Input-Output Interactions and Optimal Monetary Policy*

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Abstract

This paper deals with the implications of factor demand linkages for monetary policy design in a two-sector dynamic general equilibrium model. Part of the output of each sector serves as a production input in both sectors, in accordance with a realistic input-output structure. Strategic complementarities induced by factor demand linkages significantly alter the transmission of shocks and amplify the loss of social welfare under optimal monetary policy, compared to what is observed in standard two-sector models. The distinction between value added and gross output that naturally arises in this context is of key importance to explore the welfare properties of the model economy. A flexible inflation targeting regime is close to optimal only if the central bank balances inflation and value added variability. Otherwise, targeting gross output variability entails a substantial increase in the loss of welfare.

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Introduction

This paper deals with the implications of factor demand linkages for monetary policy design. We build a dynamic stochastic general equilibrium (DSGE) model with two sectors that produce services and manufactured goods. The gross output of each sector serves either as a final consumption good, or as an intermediate input in both sectors, according to a realistic input-output structure.

Factor demand linkages are empirically relevant[1] and their importance in the transmission of both sectoral and aggregate shocks has long been recognized by the literature exploring the sources and channels of propagation of the business cycle. Horvath (1998, 2000) shows that cross-industry flows of input materials can reinforce the effect of sectoral shocks, generating aggregate fluctuations and co-movement between sectors, as originally hinted by Long and Plosser (1983).[2] More recently, Bouakez, Cardia, and Ruge-Murcia (2011) and Sudo (2008) have shown that factor demand linkages help at generating positive co-movement between non-durable and durable spending in the face of a monetary innovation, thus overcoming the limits of standard two-sector models that feature heterogeneous degrees of price stickiness across sectors.[3] However, none of these papers has taken a normative perspective. The novel contribution of the present study is to explore how monetary policy should be pursued in a model with cross-industry flows of input materials.

In our framework the monetary authority cannot attain the Pareto optimal allocation consistent with the full stabilization of output and inflation. Thus, we explore optimal monetary policy under the assumption that the policy maker can credibly commit to a policy rule derived from the minimization of a social welfare function. The loss function balances, along with sectoral inflation variability, a preference to reduce fluctuations in aggregate consumption (or, equivalently, value added). Given the natural distinction between consumption and production in the presence of input materials, it is no longer irrelevant whether the monetary authority targets the output gap or the consumption gap. This result has important implications for both the transmission of exogenous shocks and the selection of policy regimes as alternatives to the optimal policy under commitment.

Introducing factor demand linkages into an otherwise standard two-sector model amplifies the loss of social welfare and alters the transmission of shocks to the system, compared to the benchmark economy without input materials. A distinctive feature of the model is that a technology shock to either sector also affects potential output in the other sector, even if preferences over different types of consumption goods are separable.

[1] Input-output structures are a pervasive feature of industrialized economies. Most of the goods in the economy are used for both consumption purposes and as intermediates in other sectors, in accordance with dense networks of factor demand linkages. Bouakez, Cardia, and Ruge-Murcia (2009) and Holly and Petrella (2010) report extensive evidence on the importance of input-output interactions. In this respect, we should note that the cost of intermediate goods corresponds to the largest share in the total cost of production. Dale Jorgenson's data on input expenditures by US industries show that input materials (including energy) account for roughly 50% of outlays, while labor and capital account for 34% and 16%, respectively.

[2] Kim and Kim (2006) show that a similar mechanism generates widespread co-movement of economic activity across sectors. In a similar vein, Carvalho (2009) explores the network structure of intersectoral trade.

[3] Although the co-movement puzzle has been emphasized in connection with the dichotomy between durables and non-durables (Barsky et al., 2007), sectoral co-movement is an inherent feature of the business cycle (see, e.g., Hornstein and Praschnik, 1997 and Christiano and Fitzgerald, 1998) that multi-sector DSGE models need to be able to replicate.

Furthermore, factor demand linkages imply that the relative price of services not only affects the marginal rate of substitution between manufactured goods and services, but also exerts a positive (negative) impact on the real marginal cost in the manufacturing (services) sector. The relative magnitude of the second effect depends on the off-diagonal elements in the input-output matrix.

Beyond reconciling conventional two-sector DSGE models with a realistic structure of the economy, this paper detects important differences between the way monetary policy should be pursued and what is otherwise prescribed by the existing literature on multi-sector models without factor demand linkages. We compare the welfare properties of the model under alternative policy regimes and show that a flexible inflation targeting regime delivers a welfare loss close to that attained under the optimal policy. Most importantly, the central bank attains a smaller loss when fluctuations in aggregate or core inflation are balanced with those in real value added, compared to the loss induced by targeting gross output. We also consider the case of asymmetric price stickiness, which implies a natural divergence between core and aggregate inflation. Although such a difference is still relevant within our framework, targeting either core or aggregate inflation makes little difference in terms of welfare loss. By contrast, what matters is the term accounting for real volatility: in this respect, targeting the consumption gap entails substantial benefits compared to targeting the production gap. These results emphasize the distinction between consumption and production that naturally arises in this class of models.

The remainder of the paper is laid out as follows: Section 1 introduces the theoretical setting; Section 2 discusses the calibration of the model economy; Section 3 explores the Pareto optimal outcome; Section 4 studies the implementation of the optimal monetary policy under commitment and compares the resulting loss of social welfare with that attainable under a number of alternative policy regimes. Section 5 concludes.

1 The Model

We develop a DSGE model with two sectors that produce manufactured goods and services, respectively.[4] The model economy is populated by a large number of infinitely-lived households. Each of these is endowed with one unit of time and derives utility from the consumption of services, manufactured goods and leisure. As in Bouakez, Cardia, and Ruge-Murcia (2011) the two sectors of production are connected through factor demand linkages.[5] Goods produced in each sector serve either as a final consumption good, or as an intermediate production input in both sectors. The net flow of intermediate goods between sectors depends on the input-output structure of the supply side.

[4] Both types of consumption goods are non-durable. Petrella and Santoro (2010) study optimal monetary policy in a similar setting, assuming that consumers have preferences defined over both durable and non-durable goods.

[5] Throughout the paper we will refer to factor demand linkages as indicating cross-industry flows of input materials. If a specific feature of the framework is exclusively determined by the use of intermediate goods in the production process (i.e., inter-sectoral relationships are not essential) we will explicitly refer to input materials.

