting. We determine the reaction coefficients in (26) so that $\phi_\pi \in [1.1, 5.1]$ and $\phi_y \in [0, 5]$.[25] Thus, we search for the combinations of ϕ_π and ϕ_y that minimize the unconditional welfare measure in the decentralized equilibrium of the model economy. The resulting coefficients are reported, together with the (percentage) excess loss with respect to the optimal policy under timeless-perspective commitment, in the top panel of Table 1. The exercise is repeated for alternative production economies and conditional on different sources of exogenous perturbation.

Insert Table 1 here

It is interesting to note that in the presence of cost-push shocks we generally observe a positive policy rate response to the output gap in the two models with input materials, while in the baseline model with $\alpha_n = \alpha_d = 0$ it is desirable to set $\phi_y = 0$. This result is driven by the interplay between durables and sectoral production linkages. Compared with models where consumers' utility only depends on non-durable consumption, durable goods introduce additional volatility in the system, as documented by Erceg and Levin (2006). Moreover, factor demand linkages magnify the response of real activity to sectoral cost-shifters. The policy maker needs to account for such an additional source of volatility. The task is made accomplishable by the fact that intermediate goods induce a dampened response of sectoral inflations to the real marginal costs, which implies a relaxation of the trade-off between inflation and output stabilization. To see this, note that the real marginal cost of sector i is a homogeneous function of degree $1 - \alpha_i\left(1 - \gamma_{ji}\right) < 1$, with

[25] These ranges of variation are selected so as to retain the property of implementability for the selected policy rule, avoiding to allow for unreasonably high response coefficients to inflation and the output gap.

$i, j = \{n, d\}$ and $i \neq j$. It is instructive to see what goes on in the background of the picture presented so far. The bottom panel of Table 1 reports the ratio between the standard deviations of the sectoral variables under the optimal instrumental rule and the optimal policy under commitment (conditional on both shocks). The rule stabilizes sectoral expenditure more than commitment does, at the cost of inducing too much variability in the sectoral rates of inflation. This is because, by construction, the interest rate rule does not allow the Central Bank to respond to aggregate inflation as much as it would be desirable from the timeless perspective.

When dealing with asymmetric stickiness, we examine the performance of the following rules:

$$i_t = i_t^* + \phi_\pi \pi_t + \phi_y \widetilde{y}_t, \tag{27}$$

$$i_t = i_t^* + \phi_\pi \pi_t^c + \phi_y \widetilde{y}_t, \tag{28}$$

where $\pi_t^c = \varpi \pi_t^n + (1 - \varpi) \pi_t^d$ is an index of core inflation, according to which sectoral inflations are weighed depending on both sectors' relative size and degree of price stickiness.

Insert Table 2 here

Once again, the Central Bank does attach a positive response coefficient to the output gap in the two models with input materials (see Table 2), though the optimal ϕ_y is generally greater than what we observe under symmetric price stickiness. This can be explained by the fact that durable prices are relatively more flexible and allow the Central Bank to partly shift its focus on real activity. However, under asymmetric stickiness and over the selected intervals for the reaction coefficients, the performance of (27) and (28) relative to timeless-perspective commitment is not as good as that observed under symmetric stickiness. More surprisingly, the rules that respond to core inflation tend to perform worse than those reacting to aggregate inflation.[26] To uncover this fact, we

[26] As expected on *a priori* grounds, the opposite holds true in the model where both input materials and cost-push shocks are ruled out (Woodford, 2003).

examine the influence of the relative weights attached to sectoral inflation on the loss of social welfare. Along with computing $\{\phi_\pi, \phi_y\}$, the exercise portrayed in Figure 4 determines the optimal weights attached to the sectoral rates of inflation. Thus, we report the excess loss of welfare under the interest rate rule (with respect to the policy under commitment) as a function of the weight attached to durable goods inflation. The exercise is performed both for the model economy with factor demand linkages and that without intermediate goods. Compared with the case of core inflation, increasing the weight attached to durable inflation helps at reducing the distance between the losses under alternative policies. In fact, in the model with factor demand linkages the optimal weight of durable inflation is surprisingly close to that assumed in the aggregate inflation index, i.e. $1 - \phi$. Intermediate goods reduce the slope of the New Keynesian Phillips, as compared with an economy in which production is carried out just by means of the labor input. This implies that the impact of the real marginal cost on current inflation is attenuated, relative to the scenario with no input materials, and more so when cross-industry flows of input materials are in place. In the presence of sectoral linkages, raising the weight attached to π_t^d reduces the volatility of both π_t^d and π_t^n (relative to their counterparts under timeless-perspective commitment) up to the point consistent with the optimal weight.[27] This is because reducing the variability of durable inflation has a beneficial impact on the marginal cost of producing non-durables.

Insert Figure 4 here

In standard two-sector models the optimal monetary policy literature suggests that the relative weight of a particular sector's inflation should depend on the relative degree of nominal rigidity in price-setting and the relative expenditure share (Woodford, 2003; Benigno, 2004). We show that even when a sector is characterized by a relatively lower degree of rigidity in price-setting, the effective slope of the supply schedule may be crucially affected by the presence of input materials. In this context the calibration of the input-output matrix is an element that needs to be carefully accounted for to compute measures of (final goods) price inflation that may assume some relevance from

[27] After this point, only the relative standard deviation of π_t^d keeps decreasing.

the perspective of policy-making. In the specific setting under examination, even if the durable goods sector is characterized by higher frequency of price-setting, the index of core inflation attaches too much importance to non-durable inflation.

Our analysis is somehow related to that of Jeske and Liu (2012), though they explicitly treat durables as housing. They show that asymmetries in the factor intensity of sectoral technologies affect the optimal weight attached to rental price inflation. Specifically, they assume no labor employed in the production of housing, so that the optimal weight results much lower than that attached in the computation of the consumer price index. By contrast, we prescribe attaching greater importance to durable inflation than what is implied by a standard index of core inflation. Even if the production of durables is more labor intensive than non-durable production, calibrating the input-output matrix involves a high share of non-durable intermediate goods employed in the production of durables, so that nominal rigidity in non-durables price-setting is passed onto the marginal cost of producing durables.

6 Conclusions

We have examined the normative implications of a two-sector economy with durable and non-durable consumption goods and sectoral production linkages between sectors. The interplay between these realistic features of modern industrialized economies has non-negligible implications for the formulation of policies aimed at reducing real and nominal fluctuations.

We explore the capability of simple interest rate rules to mimic the optimal policy under timeless-perspective commitment. A clear advantage of these rules is to avoid considering stringent informational requirements as for the optimal policy. Compared with otherwise standard models where only labor is employed in the production process, input materials imply a dampened response of sectoral inflations to the real marginal costs. This feature of the model relaxes the conventional trade-off between inflation and output stabilization, so that the policy maker may account for the volatility induced by durable expenditure. A key result is that input materials do matter when it comes to aggregating sectoral inflation rates into an overall index of price inflation. Conventional measures of core inflation do not account for the impact of input materials on the slope of the aggregate supply function, thus attaching too much importance to the rate of inflation of the sector with higher rigidity in price-setting. Input-output interactions exacerbate such a discrepancy, as they induce further attenuation of the pass-through from the real marginal cost to the rate of inflation.

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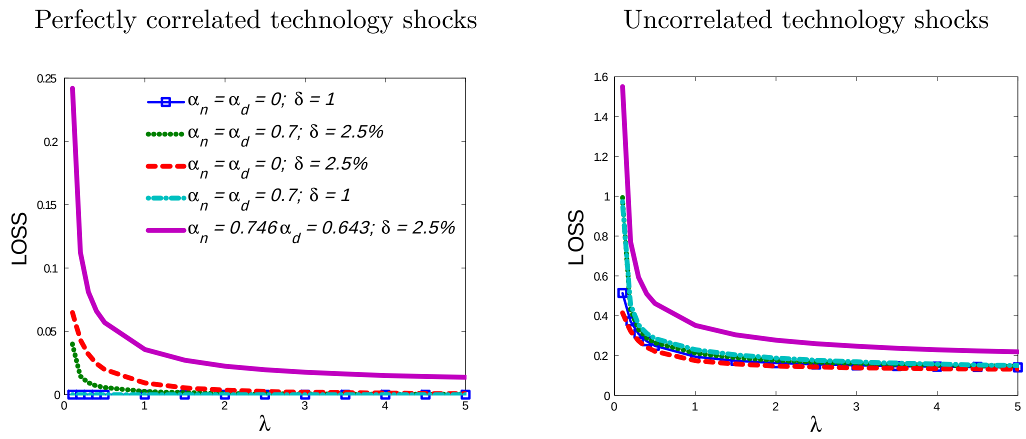
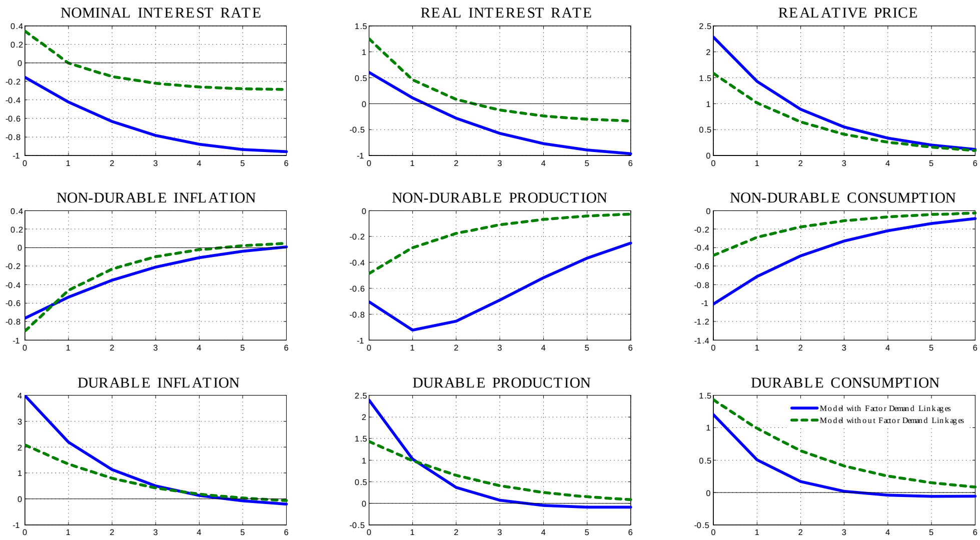
Figures and Tables