1.1 Producers

Consider an economy that consists of two distinct sectors producing services (sector s) and manufactured goods (sector m). Each sector is composed of a continuum of firms producing differentiated products. Let Y_t^s (Y_t^m) denote gross output of the services (manufacturing) sector:

$$Y_t^i = \left[\int_0^1 \left(Y_{ft}^i \right)^{\frac{\varepsilon_t^i - 1}{\varepsilon_t^i}} df \right]^{\frac{\varepsilon_t^i}{\varepsilon_t^i - 1}}, \qquad i = \{s, m\} \tag{1}$$

where ε_t^i denotes the time-varying elasticity of substitution between differentiated goods in the production composite of sector $i = \{s, m\}$. Each production composite is produced in the "aggregator" sector operating under perfect competition. It is possible to show that a generic firm f in sector i faces the following demand schedule:

$$Y_{ft}^i = \left(\frac{P_{ft}^i}{P_t^i} \right)^{-\varepsilon_t^i} Y_t^i, \qquad i = \{s, m\} \tag{2}$$

where P_t^i is the price of the composite good in the ith sector. Using (1) and (2), the relationship between the firm-specific and the sector-specific price is:

$$P_t^i = \left[\int_0^1 \left(P_{ft}^i \right)^{1-\varepsilon_t^i} df \right]^{\frac{1}{1-\varepsilon_t^i}}, \qquad i = \{s, m\}. \tag{3}$$

Sectors are related by factor demand linkages. Part of the output of each sector serves as an intermediate input in both sectors. The allocation of output produced in the ith sector is such that:

$$Y_t^i = C_t^i + M_t^{is} + M_t^{im}, \qquad i = \{s, m\} \tag{4}$$

where C_t^i denotes the amount of consumption goods produced by sector i, while M_t^{is} (M_t^{im}) is the amount of goods produced in sector i and used as input materials in sector s (m).

The production technology of a generic firm f in sector i is:

$$Y_{ft}^i = Z_t^i \left[\frac{\left(M_{ft}^{si} \right)^{\gamma_{si}} \left(M_{ft}^{mi} \right)^{\gamma_{mi}}}{\gamma_{si}^{\gamma_{si}} \gamma_{mi}^{\gamma_{mi}}} \right]^{\alpha_i} \left(L_{ft}^i \right)^{1-\alpha_i}, \qquad i = \{s, m\} \tag{5}$$

where Z_t^i ($i = \{s, m\}$) is a sector-specific productivity shock, L_{ft}^i denotes the number of hours worked in the fth firm of sector i, M_{ft}^{ji} ($j = \{s, m\}$) denotes material inputs produced in sector j and supplied to firm f in sector i. Moreover, γ_{ij} ($i, j = \{s, m\}$) denotes the generic element of the input-output matrix, Γ, and corresponds to the steady state share of total intermediate goods used in the production of sector j and supplied by sector i. The input-output matrix is normalized, so that the elements of each column sum up to one: $\sum_{j=\{s,m\}} \gamma_{js} = 1$ (and $\sum_{j=\{s,m\}} \gamma_{jm} = 1$).

Material inputs are combined according to a CES aggregator:

$$M_{ft}^{ji} = \left[\int_0^1 \left(M_{kf,t}^{ji} \right)^{\left(\varepsilon_t^j - 1\right)/\varepsilon_t^j} dk \right]^{\varepsilon_t^j / \left(\varepsilon_t^j - 1\right)}, \tag{6}$$

where $\left\{M^{ji}_{kf,t}\right\}_{k\in[0,1]}$ is a sequence of intermediate inputs produced in sector j by firm k, which are employed in the production process of firm f in sector i.

Firms in both sectors set prices given the demand functions reported in (2). They are also assumed to adjust their price with probability $1-\theta_i$ in each period. When they are able to do so, they set the price that maximizes expected profits:

$$\max_{P^i_{ft}} E_t \sum_{n=0}^{\infty} (\beta\theta_i)^n \Omega_{t+n} \left[P^i_{ft+n}\left(1+\tau_i\right) - MC^i_{ft+n}\right] Y^i_{ft+n}, \qquad i=\{s,m\} \tag{7}$$

where Ω_t is the stochastic discount factor (consistent with households' maximizing behavior, which is described in the next subsection), τ_i is a subsidy to producers in sector i, while MC^i_{ft} denotes the marginal cost of production of firm f in sector i. The optimal pricing choice, given the sequence $\{P^s_t, P^m_t, Y^s_t, Y^m_t\}$, reads as:

$$\overline{P}^i_{ft} = \frac{\varepsilon^i_t}{\left(\varepsilon^i_t - 1\right)\left(1+\tau_i\right)} \frac{E_t \sum_{n=0}^{\infty}(\beta\theta_i)^n \Omega_{t+n} MC^i_{ft+n} Y^i_{ft+n}}{E_t \sum_{n=0}^{\infty}(\beta\theta_i)^n \Omega_{t+n} Y^i_{ft+n}}, \qquad i=\{s,m\}. \tag{8}$$

Note that assuming time-varying elasticities of substitution translates into sectoral cost-push shocks that allow us to account for sector-specific shift parameters in the supply schedules.

In every period each firm solves a cost minimization problem to meet demand at its stated price. The first order conditions from this problem result in the following relationships:

$$MC^i_{ft} Y^i_{ft} = \frac{W^i_t L^i_{ft}}{1-\alpha_i} = \frac{P^s_t M^{si}_{ft}}{\alpha_i \gamma_{si}} = \frac{P^m_t M^{mi}_{ft}}{\alpha_i \gamma_{mi}}, \qquad i=\{s,m\}. \tag{9}$$

It is useful to express the sectoral real marginal cost as a function of the relative price, $Q_t = P^s_t / P^m_t$, and the sectoral real wage:

$$\frac{MC^s_t}{P^s_t} = \frac{\overline{\phi}^s \left(Q_t^{-\gamma_{ms}}\right)^{\alpha_s} \left(RW^s_t\right)^{1-\alpha_s}}{Z^s_t}, \tag{10}$$

$$\frac{MC^m_t}{P^m_t} = \frac{\overline{\phi}^m \left(Q_t^{\gamma_{sm}}\right)^{\alpha_m} \left(RW^m_t\right)^{1-\alpha_m}}{Z^m_t}, \tag{11}$$

where, for $i=\{s,m\}$, $RW^i_t = W^i_t/P^i_t$ is the real wage in sector i and $\overline{\phi}^i$ is a convolution of the production parameters ($\overline{\phi}^i = \alpha_i^{\alpha_i}\left(1-\alpha_i\right)^{1-\alpha_i}$).