FIGURE 1: WELFARE LOSS UNDER VARYING DEGREES OF LABOR MOBILITY



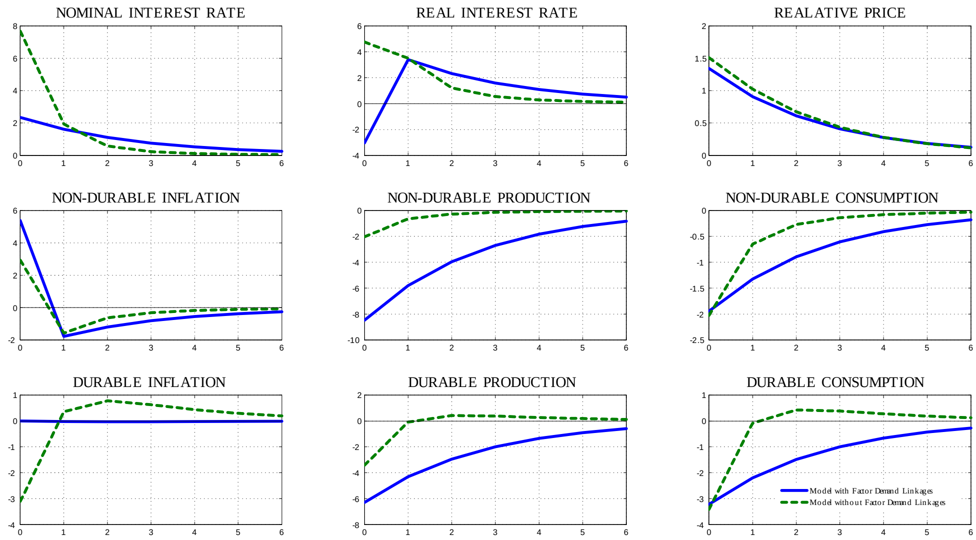
Notes: We report the loss of welfare under timeless-perspective commitment, computed as a percentage of steady state aggregate consumption (multiplied by 100) for various model economies and conditional on different shock configurations. The left-hand panel reports the loss of welfare under perfectly correlated technology shocks, while in the right-hand panel we consider uncorrelated disturbances. In both cases we rule out cost-push shocks.

FIGURE 2: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE NON-DURABLE GOODS SECTOR



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.

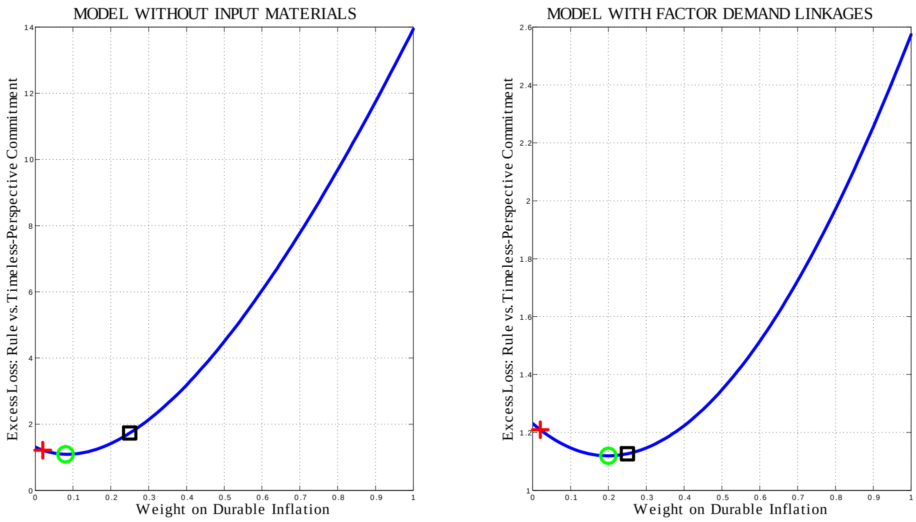
FIGURE 3: IMPULSE RESPONSES TO A COST-PUSH SHOCK IN THE NON-DURABLE GOODS SECTOR



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.

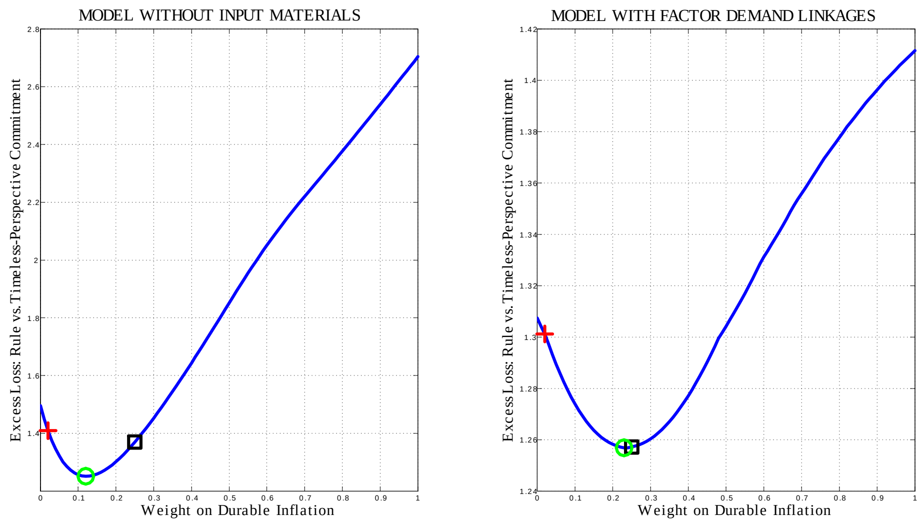
FIGURE 4a: EXCESS LOSS AS A FUNCTION OF THE WEIGHT ATTACHED TO DURABLE INFLATION

(CONDITIONAL ON SECTORAL TECHNOLOGY SHOCKS)



Notes: The figure portrays the percentage excess loss under the optimally computed interest rate rule and that under timeless-perspective commitment, as a function of the weight attached to durable inflation. The cross denotes the weight consistent with a rule that responds to core inflation, while the square denotes the weight consistent with targeting aggregate inflation. The circle denotes the weight that minimizes the distance between the two losses.

FIGURE 4b: EXCESS LOSS AS A FUNCTION OF THE WEIGHT ATTACHED TO DURABLE INFLATION (CONDITIONAL ON BOTH TECHNOLOGY AND COST-PUSH SHOCKS)



Notes: The figure portrays the percentage excess loss under the optimally computed interest rate rule and that under timeless-perspective commitment, as a function of the weight attached to durable inflation. The cross denotes the weight consistent with a rule that responds to core inflation, while the square denotes the weight consistent with targeting aggregate inflation. The circle denotes the weight that minimizes the distance between the two losses.

TABLE 1a. INCREMENTAL LOSS: RULE VS. TIMELESS-PERSPECTIVE COMMITMENT (SYMMETRIC STICKINESS)			
	Tech. Shocks	Cost Push Shocks	Both Shocks
No Input Materials	$\phi_\pi = 5.1, \phi_y = 5$	$\phi_\pi = 5.1, \phi_y = 0$	$\phi_\pi = 5.1, \phi_y = 0$
	0.5638	12.3468	10.9814
FDL	$\phi_\pi = 5.1, \phi_y = 5$	$\phi_\pi = 5.1, \phi_y = 0.4$	$\phi_\pi = 5.1, \phi_y = 0.4$
	3.4606	22.9243	20.2631

TABLE 1b. STANDARD DEVIATION: RULE VS. TIMELESS-PERSPECTIVE COMMITMENT (SYMMETRIC STICKINESS)				
	Non-durable Exp.	Durable Exp.	Non-durable Infl.	Durable Infl.
No Input Materials	0.977	0.974	1.098	1.022
FDL	0.908	0.478	1.137	1.096

Notes: We assume symmetric price stickiness, with the average duration of prices set at 4 quarters. Table 1a reports, conditional on different sources of exogenous perturbation and for different model economies (i.e., no input materials in the production function vs. factor demand linkages between sectors), the reaction coefficients in the interest rate rule $i_t^* + \phi_\pi \pi_t + \phi_y \widetilde{y}_t$ that minimize the loss of social welfare (17), as well as the percentage excess loss under the optimally computed rule and that under timeless-perspective commitment. Table 1b reports the standard deviations of sectoral expenditures and inflation rates under the optimal interest rate rule relative to that under timeless-perspective commitment, conditional on the realization of both technology and cost-push shocks.

TABLE 2a. INCREMENTAL LOSS: RULE VS. TIMELESS-PERSPECTIVE COMMITMENT (ASYMMETRIC STICKINESS)			
	Tech. Shocks	Cost Push Shocks	Both Shocks
No Input Materials (Core Inflation)	ϕ_π= 5.1, ϕ_y= 5	ϕ_π= 5.1, ϕ_y= 0	ϕ_π= 5.1, ϕ_y= 0
	20.012	44.138	43.854
No Input Materials (Aggr. Inflation)	ϕ_π= 3.5, ϕ_y= 5	ϕ_π= 5.1, ϕ_y= 0	ϕ_π= 5.1, ϕ_y= 0
	23.700	30.546	37.159
FDL (Core Inflation)	ϕ_π= 5.1, ϕ_y= 5	ϕ_π= 4.9, ϕ_y= 0.4	ϕ_π= 5.1, ϕ_y= 0.6
	8.008	29.197	30.415
FDL (Aggr. Inflation)	ϕ_π= 5.1, ϕ_y= 5	ϕ_π= 3.5, ϕ_y= 0	ϕ_π= 5.1, ϕ_y= 0.2
	2.799	21.369	25.732

TABLE 2b. STANDARD DEVIATION: RULE VS. TIMELESS-PERSPECTIVE COMMITMENT (ASYMMETRIC STICKINESS)				
	Non-durable Exp.	Durable Exp.	Non-durable Infl.	Durable Infl.
No Input Materials (Core Inflation)	0.988	0.839	1.234	1.226
No Input Materials (Aggr. Inflation)	0.669	0.980	1.511	0.554
FDL (Core Inflation)	0.763	0.385	1.214	1.089
FDL (Aggr. Inflation)	1.144	0.684	1.115	0.902

Notes: Asymmetric price stickiness is assumed, with the average duration of the price of non-durables set at 4 quarters, whereas we reduce the duration of durable prices to 1.3 quarters. Table 2a reports, conditional on different sources of exogenous perturbation and for different model economies (i.e., no input materials in the production function vs. factor demand linkages between sectors), the reaction coefficients in the interest rate rules $i_t = i_t^* + \phi_\pi \pi_t + \phi_y \widetilde{y}_t$ and $i_t = i_t^* + \phi_\pi \pi_t^c + \phi_y \widetilde{y}_t$ that minimize the loss of social welfare (17), as well as the percentage excess loss under the optimally computed rule and that under timeless-perspective commitment. The term $\pi_t^c = \phi \pi_t^n + (1 - \phi) \pi_t^d$ denotes aggregate inflation, while $\pi_t^c = \varpi \pi_t^n + (1 - \varpi) \pi_t^d$ is the index of core inflation. Table 2b reports the standard deviations of sectoral expenditures and inflation rates under the optimal interest rate rule relative to that under timeless-perspective commitment, conditional on the realization of both technology and cost-push shocks.

APPENDIX A: First Order Conditions from Households' Utility Maximization

Maximizing (1) subject to (3), (4), (5), and (6) leads to a set of first-order conditions that can be re-arranged to obtain:

$$\mu_n H_t^{1-\sigma} \left(C_t^n\right)^{-1} = \beta R_t E_t \left[\frac{\mu_n H_{t+1}^{1-\sigma} \left(C_{t+1}^n\right)^{-1}}{\Pi_{t+1}^n}\right], \tag{29a}$$

$$\begin{aligned} \frac{\mu_n H_t^{1-\sigma} P_t^d}{C_t^n P_t^n} = {} & E_t \left\{\beta \left(1-\delta\right) \mu_n \frac{H_{t+1}^{1-\sigma} P_{t+1}^d}{C_{t+1}^n P_{t+1}^n} + \right. \\ & \left. + \frac{\mu_d H_t^{1-\sigma}}{\mathfrak{D}_t \left[1 - \frac{\Xi}{D}\left(D_t - D_{t-1}\right)\right]^{-1}} + \beta \frac{\Xi}{D} \frac{\mu_d H_{t+1}^{1-\sigma}}{\mathfrak{D}_{t+1} \left(D_{t+1} - D_t\right)^{-1}} \right\}, \end{aligned} \tag{29b}$$

$$W_t^n \frac{\mu_n H_t^{1-\sigma} \left(C_t^n\right)^{-1}}{P_t^n} = \varrho \phi^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} \left(L_t^n\right)^{\frac{1}{\lambda}}, \tag{29c}$$

$$W_t^d \frac{\mu_n H_t^{1-\sigma} \left(C_t^n\right)^{-1}}{P_t^n} = \varrho \left(1-\phi\right)^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} \left(L_t^d\right)^{\frac{1}{\lambda}}. \tag{29d}$$

APPENDIX B: Some Useful Steady State Relationships

As in the competitive equilibrium real wage in each sector equals the marginal product of labor. Thus, we can derive the following relationship between the production in non-durable and durable goods in the steady state:

$$\frac{Y^n}{Y^d} = \frac{(1-\alpha_d)\phi}{\left(1-\alpha_n\right)\left(1-\phi\right)} Q^{-1}.$$

Furthermore, the following relationship between durable and non-durable consumption can be derived from the Euler conditions:

$$\frac{C^n}{C^d} = \left(1 - \beta\left(1-\delta\right)\right) \frac{\mu_n}{\mu_d} \frac{1}{\delta} Q^{-1}.$$

Moreover, the following shares of consumption and intermediate goods over total production are determined for the non-durable goods sector:

$$\begin{aligned} \frac{C^n}{Y^n} &= \frac{\left(1-\alpha_n \gamma_{nn}\right) \phi \left(1-\alpha_d\right) - \left(1-\alpha_n\right)\left(1-\phi\right) \alpha_d \gamma_{nd}}{\phi \left(1-\alpha_d\right)}, \\ \frac{M^{nn}}{Y^n} &= \alpha_n \gamma_{nn}, \\ \frac{M^{nd}}{Y^n} &= \frac{\left(1-\alpha_n\right)}{\phi} \frac{1-\phi}{1-\alpha_d} \alpha_d \gamma_{nd}. \end{aligned}$$

Analogously, for the durable goods sector:

$$
\begin{aligned}
\frac{C^d}{Y^d} &= \frac{(1-\alpha_d\gamma_{dd})(1-\phi)(1-\alpha_n)-(1-\alpha_d)\phi\alpha_n\gamma_{dn}}{(1-\phi)(1-\alpha_n)}, \\
\frac{M^{dn}}{Y^d} &= \frac{1-\alpha_d}{1-\phi}\frac{\phi}{1-\alpha_n}\alpha_n\gamma_{dn}, \\
\frac{M^{dd}}{Y^d} &= \alpha_d\gamma_{dd}.
\end{aligned}
$$

These conditions prove to be crucial in the second-order approximation of consumers' utility to eliminate linear terms. Moreover, they allow us to derive the steady state ratio of labor supply in the non-durable goods sector over the total labor supply (ϕ).

The Relative Price in the Steady State

We consider the steady state condition for the marginal cost in the non-durable goods sector:

$$
\begin{aligned}
MC^n &= \overline{\phi}_n\left[(P^n)^{\gamma_{nn}}\left(P^d\right)^{\gamma_{dn}}\right]^{\alpha_n}(W^n)^{1-\alpha_n}, \\
\overline{\phi}_n &= \alpha_n^{\alpha_n}(1-\alpha_n)^{1-\alpha_n}.
\end{aligned}
$$

As in the steady state production subsidies neutralize distortions due to imperfect competition:

$$
\begin{aligned}
P^n &= MC^n \\
&= \overline{\phi}_n\left[(P^n)^{\gamma_{nn}}\left(P^d\right)^{\gamma_{dn}}\right]^{\alpha_n}(W^n)^{1-\alpha_n}.
\end{aligned}
$$

After some trivial manipulations it can be shown that:

$$
\overline{\phi}_n Q^{-\alpha_n\gamma_{dn}}(RW^n)^{1-\alpha_n}=1.
$$

Analogously, for the durable goods sector:

$$
\overline{\phi}_d Q^{\alpha_d\gamma_{nd}}\left(RW^d\right)^{1-\alpha_d}=1.
$$

Using the fact that in steady state $W^n = W^d = W$:

$$
\begin{aligned}
\frac{RW^n}{RW^d}Q &= 1, \\
\frac{\left(\overline{\phi}_n^{-1}Q^{\alpha_n\gamma_{dn}}\right)^{\frac{1}{1-\alpha_n}}}{\left(\overline{\phi}_d^{-1}Q^{-\alpha_d\gamma_{nd}}\right)^{\frac{1}{1-\alpha_d}}}Q &= 1.
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
Q &= \left(\overline{\phi}_n^{1-\alpha_d}\overline{\phi}_d^{-(1-\alpha_n)}\right)^{\frac{1}{\varphi}}, \\
\varphi &= (1-\alpha_n)(1-\alpha_d)+\alpha_n\gamma_{dn}(1-\alpha_d)+\alpha_d\gamma_{nd}(1-\alpha_n).
\end{aligned}
$$

Notice that, when $\alpha_n = \alpha_d = 1$:

$$Q = \overline{\phi}_n \overline{\phi}_d^{-1}$$

as in the case considered by Huang and Liu (2005) and Strum (2008).