From (10) and (11) it is clear that the relative price exerts a direct effect on the real marginal cost of each sector, whose magnitude depends on the size of the cross-industry flows of input materials.[6] Specifically, for the ith sector the absolute impact of Q_t on

[6] Note that under a diagonal input-output matrix, which rationalizes a two-sector model with a pure roundabout structure, the relative price does not affect the real marginal cost. In this case a higher share of intermediate goods dampens the impact of the real wage on the real marginal cost, increasing strategic complementarity in price-setting among firms in the same sector. In turn, this may determine large output effects in the face of a disturbance to nominal spending (see Basu, 1995 and Woodford, 2003, pp. 170-173). This effect is still at work within the general structure we envisage. In addition, cross-industry flows of input materials induce strategic complementarities between sectors (Horvath, 1998).

MC_t^i/P_t^i is related to the "importance" of the other sector as input supplier, i.e. on the magnitude of the off-diagonal elements in the input-output matrix (γ_{sm} and γ_{ms}). This is a distinctive feature of the framework we deal with. By contrast, in traditional multi-sector models without factor demand linkages (e.g., Aoki, 2001), the relative price only affects the real marginal cost indirectly, through the marginal rate of substitution between different consumption goods.[7]

1.2 Consumers

Households derive income from working in firms in the two sectors, investing in bonds, and from the stream of profits generated by the production sectors. They have preferences defined over a composite of services (C_t^s), manufactured goods (C_t^m) and labor (L_t). They maximize the expected present discounted value of their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{H_t^{1-\sigma}}{1-\sigma} - \varrho \frac{L_t^{1+v}}{1+v} \right], \tag{12}$$

where $H_t = (C_t^s)_t^{\mu_s} (C_t^m)^{\mu_m}$, μ_s and $\mu_m (= 1-\mu_s)$ denote the expenditure shares on services and manufactured goods, β is the discount factor, σ is the inverse of the intertemporal elasticity of substitution, v is the inverse of the Frisch elasticity of labor supply.

The following sequence of (nominal) budget constraints applies:

$$\sum_{i=\{s,m\}} P_t^i C_t^i + B_t = R_{t-1} B_{t-1} + \sum_{i=\{s,m\}} W_t^i L_t^i + \sum_{i=\{s,m\}} \Psi_t^i - T_t, \tag{13}$$

where B_t denotes a one-period risk-free nominal bond remunerated at the gross risk-free rate R_t, and T_t denotes a lump-sum tax paid to the government. The term $\Psi_t^s + \Psi_t^m$ captures the nominal flow of dividends from both sectors of production.

We assume that labor can be either supplied to sector s or m according to a CES aggregator:

$$L_t = \left[\phi^{-\frac{1}{\lambda}} \left(L_t^s \right)^{\frac{1+\lambda}{\lambda}} + (1-\phi)^{-\frac{1}{\lambda}} \left(L_t^m \right)^{\frac{1+\lambda}{\lambda}} \right]^{\frac{\lambda}{1+\lambda}}, \tag{14}$$

where λ denotes the elasticity of substitution in labor supply, and ϕ is the steady state ratio of labor supply in the services sector over the total supply of labor (i.e., $\phi = L^s/L$). This functional form allows us to account for different degrees of labor mobility between sectors, depending on λ.[8] To see this, we report the following relationship, which can be retrieved from the first order conditions of consumers' utility maximization with respect

[7] Similarly, in a model with vertical input linkages the relative price only exerts a direct effect on the real marginal cost of the final goods sector. Nevertheless, it can still be related to the real marginal cost of the intermediate goods sector through the marginal rate of substitution between consumption and leisure (see Huang and Liu, 2005 and Strum, 2009).

[8] Empirical evidence suggests that labor is not perfectly mobile across sectors. Davis and Haltiwanger (2001) support this view, finding limited labor mobility across sectors in response to monetary and oil shocks.

to L_t^s and L_t^m:[9]

$$\left(\frac{\phi}{1-\phi}\right)^{-\frac{1}{\lambda}}\left(\frac{L_t^s}{L_t^m}\right)^{\frac{1}{\lambda}}=\frac{W_t^s}{W_t^m}. \tag{15}$$

For $\lambda = 0$ labor is prevented from moving across sectors and its relative supply is constantly equal to the steady state level. By contrast, for $\lambda = \infty$ workers devote all time to the sector paying the highest wage. Hence, at the margin, all sectors pay the same hourly wage, so that households are indifferent to allocating their time to work for one sector or the other. Thus, perfect labor mobility is attained. For $\lambda < \infty$ hours worked are not perfect substitutes. An interpretation of this is that workers have a preference for diversity of labor and would prefer working closer to an equal number of hours in each sector even in the presence of wage differences across sectors.[10]

1.3 The Government and the Monetary Authority

The government serves two purposes in the economy. First, it delegates monetary policy to an independent central bank. We assume that the short-term nominal interest rate is used as the instrument of monetary policy and that the policy maker is able to precommit to a time-invariant rule. In Section 4 we study monetary policy under alternative policy regimes characterized by different loss functions that the central banker commits to minimize.

The second task of the government consists of taxing households and providing subsidies to firms to eliminate distortions arising from monopolistic competition in the markets for both classes of consumption goods. This task is pursued via lump-sum taxes that maintain a balanced fiscal budget.

1.4 Market Clearing

The allocation of output produced by each sector requires that sectoral gross output is partly sold on the markets for consumption goods, while a proportion is sold on the markets for input materials. Therefore, (4) must be met in each sector. Consequently, aggregate production is greater than aggregate consumption, which in this setting can be regarded as the empirically relevant definition of real value added (or GDP).[11]

2 Solution and Calibration

To solve the model, we log-linearize behavioral equations and resource constraints around the non-stochastic steady state and then take the deviation from their counterparts under flexible prices. The difference between log-variables under sticky prices and their

[9] The first order conditions for the consumers' problem are reported in Appendix A.

[10] Horvath (2000) motivates a similar specification based on the desire to capture some degree of sector-specificity to labor while not deviating from the representative consumer/worker assumption. However, an important difference between (14) and the CES aggregator used by Horvath (2000) is that the former allows us to neutralize the impact of labor market frictions in the steady state.