APPENDIX C: Equilibrium Dynamics in the Efficient Equilibrium

In this appendix we outline the solution method of the linear model under the efficient equilibrium. This is obtained when both prices are flexible and elasticities of substitution are constant. Let us start from the pricing rule under flexible prices:

$$\begin{aligned} P_t^{n^*} &= \frac{\Theta^n}{1+\tau_n} MC_t^{n^*} \\ &= \frac{\Theta^n}{1+\tau_n} \frac{\overline{\phi}^n \left[\left(P_t^{n^*}\right)^{\gamma_{nn}} \left(P_t^{d^*}\right)^{\gamma_{dn}} \right]^{\alpha_n} \left(W_t^*\right)^{1-\alpha_n}}{Z_t^n} \end{aligned}$$

$$\begin{aligned} P_t^{d^*} &= \frac{\Theta^d}{1+\tau_d} MC_t^{d^*} \\ &= \frac{\Theta^d}{1+\tau_d} \frac{\overline{\phi}^d \left[\left(P_t^{n^*}\right)^{\gamma_{nd}} \left(P_t^{d^*}\right)^{\gamma_{dd}} \right]^{\alpha_d} \left(W_t^*\right)^{1-\alpha_d}}{Z_t^d} \end{aligned}$$

where Θ^n and Θ^d denote the mark-up terms. In log-linear form the conditions above reduce to:

$$\begin{aligned} (1-\alpha_n)\, rw_t^{n*} &= z_t^n + \alpha_n \gamma_{dn} q_t^* && (30) \\ (1-\alpha_d)\, rw_t^{d*} &= z_t^d - \alpha_d \gamma_{nd} q_t^* && (31) \end{aligned}$$

We now recall some conditions under flexible prices from the linearized system:

$$
\begin{aligned}
c_t^{d*} &= \frac{1}{\delta} d_t^* - \frac{1-\delta}{\delta} d_{t-1}^*, && (32)\\
rw_t^{n*} &= -\gamma c_t^{n*} - (1-\sigma)\,\mu_d d_t^* + \left[v\,(1-\phi) - \frac{1}{\lambda} \right] l_t^{d*} \\
&\quad + \left(\vartheta\phi + \frac{1}{\lambda} \right) l_t^{n*}, && (33)\\
l_t^{n*} &= \lambda \left(rw_t^{n*} - rw_t^{d*} + q_t^* \right) + l_t^{d*}, && (34)\\
y_t^{n*} &= \frac{C^n}{Y^n} c_t^{n*} + \frac{M^{nn}}{Y^n} m_t^{nn*} + \frac{M^{nd}}{Y^n} m_t^{nd*}, && (35)\\
y_t^{d*} &= \frac{C^d}{Y^d} x_t^* + \frac{M^{dn}}{Y^d} m_t^{dn*} + \frac{M^{dd}}{Y^d} m_t^{dd*}, && (36)\\
0 &= rw_t^{n*} + l_t^{n*} - y_t^{n*}, && (37)\\
0 &= rw_t^{d*} + l_t^{d*} - y_t^{d*}, && (38)\\
0 &= m_t^{nn*} - y_t^{n*}, && (39)\\
0 &= m_t^{nd*} + q_t^* - y_t^{d*}, && (40)\\
0 &= m_t^{dn*} - q_t^* - y_t^{n*}, && (41)\\
0 &= m_t^{dd*} - y_t^{d*}, && (42)
\end{aligned}
$$

where $\vartheta = \left(v - \frac{1}{\lambda}\right)$, $\gamma = (1-\sigma)\,\mu_n - 1$ and $\phi = \frac{L^n}{L}$. We substitute (30) and (31) into (36) and (37) respectively:

$$
\begin{aligned}
l_t^{n*} &= y_t^{n*} - \frac{1}{1-\alpha_n} z_t^n - \frac{\alpha_n \gamma_{dn}}{1-\alpha_n} q_t^*, && (43)\\
l_t^{d*} &= y_t^{d*} - \frac{1}{1-\alpha_d} z_t^d + \frac{\alpha_d \gamma_{nd}}{1-\alpha_d} q_t^*. && (44)
\end{aligned}
$$

We can use conditions (34), (35), and (39)-(42), to obtain:

$$
y_t^{n*} = \frac{C^n}{Y^n} c_t^{n*} + \frac{M^{nn}}{Y^n} y_t^{n*} + \frac{M^{nd}}{Y^n} \left(y_t^{d*} - q_t^* \right)
$$

and

$$
y_t^{d*} = \frac{C^d}{Y^d} c_t^{d*} + \frac{M^{dn}}{Y^d} \left(q_t^* + c_t^{g*} \right) + \frac{M^{dd}}{Y^d} y_t^{d*}.
$$

We can find a VAR solution to this system, so that we can express y_t^{n*} and y_t^{d*}as a function of c_t^{n*}, c_t^{d*} and q_t^* :

$$
\mathbf{A} \begin{bmatrix} y_t^{n*} \\ y_t^{d*} \end{bmatrix} = \mathbf{B} \begin{bmatrix} c_t^{n*} \\ c_t^{d*} \end{bmatrix} + \boldsymbol{\Upsilon} q_t^*,
$$

where

$$\begin{aligned}
\mathbf{A} &= \begin{bmatrix} 1-\frac{M^{nn}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & 1-\frac{M^{dd}}{Y^d} \end{bmatrix} = \begin{bmatrix} \frac{C^n}{Y^n}+\frac{M^{nd}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & \frac{C^d}{Y^d}+\frac{M^{dn}}{Y^d} \end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} \frac{C^n}{Y^n} & 0 \\ 0 & \frac{C^d}{Y^d} \end{bmatrix}, \\
\boldsymbol{\Upsilon} &= \begin{bmatrix} -\frac{M^{nd}}{Y^n} \\ \frac{M^{dn}}{Y^d} \end{bmatrix}.
\end{aligned}$$

Thus, we obtain:

$$\begin{bmatrix} y_t^{n*} \\ y_t^{d*} \end{bmatrix} = \mathbf{A}^{-1}\mathbf{B} \begin{bmatrix} c_t^{n*} \\ c_t^{d*} \end{bmatrix} + \mathbf{A}^{-1}\boldsymbol{\Upsilon} q_t^*,$$

or equivalently:

$$\begin{aligned}
y_t^{n*} &= \psi_1 c_t^{n*} + \psi_2 c_t^{d*} + \psi_5 q_t^*, \\
y_t^{d*} &= \psi_3 c_t^{n*} + \psi_4 c_t^{d*} + \psi_6 q_t^*.
\end{aligned}$$

Clearly, interdependence among sectors reflects the presence of cross-industry flows of input materials that imply $\psi_2, \psi_3, \psi_5, \psi_6 \neq 0$ and $\psi_1, \psi_4 \neq 1$. Plugging these expressions into (43) and (44) we obtain:

$$l_t^{n*} = \psi_1 c_t^{n*} + \psi_2 c_t^{d*} - \frac{1}{1-\alpha_n} z_t^n + \left(\psi_5 - \frac{\alpha_n \gamma_{dn}}{1-\alpha_n}\right) q_t^* \tag{45}$$

$$l_t^{d*} = \psi_3 c_t^{n*} + \psi_4 c_t^{d*} - \frac{1}{1-\alpha_d} z_t^d + \left(\frac{\alpha_d \gamma_{nd}}{1-\alpha_d} + \psi_6\right) q_t^* \tag{46}$$

Thus, we can substitute everything into (33) and (30):

$$\frac{1+v\phi}{1-\alpha_n} z_t^n + \xi_1 z_t^d = \xi_2 c_t^{n*} - (1-\sigma)\,\mu_d d_t^* + \xi_3 c_t^{d*} + \xi_4 q_t^*, \tag{47}$$

where:

$$\begin{aligned}
\xi_1 &= \frac{v(1-\phi)\lambda - 1}{(1-\alpha_n)\,\lambda}, \\
\xi_2 &= \frac{\lambda\left(v\phi\psi_1 - \gamma\right) + \psi_3\left[v(1-\phi)\lambda - 1\right]}{\lambda}, \\
\xi_3 &= \frac{\lambda v\phi\psi_2 + \psi_4\left[v(1-\phi)\lambda - 1\right]}{\lambda}, \\
\xi_4 &= -\frac{\alpha_n \gamma_{dn}\,(1+v\phi)}{1-\alpha_n} + \left[\frac{v(1-\phi)\lambda - 1}{(1-\alpha_d)\,\lambda}\left(\frac{\alpha_d \gamma_{nd}}{1-\alpha_d} + \psi_6\right) + \psi_5 v\phi\right].
\end{aligned}$$

In turn, we can plug (45), (46), (30) and (31) into (33):[28]

$$\xi_5 q_t^* = \frac{1}{1-\alpha_n} z_t^n - \frac{1}{1-\alpha_d} z_t^d - \frac{\psi}{(1+\lambda)} \left(c_t^{n*} - c_t^{d*} \right) \tag{48}$$

where

$$\xi_5 = \frac{\psi_5 - \psi_6 - \lambda}{1+\lambda} - \left[\frac{\alpha_n \gamma_{dn}}{(1-\alpha_n)} + \frac{\alpha_d \gamma_{nd}}{(1-\alpha_d)} \right].$$

Conditions (47) and (48), together with the Euler conditions for the durable and the non-durable goods, and the law of accumulation for durable goods, allow us to determine a system of linear difference equations from which we derive equilibrium dynamics under flexible prices.