[11] See also Nakamura and Steinsson (2008b) on the distinction between value added and gross output in multi-sector models with input materials.

linearized steady state is denoted by the symbol "$\hat{}$", while we use "$*$" to denote percent deviations of variables in the efficient equilibrium (i.e., flexible prices and constant elasticities of substitution) from the corresponding steady state value. Finally, we use "$\tilde{}$" to denote the difference between linearized variables under sticky prices and their counterparts in the efficient equilibrium.[12]

It is useful to report the system of dynamic equations describing the log-linearized economy:

$$\widetilde{c}_t^s = \gamma^{-1}\left(\widehat{r}_t - E_t\pi_{t+1}^s - rr_t^*\right) + E_t\widetilde{c}_{t+1}^s + \gamma^{-1}\mu_m\left(1-\sigma\right)\left(E_t\widetilde{c}_{t+1}^m - \widetilde{c}_t^m\right), \tag{16}$$

$$\widetilde{c}_t^s = \widetilde{c}_t^m - \widetilde{q}_t, \tag{17}$$

$$\widetilde{rw}_t^s = -\gamma\widetilde{c}_t^s - (1-\sigma)\mu_m\widetilde{c}_t^m + \vartheta(1-\phi)\widetilde{l}_t^m + \left(\vartheta\phi + \lambda^{-1}\right)\widetilde{l}_t^s, \tag{18}$$

$$\widetilde{l}_t^s = \lambda\left(\widetilde{rw}_t^s - \widetilde{rw}_t^m + \widetilde{q}_t\right) + \widetilde{l}_t^m, \tag{19}$$

$$\widetilde{y}_t^s = \alpha_s\gamma_{ss}\widetilde{m}_t^{ss} + \alpha_s\gamma_{ms}\widetilde{m}_t^{ms} + (1-\alpha_s)\widetilde{l}_t^s, \tag{20}$$

$$\widetilde{y}_t^m = \alpha_m\gamma_{sm}\widetilde{m}_t^{sm} + \alpha_m\gamma_{mm}\widetilde{m}_t^{mm} + (1-\alpha_m)\widetilde{l}_t^m, \tag{21}$$

$$\widetilde{y}_t^s = \left(C^s/Y^s\right)\widetilde{c}_t^s + \left(M^{ss}/Y^s\right)\widetilde{m}_t^{ss} + \left(M^{sm}/Y^s\right)\widetilde{m}_t^{sm}, \tag{22}$$

$$\widetilde{y}_t^m = \left(C^m/Y^m\right)\widetilde{c}_t^m + \left(M^{ms}/Y^m\right)\widetilde{m}_t^{ms} + \left(M^{mm}/Y^m\right)\widetilde{m}_t^{mm}, \tag{23}$$

$$\widetilde{rmc}_t^s = \widetilde{rw}_t^s + \widetilde{l}_t^s - \widetilde{y}_t^s, \tag{24}$$

$$\widetilde{rmc}_t^m = \widetilde{rw}_t^m + \widetilde{l}_t^m - \widetilde{y}_t^m, \tag{25}$$

$$\widetilde{rmc}_t^s = \widetilde{m}_t^{ss} - \widetilde{y}_t^s, \tag{26}$$

$$\widetilde{rmc}_t^m = \widetilde{m}_t^{sm} + \widetilde{q}_t - \widetilde{y}_t^m, \tag{27}$$

$$\widetilde{rmc}_t^s = \widetilde{m}_t^{ms} - \widetilde{q}_t - \widetilde{y}_t^s, \tag{28}$$

$$\widetilde{rmc}_t^m = \widetilde{m}_t^{mm} - \widetilde{y}_t^m, \tag{29}$$

$$\pi_t^s = \beta E_t\pi_{t+1}^s + \kappa_s\widetilde{rmc}_t^s + \eta_t^s, \tag{30}$$

$$\pi_t^m = \beta E_t\pi_{t+1}^m + \kappa_m\widetilde{rmc}_t^m + \eta_t^m, \tag{31}$$

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^s - \pi_t^m - \Delta q_t^*, \tag{32}$$

where $\gamma = (1-\sigma)\mu_s - 1$, $\vartheta = \left(v - \frac{1}{\lambda}\right)$, $\kappa_i = (1-\beta\theta_i)(1-\theta_i)\theta_i^{-1}$ $(i = \{s, m\})$ and η_t^s and η_t^m are reduced-form expressions for the time-varying cost-shift parameters in the sectoral New Keynesian Phillips curves. In Section 4 we will consider alternative monetary policy rules to close the model.

The model is calibrated at a quarterly frequency. We assume that the discount factor $\beta = 0.993$. We set $\sigma = 1$, a value in line with Ngai and Pissarides (2007), that implies separability in the utility derived from different consumption goods. Following Horvath (2000), we set the expenditure share on services (μ_s) to 0.62. He measures the consumption weights as the average expenditure shares in the National Income and Product Accounts (NIPA), from 1959 to 1995. The inverse of the Frisch elasticity of

[12] The steady state conditions are reported in Appendix B. We omit the time subscript to denote variables in the steady state. Appendix C presents the economy under flexible prices.

labor supply (v) is set to 3, while $\lambda = 1$, which reflects limited labor mobility.[13] As to the parameters included in the production technologies of the two sectors, we refer to Bouakez, Cardia, and Ruge-Murcia (2009) and set $\alpha_s = 0.50$ and $\alpha_m = 0.75$. The entries of the input-output matrix are set in accordance with the input-use table of the US economy: $\gamma_{ss} = 0.86$ and $\gamma_{sm} = 0.41$.[14] These numbers are in line with the entries of the US input-use table. These values imply a positive net flow of input materials from the services sector to the manufacturing sector.[15] At different stages of the analysis we allow for both symmetric and asymmetric degrees of nominal rigidity across sectors. In the symmetric case we set $\theta_s = \theta_m = 0.75$. Finally, we assume that sectoral elasticities of substitution have a steady state value equal to 11.

As discussed above, the system features two sector-specific technology shocks, z_t^s and z_t^m and two cost-push shocks, η_t^s and η_t^m. Exogenous variables are assumed to follow a first-order stationary VAR with *iid* innovations and diagonal covariance matrix. We set the parameters capturing the persistence and variance of the productivity growth stochastic processes so that $\rho^{z^s} = \rho^{z^m} = 0.95$ and $\sigma^{z^s} = \sigma^{z^m} = 0.02$, respectively. As to the cost-push shocks, we follow Jensen (2002), Walsh (2003) and Strum (2009), and assume that these are purely transitory, with $\sigma^{\eta^s} = \sigma^{\eta^m} = 0.02$.

3 The Pareto Optimum

Removing sources of distortion in the labor market (imperfect labor mobility) and the goods market (monopolistic competition) represents a desirable situation for a benevolent central banker. At this stage of the analysis we are interested in understanding whether, after removing these distortions (in a variant economy without cost-push shocks), the monetary authority can attain a first best allocation where inflation and the output gap in both sectors are jointly stabilized. The answer to this question is negative for general parameter values and shock processes. The following proposition formalizes our results.

Proposition 1 *In the model with sticky prices and perfect labor mobility across sectors, there exists no monetary policy that can attain the Pareto optimal allocation unless the shock buffeting the services sector equals the one buffeting the manufacturing sector, scaled by a factor* $\zeta = (1-\alpha_s)/(1-\alpha_m)$.[16]
Proof. *See Appendix D.* ■

This result is in line with Huang and Liu (2005), who emphasize that vertical trade linkages cause both aggregate output and the relative price to fluctuate in response to productivity shocks, unless these are identical, in which case only output would fluctuate.

[13] This value is in line with the calibration proposed by Horvath (2000) and Bouakez, Cardia, and Ruge-Murcia (2011). The literature on two-sector models has generally considered either perfect mobility (e.g., Huang and Liu, 2005 and Strum, 2009) or immobility (e.g., Aoki, 2001 and Erceg and Levin, 2006). A value of $\lambda = 1$ allows us to capture slow equalization of nominal wages, which is consistent with the empirical evidence (e.g., Davis and Haltiwanger, 2001).

[14] These shares have been computed using the table "The Use of Commodities by Industries" for 1992, produced by the Bureau of Economic Activity (BEA). The input-use table has displayed remarkable stability since after the 80's.

[15] Moreover, they imply that the marginal impact of changes in the relative price on the real sectoral marginal cost is, in absolute value, higher for the manufacturing sector, as $\alpha_m > \alpha_s$ and $\gamma_{sm} \gg \gamma_{ms}$.

[16] Allowing for imperfect labor mobility would only further constrain the ability of the monetary authority to neutralize technology shocks.

Therefore, the monetary authority is faced with a trade-off, as it can stabilize either the output gap or the relative price gap, but not both. It is interesting to note that more restrictive conditions are required for the full stabilization of the framework we envisage. Once we assume that input materials are used by both sectors and that productivity shocks are Hicks neutral,[17] not only sectoral innovations need to be perfectly correlated, but the production technologies need to be the same across sectors.[18]

4 Optimal Monetary Policy

As the central bank cannot attain the Pareto optimal allocation we turn our attention to policy strategies capable of attaining second best outcomes. We explore equilibrium dynamics under the assumption that the policy maker can credibly commit to a rule derived from the minimization of his objective function. The optimal policy consists of maximizing the conditional expectation of intertemporal household utility subject to private sector's behavioral equations and resource constraints, as discussed by Woodford (2003).[19]

To evaluate social welfare we take a second-order Taylor approximation to the representative household's lifetime utility.[20] Our procedure follows the standard analysis of Rotemberg and Woodford (1998), adapted to account for the presence of factor demand linkages. The resulting intertemporal social loss function reads as:

$$\begin{aligned}\mathcal{SW}_0 \;\approx\; & -\frac{U_H(H)\,H}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{(\sigma+\nu)\left(\mu_s\widetilde{c}_t^s+\mu_m\widetilde{c}_t^m\right)^2+\varsigma\left[\varpi\left(\pi_t^s\right)^2+(1-\varpi)\left(\pi_t^m\right)^2\right]\right\}\\ & +\text{t.i.p.}+O\left(\|\xi\|^3\right), \qquad (33)\end{aligned}$$

where $\varpi=\phi\varepsilon^s\left(\kappa_s\varsigma\right)^{-1}$, $\varsigma=\phi\varepsilon^s\left(\kappa_s\right)^{-1}+(1-\phi)\,\varepsilon^m\left(\kappa_m\right)^{-1}$, t.i.p. collects the terms independent of policy stabilization and $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher. According to (33) the welfare criterion assumed by the central bank balances, along with sectoral inflation variability, fluctuations in aggregate consumption (or, equivalently, value added). This is a distinctive feature of the model under scrutiny, as the presence of input materials implies a non-trivial distinction between output and con-

[17] Instead, Huang and Liu (2005) assume that technological progress is Harrod neutral in the final goods sector. Under this assumption the Pareto optimum only requires the sectoral technology shocks to be perfectly correlated.

[18] It is also useful to compare our result to Aoki (2001). In his setting, complete stabilization of the two-sector economy is achieved whenever the central bank stabilizes core inflation. Here, instead, factor demand linkages are such that the relative price may differ from its level under flexible prices even if the central bank stabilizes inflation in the stickier sector, unless $\alpha_s=\alpha_m$ and shocks are perfectly correlated. In this case symmetric pass-through of the shocks onto the two sectors can be appreciated and the relative price is authomatically stabilized.

[19] We pursue a "timeless perspective" approach, as in Woodford (1999). This involves ignoring the conditions that prevail at the regime's inception, thus imagining that the commitment to apply the rules deriving from the optimization problem had been made in the distant past. In this case, there is no dynamic inconsistency in terms of the central bank's own decision-making process. The system is solved for the evolution of the endogenous variables by relying on the common practice discussed, e.g., by Sims (2002).

[20] We assume that shocks that hit the economy are not big enough to lead to paths of the endogenous variables distant from their steady state levels. This means that shocks do not drive the economy too far from its approximation point and, therefore, a linear quadratic approximation to the policy problem leads to reasonably accurate solutions. Appendix E reports the derivation of the quadratic welfare function.

sumption. Therefore, it is no longer irrelevant whether the central bank targets output or consumption gap variability.

The weights of the time-varying terms in (33) can be interpreted as follows: (i) ς indexes the total degree of nominal stickiness in the economy and is inversely related to both κ_m and κ_s; (ii) ϖ accounts for the relative degree of price stickiness in the services sector. Also note that the relative importance of sector-specific inflation variability depends on the steady state ratio of labor supplied to the services sector to the total labor force (ϕ).[21]

How do factor demand linkages influence social welfare? Figure 1 reports the loss defined over the subspace of the production parameters α_s and α_m, for different shock configurations. The general pattern suggests that welfare loss increases monotonically in the share of intermediate goods used to produce services, whereas changes in the income share of input materials in the manufacturing sector exert a negligible impact.[22] Such an asymmetric impact can be ascribed to the services goods sector being the largest sector and a net supplier of input materials.