APPENDIX D: Log-linear Economy

Here we report the log-linear economy in extensive form:

[28] It can be shown that $\psi_1 - \psi_3 = -(\psi_2 - \psi_4) = \left(\frac{M^{dn}}{C^d} + \frac{M^{nd}}{C^n} + 1 \right)^{-1} = \psi < 1$.

$$
\begin{aligned}
\widetilde{c}_t^n &= \frac{1}{\gamma}\left(\widehat{r}_t - E_t\pi_{t+1}^n - rr_t^*\right) + E_t\widetilde{c}_{t+1}^n + \frac{(1-\sigma)\,\mu_d}{\gamma}E_t\Delta\widetilde{d}_{t+1}, \\
\widetilde{c}_t^n &= \frac{1}{\mu_n(1-\sigma)}\left\{[1-\mu_d(1-\sigma)]\,\widetilde{d}_t + \frac{1}{1-\beta(1-\delta)}\left[(\mu_n(1-\sigma)-1)\,\widetilde{c}_t^n + \mu_d(1-\sigma)\,\widetilde{d}_t - \widetilde{q}_t\right] + \right. \\
&\quad -\frac{(1-\delta)\,\beta}{[1-\beta(1-\delta)]}\left[(\mu_n(1-\sigma)-1)\,\widetilde{c}_{t+1}^n + \mu_d(1-\sigma)\,\widetilde{d}_{t+1} - \widetilde{q}_{t+1}\right] + \\
&\quad \left. +\Xi\left(\widetilde{d}_t - \widetilde{d}_{t-1}\right) - \beta\Xi\left(\widetilde{d}_{t+1} - \widetilde{d}_t\right)\right\} \\
\widetilde{c}_t^d &= \frac{1}{\delta}\widetilde{d}_t - \frac{1-\delta}{\delta}\widetilde{d}_{t-1}, \\
\widetilde{rw}_t^n &= -\gamma\widetilde{c}_t^n - (1-\sigma)\,\mu_d\widetilde{d}_t + \vartheta(1-\phi)\,\widetilde{l}_t^d + \left(\vartheta\phi + \frac{1}{\lambda}\right)\widetilde{l}_t^n, \\
\widetilde{l}_t^n &= \lambda\left(\widetilde{rw}_t^n - \widetilde{rw}_t^d + \widetilde{q}_t\right) + \widetilde{l}_t^d, \\
\pi_t^n &= \beta E_t\pi_{t+1}^n + \frac{(1-\beta\theta_n)(1-\theta_n)}{\theta_n}\widetilde{rmc}_t^n + \eta_t^n, \\
\pi_t^d &= \beta E_t\pi_{t+1}^d + \frac{(1-\beta\theta_d)(1-\theta_d)}{\theta_d}\widetilde{rmc}_t^d + \eta_t^d, \\
\widetilde{y}_t^n &= \alpha_n\gamma_{nn}\widetilde{m}_t^{nn} + \alpha_n\gamma_{dn}\widetilde{m}_t^{dn} + (1-\alpha_n)\,\widetilde{l}_t^n, \\
\widetilde{y}_t^d &= \alpha_d\gamma_{nd}\widetilde{m}_t^{nd} + \alpha_d\gamma_{dd}\widetilde{m}_t^{dd} + (1-\alpha_d)\,\widetilde{l}_t^d, \\
\widetilde{y}_t^n &= \frac{C^n}{Y^n}\widetilde{c}_t^n + \frac{M^{nn}}{Y^n}\widetilde{m}_t^{nn} + \frac{M^{nd}}{Y^n}\widetilde{m}_t^{nd}, \\
\widetilde{y}_t^d &= \frac{C^d}{Y^d}\widetilde{c}_t^d + \frac{M^{dn}}{Y^d}\widetilde{m}_t^{dn} + \frac{M^{dd}}{Y^d}\widetilde{m}_t^{dd}, \\
\widetilde{rmc}_t^n &= \widetilde{rw}_t^n + \widetilde{l}_t^n - \widetilde{y}_t^n, \\
\widetilde{rmc}_t^d &= \widetilde{rw}_t^d + \widetilde{l}_t^d - \widetilde{y}_t^d, \\
\widetilde{rmc}_t^n &= \widetilde{m}_t^{nn} - \widetilde{y}_t^n, \\
\widetilde{rmc}_t^d &= \widetilde{m}_t^{nd} + \widetilde{q}_t - \widetilde{y}_t^d, \\
\widetilde{rmc}_t^n &= \widetilde{m}_t^{dn} - \widetilde{q}_t - \widetilde{y}_t^n, \\
\widetilde{rmc}_t^d &= \widetilde{m}_t^{dd} - \widetilde{y}_t^d, \\
\widetilde{q}_t &= \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*.
\end{aligned}
$$

where $\gamma = (1-\sigma)\,\mu_n - 1$.

APPENDIX E: Relative Price in the Efficient Equilibrium with Perfect labor Mobility

We now define the efficient equilibrium in the model with no frictions in both the goods and the labor market. On the labor market this condition, obtained for $\lambda \to \infty$, ensures that nominal salaries are equalized across sectors of the economy:

$$W_t^{n^*} = W_t^{d^*} = W_t^*. \tag{49}$$

Moreover, given the production subsidies that eliminate sectoral distortions due to monopolistic competition:

$$P_t^{n^*} = MC_t^{n^*} \qquad\qquad P_t^{d^*} = MC_t^{d^*}. \tag{50}$$

Conditions (49) and (50) imply that:

$$P_t^{n^*} = \left(\overline{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(P_t^{d^*}\right)^{\frac{\alpha_n\gamma_{dn}}{1-\alpha_n\gamma_{nn}}} \left(W_t^*\right)^{\frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}} \left(Z_t^n\right)^{-\frac{1}{1-\alpha_n\gamma_{nn}}}, \tag{51}$$

$$P_t^{d^*} = \left(\overline{\phi}^d\right)^{\frac{1}{1-\alpha_d\gamma_{dd}}} \left(P_t^{n^*}\right)^{\frac{\alpha_d\gamma_{nd}}{1-\alpha_d\gamma_{dd}}} \left(W_t^*\right)^{\frac{1-\alpha_d}{1-\alpha_d\gamma_{dd}}} \left(Z_t^d\right)^{-\frac{1}{1-\alpha_d\gamma_{dd}}}. \tag{52}$$

We then substitute (51) into (52) to eliminate W_t^*:

$$\left(P_t^{n^*}\right)^{\vartheta_n} = \Upsilon^{(1-\alpha_n\gamma_{nn})(1-\alpha_d)} \left(P_t^{d^*}\right)^{\vartheta_d} \left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{(1-\alpha_n)}$$

where

$$\Upsilon = \left(\overline{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(\overline{\phi}^d\right)^{-\frac{1}{1-\alpha_d}\frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}}$$

and

$$\vartheta_n = \vartheta_d = (1-\alpha_d)(1-\alpha_n\gamma_{nn}) + (\alpha_d\gamma_{nd})(1-\alpha_n).$$

Thus, after some trivial algebra we can show that the relative price reads as:

$$\begin{aligned} Q_t^* &= \frac{P_t^{n^*}}{P_t^{d^*}} = \Upsilon\left[\left(Z_t^n\right)^{-(1-\alpha_d)}\left(Z_t^d\right)^{1-\alpha_n}\right]^{\frac{1}{\varkappa+1}} \\ &= \Upsilon\left[\left(Z_t^n\right)^{-(1-\alpha_d)}\left(Z_t^d\right)^{1-\alpha_n}\right]^{\frac{1}{\varkappa+1}}. \end{aligned}$$

where

$$\varkappa = \alpha_n\alpha_d(\gamma_{nn}+\gamma_{dd}-1) - \alpha_n\gamma_{nn} - \alpha_d\gamma_{dd}.$$

APPENDIX F: Second-order Approximation of the Utility Function

Following Woodford (2003), we derive a well-defined welfare function from the utility function of the representative household:

$$\mathcal{W}_t = U\left(C_t^n, D_t\right) - V\left(L_t\right).$$

We start from a second-order approximation of the utility from consumption of durable and non-durable goods:

$$\begin{aligned} U\left(C_t^n, D_t\right) &\approx U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right)\left(C_t^n - C^n\right) + \frac{1}{2} U_{C^n C^n}\left(C^n, D\right)\left(C_t^n - C^n\right)^2 \quad (53) \\ &+ U_D\left(C^n, D\right)\left(D_t - D\right) + \frac{1}{2} U_{DD}\left(C^n, D\right)\left(D_t - D\right)^2 + \frac{1}{2}\Xi U_D\left(C^n, D\right)\left(D_t - D_{t-1}\right)^2 \\ &+ U_{C^n D}\left(C^n, D\right)\left(C_t^n - C^n\right)\left(D_t - D\right) + O\left(\|\xi\|^3\right), \quad (54) \end{aligned}$$

where $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher. Notice that:

$$\begin{aligned} U_D\left(C^n, D\right) &= \left(\mu_d C^n / \mu_n D\right) U_{C^n}\left(C^n, D\right), \\ U_{C^n C^n}\left(C^n, D\right) &= \left[\mu_n\left(1-\sigma\right) - 1\right]\left(C^n\right)^{-1} U_{C^n}\left(C^n, D\right), \\ U_{DD}\left(C^n, D\right) &= \left[\mu_d\left(1-\sigma\right) - 1\right]\left(\mu_d C^n / \mu_n D\right) U_{C^n}\left(C^n, D\right), \\ U_{C^n D}\left(C^n, D\right) &= \mu_d\left(1-\sigma\right) D^{-1} U_{C^n}\left(C^n, D\right). \end{aligned}$$

As $\frac{C_t^n - C^n}{C^n} = \widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2$, where $\widehat{c}_t^n = \log\left(\frac{C_t^n}{C^n}\right)$ is the log-deviation from steady state under sticky prices, we obtain:

$$\begin{aligned} U\left(C_t^n, D_t\right) &\approx U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right) C^n\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right] + \\ &+ \frac{1}{2}\left[\mu_n\left(1-\sigma\right) - 1\right] U_{C^n}\left(C^n, D\right) C\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]^2 + \\ &+ U_D\left(C^n, D\right) D\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \frac{1}{2}\left[\mu_d\left(1-\sigma\right) - 1\right] U_D\left(C^n, D\right) D\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right)^2 + \\ &+ \frac{1}{2}\Xi U_D\left(C^n, D\right) D\left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2 + \\ &+ \mu_d\left(1-\sigma\right) U_{C^n}\left(C^n, D\right) C^n\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \text{t.i.p.} + O\left(\|\xi\|^3\right), \end{aligned}$$

where t.i.p. collects terms independent of policy stabilization.