Insert Figure 1 about here

To appreciate the actual contribution of cross-industry input-output interactions to the loss of social welfare, we compare the loss in the model with factor demand linkages to that obtained in a model that features a pure roundabout input-output structure (i.e., $\gamma_{ss} = \gamma_{mm} = 1$).[23] Figure 2 reports the difference between the two losses over the production parameters subspace. When both technology and cost push shocks are in place, factor demand linkages between sectors induce an attenuation in the welfare loss with respect to the alternative case. Attenuation increases in both α_s and α_m, though the marginal effect of an increase in the income share of input materials used by the manufacturing sector on the loss of welfare is greater than that registered for the services sector. This reflects the role of the services sector as the main input supplier of the economy. Importantly, attenuation is less evident when only cost push shocks are considered. Thus, cross-industry flows of input materials are effective in attenuating the loss of welfare (compared to the alternative case), to the extent that technology shocks can be regarded as one of the drivers of the business cycle. In fact, factor demand linkages are such that sectoral productions under flexible prices always display positive co-movement, even in the presence of asymmetric technology shocks. This feature implies an endogenous attenuation of fluctuations in the sectoral consumption gaps that will be appreciated in further detail in the impulse-response analysis of the next section.

Insert Figure 2 about here

We are not only concerned with the direct welfare implications of factor demand linkages, but also with central banks' potential misperception about their role in the production process. Neglecting cross-industry flows of input materials is likely to generate excess loss with respect to the welfare criterion consistent with the correctly specified

[21] When $\alpha_s = \alpha_m$ it follows that $\phi = \frac{L^s}{L} = \frac{Y^s}{Y^s+Y^m}$.

[22] It could be noted that increasing loss may simply emerge as the joint outcome of increasing the importance of the production factor characterized by price stickiness (i.e., input materials), while decreasing the impact of labor, which is remunarated at a flexible wage. However, note that limited labor mobility still implies a real distortion in the labor market.

[23] Such a production structure would be similar to that employed by Basu (1995).

model economy. To address this issue we implement the optimal policy under the assumption that $\alpha_s = \alpha_m = 0$. Figure 3 graphs the (percentage) excess loss under misperception with respect to that attained under the "true" production structure. Excess loss increases in the actual intensity of use of input materials. Also note that the marginal impact of misperceiving α_s is greater than that associated with α_m. Once again, accounting for the services sector as the largest sector is the key for interpretation of this result.

Insert Figure 3 about here

4.1 Impulse-response Analysis under Optimal Monetary Policy

To isolate the contribution of factor demand linkages to the transmission of shocks under the optimal policy, we compare our baseline setting with both models where no cross-industry flows of input materials are in place and models that rule out input materials. Figure 4 reports equilibrium dynamics following a one-standard-deviation technology shock in the services sector, under different assumptions about the production structure.[24] All variables but the interest rate are reported in deviation from their frictionless level. Symmetric nominal rigidity is assumed, with $\theta_s = \theta_m = 0.75$.[25]

Insert Figure 4 about here

A technology shock in the services sector causes the production services to become relatively cheaper. However, their price is prevented from reaching the level consistent with flexible prices, thus inducing a negative consumption gap of services. Negative co-movement between the production gaps of the services and manufacturing sectors can be appreciated in the models without cross-industry flows of input materials. In these frameworks changes in the relative price only affect relative consumption of the two goods through the marginal rate of substitution. Therefore, in the face of a sectoral technology shock consumers substitute away from the consumption good produced by the sector which has not been directly hit by the shock to consume more of the good that has become relatively cheaper. Even in the model with cross-industry flows of input materials negative co-movement between the consumption gaps arises due to this substitution effect. However, factor demand linkages induce positive co-movement in the production gaps of manufactured goods and services through increased demand of input materials from both sectors.

Assuming different structures of the production economy has important effects on the way optimal monetary policy should be pursued. In the models without factor demand linkages the real interest rate (measured in units of services) initially rises, thus preventing output (and consumption) in the services sector from rising as much as it would do under flexible prices. Concurrently, the real rate of interest does not rise enough to prevent the output gap in the manufacturing sector from rising too much. By contrast, in the model

[24] The responses to a sectoral innovation (i.e., a technology or a cost-push shock) in the manufacturing sector mirror in the opposite direction those induced by an analogous shock in the services sector. For this reason, and for brevity of exposition, we skip their description. These results are available upon request from the authors.

[25] As in Strum (2009) we opt for this choice to prevent the central bank from focusing exclusively on the stickier sector in the formulation of its optimal policy, as predicted by Aoki (2001). In the next subsection we draw implications from the model under asymmetric degrees of nominal rigidity across sectors.

with factor demand linkages the real interest rate initially decreases, gradually converging to its equilibrium level thereafter. The resulting real interest rate gap is significantly smaller than that appreciated in the models without factor demand linkages. This effect is intimately related to the existence of cross-industry flows of input materials that induce the consumption of manufactured goods under flexible prices to increase, thus helping to reduce their consumption gap. Such an endogenous mechanism is not at work in the models without factor demand linkages, where the consumption of manufactured goods under flexible prices is not affected by the shock, as a result of setting $\sigma = 1$ (which implies separability of households' preferences in the consumption of different goods).

As to the response of prices, the inter-sectoral intermediate input channel is responsible for attenuating deflationary pressures in the services sector while inducing higher inflation in the manufacturing sector, compared to the model without input materials. It is worth recalling that, in the presence of cross-industry flows of input materials, the relative price does not only have a direct effect on the marginal rate of substitution between manufactured goods and services. As shown by equations (10) and (11), Q_t also exerts a positive (negative) effect on the real marginal cost in the manufacturing (services) sector. Thus, the positive relative price gap reinforces the conventional inflationary effect on manufacturing inflation through its influence on the real marginal cost. This effect, combined with the expansionary policy pursued by the central bank, determines stronger inflationary pressures at the aggregate level in the model with factor demand linkages, compared to the economy without intermediate inputs. When comparing our benchmark economy to that featuring roundabout production, deflation in the services sector is still attenuated due to the direct impact of the relative price on firms' real cost of production. As to the manufacturing sector, the model with pure roundabout technology produces much stronger inflationary pressures, which are driven by the substantial increase in the production gap.