Next, we introduce a second-order approximation to the transition law for the stock of durables. This will substitute out the linear term for durables in the expression above (see Erceg and Levin, 2006). The law of motion reads as:

$$D_t = \left(1-\delta\right) D_{t-1} + X_t.$$

For a general function $F\left(Y, X\right)$ the second-order Taylor approximation can be written as:

$$\begin{aligned} F\left(Y, X\right) &\approx F_Y\left(Y, X\right) Y y + F_X\left(Y, X\right) X x + \frac{1}{2}\left(F_X\left(Y, X\right) X + F_{XX}\left(Y, X\right) X^2\right) x^2 \\ &+ \frac{1}{2}\left(F_Y\left(Y, X\right) Y + F_{YY}\left(Y, X\right) Y^2\right) y^2 + F_{YX}\left(Y, X\right) Y X x y. \end{aligned}$$

Now, we can rewrite the accumulation equation as:

$$F(D_{t-1}, C_t^d) = \log\left[\left(1-\delta\right) D_{t-1} + C_t^d\right].$$

Therefore:

$$
\begin{aligned}
F_D &= \frac{(1-\delta)}{(1-\delta)D+C^d} = \frac{(1-\delta)}{(1-\delta)D+\delta D} = \frac{(1-\delta)}{D}, \\
F_{C^d} &= \frac{1}{(1-\delta)D+C^d} = \frac{1}{D}, \\
F_{DD} &= -\frac{(1-\delta)^2}{\left[(1-\delta)D+C^d\right]^2} = -\frac{(1-\delta)^2}{D^2}, \\
F_{C^dC^d} &= -\frac{1}{\left[(1-\delta)D+C^d\right]^2} = -\frac{1}{D^2}, \\
F_{DC^d} &= -\frac{1-\delta}{\left[(1-\delta)D+C^d\right]^2} = -\frac{1-\delta}{D^2}.
\end{aligned}
$$

Considering that in the steady state $C^d = \delta D$:

$$
\begin{aligned}
\widehat{d}_t \approx\ & \frac{(1-\delta)}{D} D\widehat{d}_{t-1} + \frac{1}{D}\delta D\widehat{c}_t^d + \\
& + \frac{1}{2}\left[\frac{(1-\delta)}{D}D - \frac{(1-\delta)^2}{D^2}D^2\right]\widehat{d}_{t-1}^2 + \\
& + \frac{1}{2}\left(\frac{1}{D}D - \frac{1}{D^2}D^2\right)\left(\widehat{c}_t^d\right)^2 - \frac{1-\delta}{D^2}\widehat{d}_{t-1}\widehat{x}_t \\
\approx\ & (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\delta}{2}\widehat{d}_{t-1}^2 + \frac{(1-\delta)\delta}{2}\left(\widehat{c}_t^d\right)^2 - \frac{(1-\delta)\delta}{2}\widehat{c}_t^d\widehat{d}_{t-1} \\
\approx\ & (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\delta}{2}\left(\widehat{d}_{t-1} - \widehat{c}_t^d\right)^2.
\end{aligned}
$$

Thus:

$$
\widehat{d}_t \approx (1-\delta)\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \psi_t, \tag{55}
$$

where:

$$
\begin{aligned}
\widehat{\psi}_t &= \frac{(1-\delta)\delta}{2}\left(\widehat{c}_t^d - \widetilde{d}_{t-1}\right)^2 \\
&= \frac{(1-\delta)}{2\delta}\left(\widehat{d}_t - \widetilde{d}_{t-1}\right)^2.
\end{aligned}
$$

Now, let us iterate backward (55), to obtain:

$$
\sum_{t=0}^{\infty}\beta^t\widehat{d}_t = \frac{1}{1-\beta(1-\delta)}d_0 + \sum_{t=0}^{\infty}\beta^t\left[\frac{\delta}{1-\beta(1-\delta)}\widehat{c}_t^d + \frac{1}{1-\beta(1-\delta)}\widehat{\psi}_t\right].
$$

In turn, the term on the RHS will replace the one on the LHS into the intertemporal loss function.

The next step is to derive a second-order approximation for labor disutility. Recall that:

$$
\widehat{l}_t = \phi\widehat{l}_t^n + (1-\phi)\widehat{l}_t^d.
$$

Therefore the second-order approximation reads:

$$\begin{aligned}
V\left(L_t\right) &\approx V_L\left(L\right)L\left[\phi\widehat{l}_t^n+\left(1-\phi\right)\widehat{l}_t^d+\frac{\phi\left(1+2v\phi\right)}{2}\left(\widehat{l}_t^n\right)^2+\frac{\left(1-\phi\right)\left[1+2v\left(1-\phi\right)\right]}{2}\left(\widehat{l}_t^d\right)^2\right]+ \\
&+\text{t.i.p.}+O\left(\|\xi\|^3\right).
\end{aligned}$$

After these preliminary steps, we need to find an expression for $\widehat{l}_t^n$ and $\widehat{l}_t^d$. Given the definition of the marginal cost, in equilibrium we get:

$$\begin{aligned}
L_t^n &= \frac{\left(1-\alpha_n\right)MC_t^n}{W_t^n}\int_0^1 Y_{jt}^n dj=\frac{\left(1-\alpha_n\right)\overline{\phi}^n}{Z_t^n}\left(\frac{Q_t^{-\gamma_{dn}}}{RW_t^n}\right)^{\alpha_n}Y_t^n\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj, \\
L_t^d &= \frac{\left(1-\alpha_d\right)MC_t^d}{W_t^d}\int_0^1 Y_{kt}^d dk=\frac{\left(1-\alpha_d\right)\overline{\phi}^d}{Z_t^d}\left(\frac{Q_t^{\gamma_{nd}}}{RW_t^d}\right)^{\alpha_d}Y_t^d\int_0^1\left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d}dk.
\end{aligned}$$

Thus, we can report the linear approximation of the expressions above:

$$\begin{aligned}
\widehat{l}_t^n &= -\alpha_n\gamma_{dn}\widehat{q}_t-\alpha_n\widehat{rw}_t^n-z_t^n+\widehat{y}_t^n+S_{nt}, \\
\widehat{l}_t^d &= \alpha_d\gamma_{nd}\widehat{q}_t-\alpha_d\widehat{rw}_t^d-z_t^d+\widehat{y}_t^d+S_{dt},
\end{aligned}$$

where:

$$S_{nt}=\log\left[\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj\right]\qquad S_{dt}=\log\left[\int_0^1\left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d}dk\right] \tag{56}$$

If we set $\widehat{p}_{jt}^n$ to be the log-deviation of $\frac{P_{jt}^n}{P_t^n}$ from its steady state, which means that a second-order Taylor expansion of $\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj$ reads as:

$$\begin{aligned}
\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj &\approx \int_0^1\left[1-\varepsilon^n\widehat{p}_{jt}^n-\varepsilon^n\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n+\frac{1}{2}\left(\varepsilon^n\right)^2\left(\widehat{p}_{jt}^n\right)^2\right]dj+O\left(\|\xi\|^3\right) \\
&= 1-\varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n-\varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n+\frac{1}{2}\left(\varepsilon^n\right)^2\mathbf{E}_i\left(\widehat{p}_{jt}^n\right)^2+O\left(\|\xi\|^3\right),
\end{aligned}$$

where $\mathbf{E}_i\widehat{p}_{jt}^n\equiv\int_0^1\widehat{p}_{jt}^n dj$ and $\mathbf{E}_i\left(\widehat{p}_{jt}^n\right)^2\equiv\int_0^1\left(\widehat{p}_{jt}^n\right)^2dj$. At this stage, we need an expression for $\mathbf{E}_i\widehat{p}_{jt}^n$. Let us start from

$$P_t^n=\left[\int_0^1\left(P_{jt}^n\right)^{1-\varepsilon_t^n}dj\right]^{\frac{1}{1-\varepsilon_t^n}},$$

which can be re-arranged as:

$$1 \equiv \int_0^1 \left(\frac{P_{jt}^n}{P_t^n} \right)^{1-\varepsilon_t^n} dj.$$

Following the procedure above, it can be shown that:

$$\left(\frac{P_{jt}^n}{P_t^n} \right)^{1-\varepsilon_t^n} \approx 1 + (1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Substituting this into the preceding equations yields:

$$0 = \int_0^1 \left[(1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2 \right] dj + O\left(\|\xi\|^3\right),$$

which reduces to:

$$\mathbf{E}_i \widehat{p}_{jt}^n = \frac{\varepsilon^n - 1}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Thus:

$$\int_0^1 \left(\frac{P_{jt}^n}{P_t^n} \right)^{-\varepsilon_t^n} dj = 1 + \frac{\varepsilon^n}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Now, notice that:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 = \mathbf{E}_i \left[\left(p_{jt}^n\right)^2 - 2p_{jt}^n p_t^n + \left(p_t^n\right)^2 \right] + O\left(\|\xi\|^3\right),$$

where lower case letters denote the log-value of the capital letters. Here we can use a first-order approximation of $p_t^n = \int_0^1 p_{jt}^n dj$, as this term is multiplied by other first-order terms each time it appears. With this, we have a second-order approximation:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 \equiv var_j p_{jt}^n.$$

Therefore, the second-order approximation can be represented as:

$$S_{nt} = \frac{\varepsilon^n}{2} var_j p_{jt}^n + O\left(\|\xi\|^3\right).$$

Analogous steps in the sector producing durable goods lead us to:

$$S_{dt} = \frac{\varepsilon^d}{2} var_k p_{kt}^d + O\left(\|\xi\|^3\right).$$

Following Woodford (2003, Ch. 6, Proposition 6.3), we can obtain a correspondence between cross-sectional price dispersions in the two sectors and their inflation rates:

$$
\begin{aligned}
var_j p_{jt}^n &= \theta_n var_j p_{jt-1}^n + \frac{\theta_n}{1-\theta_n}\left(\pi_t^n\right)^2 + O\left(\|\xi\|^3\right), \\
var_k p_{kt}^d &= \theta_d var_k p_{kt-1}^d + \frac{\theta_d}{1-\theta_d}\left(\pi_t^d\right)^2 + O\left(\|\xi\|^3\right).
\end{aligned}
$$

Iterating these expressions forward leads to:

$$
\sum_{t=0}^{\infty} \beta^t var_j p_{jt}^n = \left(\kappa_n\right)^{-1} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^n\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{57}
$$

$$
\sum_{t=0}^{\infty} \beta^t var_k p_{kt}^d = \left(\kappa_d\right)^{-1} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^d\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{58}
$$

where

$$
\begin{aligned}
\kappa_n &= \frac{\left(1-\beta\theta_n\right)\left(1-\theta_n\right)}{\theta_n}, \\
\kappa_d &= \frac{\left(1-\beta\theta_d\right)\left(1-\theta_d\right)}{\theta_d}.
\end{aligned}
$$

After these preliminary steps, we can write $\mathcal{W}_t$ as:

$$
\begin{aligned}
\mathcal{W}_t \approx\ & U_{C^n}\left(C^n, D\right) C^n \left\{ \widehat{c}_t^n + \frac{1}{2}\left[\mu_n\left(1-\sigma\right)\right]\left(\widehat{c}_t^n\right)^2 + \left(\mu_d/\mu_n\right)\widehat{d}_t + \right. \\
& \left. + \frac{1}{2}\left[\mu_d\left(1-\sigma\right)\right]\left(\mu_d/\mu_n\right)\widehat{d}_t^2 + \mu_d\left(1-\sigma\right)\widehat{c}_t^n\widehat{d}_t + \frac{1}{2}\Xi\left(\mu_d/\mu_n\right)\left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2 \right\} + \\
& - V_L\left(L\right) L \left\{ \phi\widehat{l}_t^n + \left(1-\phi\right)\widehat{l}_t^d + \right. \\
& \left. + \left(\frac{1+v}{2}\right)\left[\phi^2\left(\widehat{l}_t^n\right)^2 + \left(1-\phi\right)\left(\widehat{l}_t^d\right)^2 + 2\phi\left(1-\phi\right)\widehat{l}_t^n\widehat{l}_t^d\right] \right\} + \\
& + \text{t.i.p.} + O\left(\|\xi\|^3\right).
\end{aligned}
$$

We now consider the linear terms in $\mathcal{W}_t$, which are collected under $\mathcal{LW}_t$:

$$
\begin{aligned}
\mathcal{LW}_t =\ & \frac{U_{C^n}\left(C^n, D\right) C^n}{\mu_n}\left\{\mu_n\widehat{c}_t^n + \mu_d\widehat{d}_t\right\} + \\
& - \left\{V_L\left(L\right) L\phi\left(-\alpha_n\gamma_{dn}\widehat{q}_t - \alpha_n\widehat{rw}_t^n + \widehat{y}_t^n\right) + \right. \\
& \left. + \left(1-\phi\right)\left(\alpha_d\gamma_{nd}\widehat{q}_t - \alpha_d\widehat{rw}_t^d + \widehat{y}_t^d\right)\right\} + \\
& + \text{t.i.p.} + O\left(\|\xi\|^2\right).
\end{aligned}
$$

We substitute for the real wage from marginal cost expressions to get:

$$
\begin{aligned}
\mathcal{LW}_t &= \frac{U_{C^n}(C^n, D)\, C^n}{\mu_n}\left\{\mu_n \widehat{c}_t^n + \mu_d \widehat{d}_t\right\} + \\
&\quad -V_L(L)\, L\phi\left(\frac{1}{1-\alpha_n}\widehat{y}_t^n - \frac{\alpha_n\gamma_{nn}}{1-\alpha_n}\widehat{m}_t^{nn} - \frac{\alpha_n\gamma_{dn}}{1-\alpha_n}\widehat{m}_t^{dn}\right) + \\
&\quad -V_L(L)\, L(1-\phi)\left(\frac{1}{1-\alpha_d}\widehat{y}_t^d - \frac{\alpha_d\gamma_{nd}}{1-\alpha_d}\widehat{m}_t^{nd} - \frac{\alpha_d\gamma_{dd}}{1-\alpha_d}\widehat{m}_t^{dd}\right) + \\
&\quad +\text{t.i.p.} + O\left(\|\xi\|^2\right).
\end{aligned}
$$

After substituting the second-order approximation for the accumulation equation of durables we get:

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \mathcal{LW}_t &= U_{C^n}(C^n, D)\, C^n \sum_{t=0}^{\infty}\beta^t\left\{\widehat{c}_t^n + \frac{\delta}{1-\beta(1-\delta)}\frac{\mu_d}{\mu_n}\widehat{c}_t^d\right\} + \\
&\quad -V_L(L)\, L\sum_{t=0}^{\infty}\beta^t\left\{\phi\left(\frac{1}{1-\alpha_n}\widehat{y}_t^n - \frac{\alpha_n\gamma_{nn}}{1-\alpha_n}\widehat{m}_t^{nn} - \frac{\alpha_n\gamma_{dn}}{1-\alpha_n}\widehat{m}_t^{dn}\right) + \right. \\
&\quad \left. +(1-\phi)\left(\frac{1}{1-\alpha_d}\widehat{y}_t^d - \frac{\alpha_d\gamma_{nd}}{1-\alpha_d}\widehat{m}_t^{nd} - \frac{\alpha_d\gamma_{dd}}{1-\alpha_d}\widehat{m}_t^{dd}\right)\right\} + \\
&\quad +\text{t.i.p.} + O\left(\|\xi\|^2\right).
\end{aligned} \tag{59}
$$

Notice that the following linear approximations for the market clearing conditions hold:

$$
\begin{aligned}
\widehat{y}_t^n &= \frac{1-\alpha_n}{\phi}\mu_n\widehat{c}_t^n + \alpha_n\gamma_{nn}\widehat{m}_t^{nn} + \frac{(1-\alpha_n)(1-\phi)}{\phi(1-\alpha_d)}\alpha_d\gamma_{nd}\widehat{m}_t^{nd}, \\
\widehat{y}_t^d &= \frac{\delta\mu_d(1-\alpha_d)}{(1-\phi)[1-\beta(1-\delta)]}\widehat{c}_t^d + \frac{(1-\alpha_d)\phi}{(1-\phi)(1-\alpha_n)}\alpha_n\gamma_{dn}\widehat{m}_t^{dn} + \alpha_d\gamma_{dd}\widehat{m}_t^{dd}.
\end{aligned}
$$

It can be shown that, in the steady state, the following relationships hold:

$$
V_{L^n}(L^n)\, L^n = \phi V_L(L)\, L \qquad\qquad V_{L^d}\left(L^d\right) L^d = (1-\phi)\, V_L L(L)
$$

Moreover, the presence of production subsidies allows us to express the steady state marginal rate of substitution between labor supply and consumption of non-durable goods as:

$$
\begin{aligned}
\frac{-V_{L^n}(L^n)}{U_{C^n}(C^n)} &= \frac{Y^n(1-\alpha_n)}{L^n}, \\
\frac{-V_{L^d}\left(L^d\right)}{U_{C^n}(C^n)} &= \frac{Y^d(1-\alpha_d)}{L^d Q}.
\end{aligned}
$$

It is now convenient to express the marginal utility from non-durable consumption in terms of the marginal utility derived from total consumption:

$$
U_{C^n}(C^n) = U_H(H)\, H\mu_n.
$$

Therefore, we can re-write (59) as:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{LW}_t &= U_H(H)H\sum_{t=0}^{\infty}\beta^t\left\{\left(\mu_n\widehat{c}_t^n+\frac{\delta\mu_d}{1-\beta(1-\delta)}\widehat{c}_t^d\right)+\right.\\&-\mu_n\left(\frac{C^n}{Y^n}\right)^{-1}(1-\alpha_n)\left[-\alpha_n\gamma_{dn}\widehat{q}_t-\alpha_n\widehat{rw}_t^n-z_t^n+\widehat{y}_t^n\right]+\\&\left.-\mu_n\left(\frac{C^n}{Y^d}\right)^{-1}(1-\alpha_d)Q^{-1}\left[\alpha_d\gamma_{nd}\widehat{q}_t-\alpha_d\widehat{rw}_t^d-z_t^d+\widehat{y}_t^d\right]\right\}+\\&+\text{t.i.p.}+O\left(\|\xi\|^2\right).\end{aligned}$$