Figure 5 reports equilibrium dynamics following a cost-push shock in the services sector. Both the model with factor demand linkages and that with roundabout production display an attenuation in the deflationary response of the inflation rate of manufactured goods, compared to the model without input materials. However, whereas in the roundabout setting attenuation is induced by a lower slope of the NKPC - a result already observed by Basu (1995) - a distinctive feature of the model with cross-industry flows of input materials is the effect of the relative price on the marginal cost of firms producing manufactured goods. A positive $\widetilde{q}_t$ partially counteracts the deflationary effect that operates through the conventional demand channel, compared to the model without input materials. Concurrently, contraction in the production of manufactured goods is magnified by the presence of factor demand linkages.

Insert Figure 5 about here

It is worth drawing attention to a subtle difference in the transmission of technology and cost-push shocks in the model with factor demand linkages. Sectoral technology shocks cause the consumption gaps of the two goods to co-move negatively. The drop in the consumption gap of the sector that experiences the positive technology shock is compensated by a rise in the demand gap of intermediate goods from the other sector. Thus, each sector experiences opposite demand effects on the markets for the consumption and intermediate goods. By contrast, a sectoral cost-push shock determines a contraction of final goods consumption in both sectors, which causes a drop in the consumption of both types of intermediate goods, thus resulting in an even greater slump in the gross output.[26] These features of the transmission mechanism have important implications for the selection of alternative policy regimes, as shown in the next section.

4.2 Optimal Monetary Policy versus Alternative Policy Regimes

We now assess the loss of welfare under the optimal policy and various alternative policies.[27] Alternative regimes admit simple loss functions, which are selected because of their suitability to be communicated to and understood by the public. We use the second-order welfare approximation (33) as a model-consistent metric. In each case we compute the expected welfare loss as the percentage of steady state aggregate consumption (and multiply the resulting term by 100).

The following period loss functions are evaluated:

Strict inflation targeting:	$\widetilde{\mathcal{W}}_t^{IT} = \left(\pi_t^{IT}\right)^2$
Gap targeting:	$\widetilde{\mathcal{W}}_t^{GT} = \left(\widetilde{x}_t^{GT}\right)^2$
Flexible inflation targeting:	$\widetilde{\mathcal{W}}_t^{FIT} = (\sigma + \nu)\, \widetilde{\mathcal{W}}_t^{GT} + \varsigma \widetilde{\mathcal{W}}_t^{IT}$

Thus, we consider both strict and flexible inflation targeting regimes, as well as consumption and output gap targeting. Inflation targeting regimes may target either core or aggregate inflation (i.e., $\pi_t^{IT} = \{\pi_t^{core}, \pi_t^{agg}\}$). In the first case the weights attached to the sectoral rates of inflation depend on the relative degree of price rigidity, as well as on the relative size of each sector and the degree of substitutability among differentiated goods ($\pi_t^{core} = \varpi \pi_t^s + (1 - \varpi)\, \pi_t^m$). In the second case the weights attached to sectoral inflations only depend on the relative size of each sector ($\pi_t^{agg} = \phi \pi_t^s + (1 - \phi)\, \pi_t^m$). Flexible inflation targeting regimes balance fluctuations in core or aggregate inflation together with a term that penalizes fluctuations in aggregate consumption or gross output (i.e., $\widetilde{x}_t^{GT} = \{\widetilde{x}_t^c, \widetilde{x}_t^p\}$, where $\widetilde{x}_t^c = \mu_s \widetilde{c}_t^s + \mu_m \widetilde{c}_t^m$ and $\widetilde{x}_t^p = \phi \widetilde{y}_t^s + (1 - \phi)\, \widetilde{y}_t^m$). The weights attached to the terms capturing variability in real activity ($\widetilde{\mathcal{W}}_t^{GT}$) and inflation variability ($\widetilde{\mathcal{W}}_t^{IT}$) are the same as those appearing in the welfare criterion derived from the second-order approximation.[28]

Note that both strict or flexible inflation targeting regimes aim at stabilizing the volatility of aggregate (or core) inflation and not the volatility of sectoral inflations separately. From a strategic viewpoint we are willing to understand whether the central bank can approximate the optimal policy outcome without taking the sectoral rates of inflation as separate objectives, as suggested by the welfare criterion we have derived. In principle, this should enable the monetary authority to provide the public with a more intelligible target. Svensson (1997) stresses the importance of assuming intermediate targets which

[26] Importantly, imperfect labor mobility exacerbates this effect, increasing the wedge between consumption and production. When aggregate demand increases, as labor cannot flow across sectors without frictions, firms need to increase intermediate inputs by more than they would do under the assumption of perfect labor mobility to meet higher demand. Consequently, fluctuations in production and consumption are wider in the presence of imperfect labor mobility.

[27] In this section we still consider the optimal monetary policy from a timeless perspective. Nevertheless, it is important to acknowledge that it would also be possible to construct a discretionary regime with a different loss function capable to outperform commitment (Dennis, 2010).

[28] Assigning the same weights to production and consumption variability in the flexible inflation targeting regime should help us at appreciating the additional loss induced by a central bank that erroneously considers production gap variability and consumption gap variability as interchangeable from the perspective of evaluating the monetary policy trade-off.

are highly correlated with the goal, easy to control, and transparent - so as to enhance communication to the public. In this sense, measures of overall inflation are more suitable than sectoral rates of inflation.[29]

Table 1 reports the welfare loss under the optimal rule and the alternative regimes. The overall loss is disaggregated into the variability of each term weighted in (33). In the first instance both technology and cost-push shocks are assumed to buffet the model economy. Moreover, we assume that sectors are symmetric in the degree of nominal rigidity ($\theta_m = \theta_s = 0.75$): in this case $\pi_t^{core} = \pi_t^{agg}$. Later in this section we will relax this assumption.

Insert Table 1 about here

As expected on a priori grounds, whenever cost-push shocks are accounted for, a flexible inflation targeting regime performs nearly as well as the optimal policy, while other policies display a poor performance. Most importantly, the central bank attains a welfare loss closer to that under the optimal policy when fluctuations in aggregate (or core) inflation are balanced with those in the aggregate value added (i.e., consumption), compared to the loss induced by controlling fluctuations in the gross output (i.e., production). Recall from Section 4.1 that sectoral shocks typically induce higher variability in aggregate production than consumption.[30] Therefore, assuming a welfare criterion that balances fluctuations in the rate of inflation with gross output variability misrepresents the actual trade-off faced by the central bank. Indeed, according to (33) inflation variability should be balanced with fluctuations in consumption rather than gross output.