It is now possible to show, given the linearized market clearing conditions in the two sectors, that $\sum_{t=0}^{\infty}\beta^t\mathcal{LW}_t=0$. The linear term in $\mathcal{W}_t$ can therefore be dropped. Thus we are left only with second-order terms:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{W}_t &\approx U_H(H)H\sum_{t=0}^{\infty}\beta^t\left\{\frac{1-\sigma}{2}\left(\mu_n\widehat{c}_t^n+\mu_d\widehat{d}_t\right)^2+\frac{1}{1-\beta(1-\delta)}\mu_d\widehat{\psi}_t+\frac{\mu_d}{2}\Xi\left(\widehat{d}_t-\widehat{d}_{t-1}\right)^2+\right.\\&-\frac{\Theta}{2}\left[\phi\varepsilon^n(\kappa_n)^{-1}(\pi_t^n)^2+(1-\phi)\varepsilon^d(\kappa_d)^{-1}\left(\pi_t^d\right)^2\right]+\\&\left.-\left(\frac{1+v}{2}\right)\Theta^{-1}\left[\mu_n\widehat{c}_t^n+\frac{\delta\mu_d}{1-\beta(1-\delta)}\widehat{c}_t^d\right]^2\right\}+\\&+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where

$$\Theta=\left(\frac{C^n}{Y^n}\right)^{-1}\frac{(1-\alpha_n)\mu_n}{\phi}=\frac{\mu_n[1-\beta(1-\delta)]+\mu_d\delta}{1-\beta(1-\delta)}.$$

We next consider the deviation of social welfare from its Pareto-optimal level:

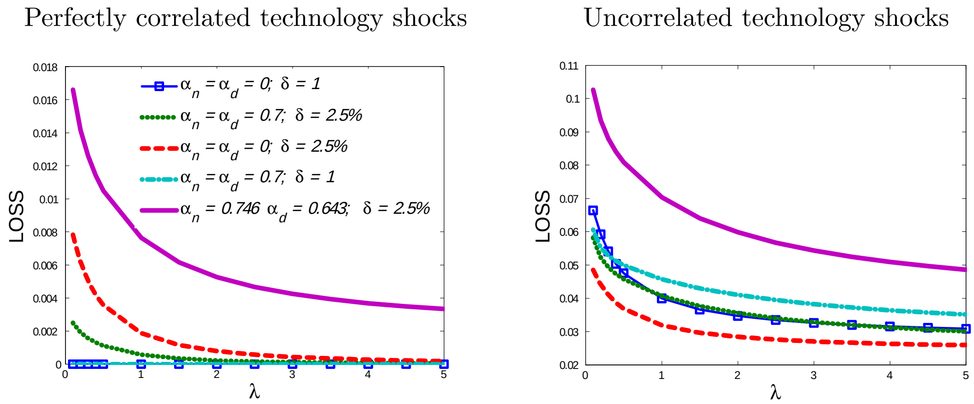
$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\widetilde{\mathcal{W}}_t &= \sum_{t=0}^{\infty}\beta^t\left(\mathcal{W}_t-\mathcal{W}_t^*\right)\approx\\&-\frac{U_H(H)H}{2}\Theta\sum_{t=0}^{\infty}\beta^t\left\{\frac{\sigma-1}{\Theta}\left(\mu_n\widetilde{c}_t^n+\mu_d\widetilde{d}_t\right)^2+\right.\\&+\left[\mu_d\Theta^{-1}\Xi+(1-\delta)(1-\omega)\delta^{-2}\right]\left(\widetilde{d}_t-\widetilde{d}_{t-1}\right)^2+\\&\left.+\varsigma\left[\varpi\left(\pi_t^n\right)^2+(1-\varpi)\left(\pi_t^d\right)^2\right]+(1+v)\left[\omega\widetilde{c}_t^n+(1-\omega)\widetilde{c}_t^d\right]^2\right\}+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where the following notation has been introduced:

$$\begin{aligned}\omega &= \frac{\mu_n[1-\beta(1-\delta)]}{\mu_n[1-\beta(1-\delta)]+\mu_d\delta},\\ \varpi &= \frac{\phi\varepsilon^n(\kappa_n)^{-1}}{\varsigma},\\ \varsigma &= \phi\frac{\varepsilon^n}{\kappa_n}+(1-\phi)\frac{\varepsilon^d}{\kappa_d}.\end{aligned}$$

APPENDIX G: Intersectoral Trade-off under Asymmetric Price Stickiness

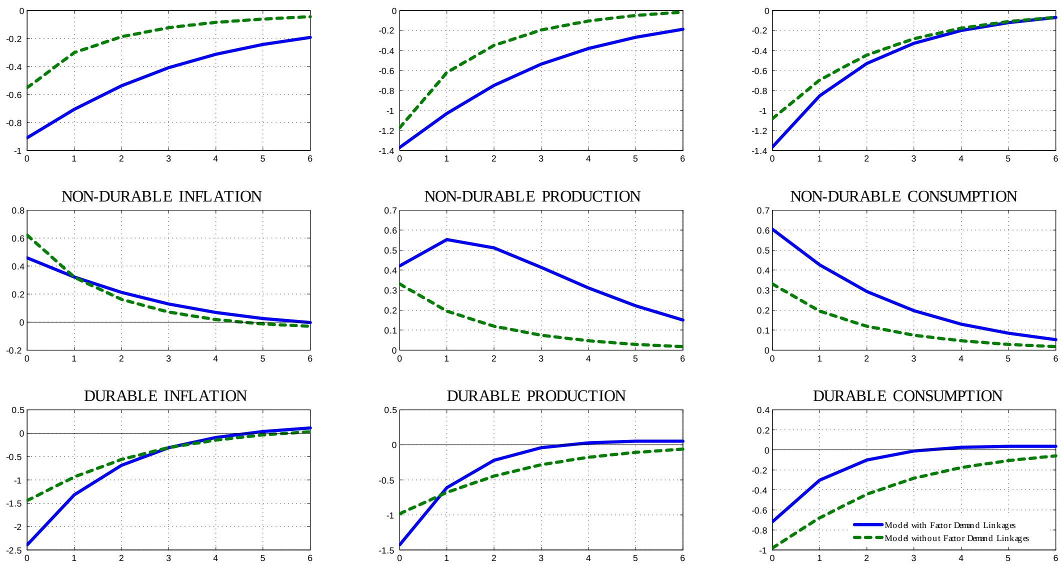
FIGURE 1.G: WELFARE LOSS UNDER VARYING DEGREES OF LABOR MOBILITY



Notes: We report the loss of welfare under timeless-perspective commitment, computed as a percentage of steady state aggregate consumption (multiplied by 100) for various model economies and conditional on different shock configurations. We impose asymmetric price stickiness, with the average duration of the price of non-durables set at 4 quarters ($\theta_n = 0.75$), whereas we reduce the duration of durable prices to 1.3 quarters ($\theta_d = 0.25$). The left-hand panel reports the loss of welfare under perfectly correlated technology shocks, while in the right-hand panel we consider uncorrelated disturbances. In both cases we rule out cost-push shocks.

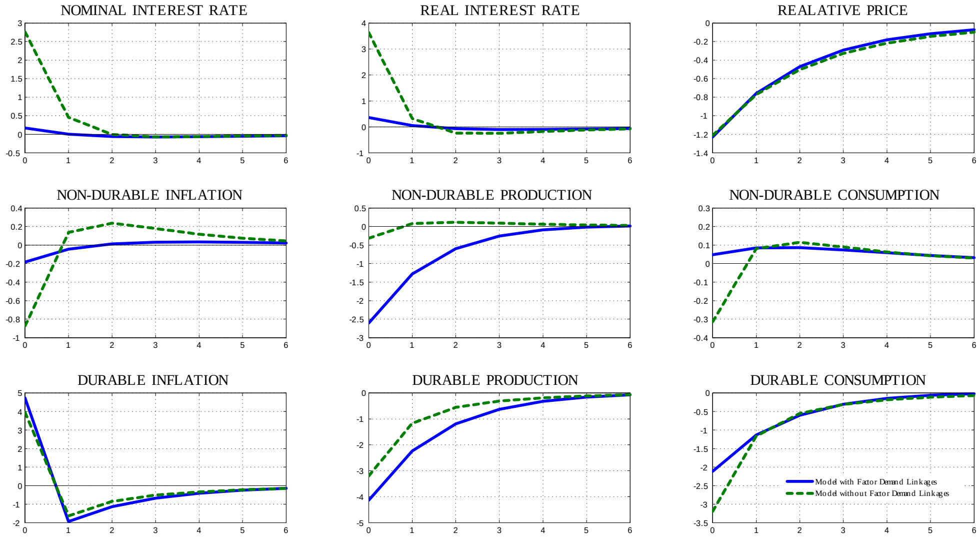
APPENDIX H: Impulse-responses to Shocks in the Durable Goods Sector

FIGURE 1.H: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE DURABLE GOODS SECTOR



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.

FIGURE 2.H: IMPULSE RESPONSES TO A COST-PUSH SHOCK IN THE DURABLE GOODS SECTOR



Notes: All variables but the nominal and real rate of interest are reported in percentage deviation from their level under flexible prices. In the model without sectoral linkages the responses of production and consumption of the same type of good are equivalent.