Insert Table 2 about here

Table 2 reports the relative performance of alternative policy regimes under different sources of exogenous perturbation. Once again, flexible inflation targeting outperforms other alternatives. This is also the case when shocks to either sector are considered separately (see Table 2, columns 3 and 4). As noted by Woodford (2003, pp. 435-443), strict inflation targeting displays a "competitive" performance only when technology shocks are the unique source of perturbation to the system, whereas accounting for cost-push shocks entails a rather poor performance. In addition, it is worth pointing out that production gap targeting outperforms consumption gap targeting only when cost-push shocks are accounted for. In this case, reducing consumption gap volatility allows the central bank to control just part of the volatility in the real marginal cost, whereas targeting the production gap partially accounts for the presence of the intermediate input channel.[31]

[29] Note that accounting for sectoral inflation volatilities entails sizeable advantages, compared to reducing the volatility of an aggregate index of price changes, whenever $\widetilde{q}_t$ has a direct influence on the loss of welfare (as, e.g., Aoki, 2001, Huang and Liu, 2005 and Strum, 2009). In this case, an explicit response to sectoral inflation rates allows the central bank to indirectly control fluctuations in the relative price. However, in our setting relative price volatility does not appear as a direct objective in the loss function. Thus, we should not expect the central bank to incur into greater losses when implementing policies that disregard sectoral inflation volatilities.

[30] A positive cost-push shock contracts the demand for both types of consumption goods, thus depressing the demand of input materials and causing an even greater slump in sectoral gross outputs. Otherwise, when a technology shock takes place, input-output interactions induce positive co-movement between sectoral productions, whereas the consumption gaps move in opposite directions and tend to offset each other.

[31] In turn, sectoral inflation volatility also benefits from this effect (see Table 1, columns 1 and 2).

Otherwise, when technology shocks are considered, consumption gap targeting outperforms production gap targeting; the reason being that the production and consumption gaps at the aggregate level tend to co-move negatively and that the sectoral consumption gaps are less volatile than their production counterparts, as hinted in the analysis of Section 4.1.

Insert Table 3 about here

Table 3 reports the loss of welfare under asymmetric price stickiness, in the form of manufactured goods prices being more flexible than the price of services ($\theta_s = 0.75$, $\theta_m = 0.25$).[32] In this case, core inflation differs from aggregate inflation, as discussed earlier. Considering core inflation targeting as an alternative to aggregate inflation targeting is somewhat related to a long-standing debate concerning the information (in terms of relative sectoral price stickiness) that the central bank can access when formulating its policy. Woodford (2003) shows that, in a two-sector model with no input materials, optimal commitment policy is nearly replicated by an inflation targeting regime, whereby the weights attached to sectoral inflations depend on the relative degree of nominal stickiness.[33] As expected, this result can only be replicated if we rule out sectoral cost-push shocks.[34] Otherwise, when assessing alternative flexible targeting regimes in the model with asymmetric price stickiness, the dichotomy between aggregate and core inflation loses much of the usual appeal in terms of comparing welfare losses. Once again, what seems relevant and inherently connected with the presence of input materials is the distinction between output and consumption. In fact, a flexible inflation targeting regime that balances consumption and (either core or aggregate) inflation variability delivers a loss of welfare systematically lower than that attained under a loss function balancing output and (either core or aggregate) inflation variability, which misrepresents the actual trade-off faced by the central bank.

5 Conclusions

We have integrated inter-sectoral factor demand linkages into a dynamic general equilibrium model with two sectors that produce services and manufactured goods. Part of the output produced in each sector is used as an intermediate input of production in both sectors, according to a realistic input-output structure of the economy. The resulting sectoral interactions have non-negligible implications for the formulation of policies aimed at reducing real and nominal fluctuations. A key role is played by the relative price, which not only acts as an allocative mechanism on the demand side (through its influence on the marginal rate of substitution between different classes of consumption goods), but also on the supply side (through its effect on the sectoral real marginal costs of production).

[32] Bils and Klenow (2004) and Nakamura and Steinsson (2008a) report evidence of higher frequency of price adjustment for manufactured goods than services.

[33] Aoki (2001) shows that the welfare-theoretic loss function consistent with a multi-sector economy with heterogeneous degrees of price stickiness assigns higher weight on the inflation variability of sectors characterized by higher nominal stickiness. This provides a theoretical rationale for seeking to stabilize an appropriately defined measure of "core" inflation rather than an equally weighted price index.

[34] Note that core inflation targeting returns the same loss as that observed under the optimal policy. However, this result only holds when the difference in the sectoral degrees of nominal rigidity is very large, as in the particular case we consider (see also Aoki, 2001).

The presence of input materials implies a non-trivial difference between aggregate consumption (or, equivalently, value added) and gross output. Such a distinction proves to be of crucial importance at different stages of the analysis. In fact, the welfare criterion consistent with the second-order approximation to households' utility reveals that the policy maker is faced with the task of stabilizing fluctuations in the sectoral rates of inflation and aggregate value added, rather than gross output. Moreover, strategic complementarities induced by factor demand linkages alter the transmission of shocks to the system, compared to what is commonly observed in otherwise standard two-sector models.

These results show how accounting for a realistic feature of multi-sector economies entails non-negligible differences with respect to the policy prescriptions retrievable for frameworks that rule out input-output interactions or consider a vertically integrated production structure. The optimal policy can be closely approximated by a flexible inflation targeting regime. However, it is of crucial importance to target consumption gap variability rather than output gap variability. This strategy allows the central bank to avoid inducing additional loss emanating from inter-sectoral complementarities.

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Tables and Figures

TABLE 1: WELFARE UNDER ALTERNATIVE POLICIES

	Loss Components			
	π_t^s	π_t^m	x_t	Total
Optimal Policy	0.4781	0.4747	0.3094	1.2621
Inflation Targeting	0.1224	0.4356	1.9676	2.5256
Consumption Gap Targeting	1.8332	0.6216	0.0000	2.4548
Production Gap Targeting	1.6088	0.5056	0.0461	2.1606
Flexible Inflation Targeting (with Consumption)	0.5413	0.4493	0.2809	1.2716
Flexible Inflation Targeting (with Output)	1.1283	0.4748	0.0652	1.6683

Notes: $x_t = \mu_s \widetilde{c}_t^s + \mu_m \widetilde{c}_t^m$. The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

TABLE 2: WELFARE UNDER ALTERNATIVE POLICIES AND DIFFERENT SHOCK CONFIGURATIONS

	Tech. Shocks	Cost Push Shocks	Ser. Sec. Shocks	Man. Sec. Shocks
Optimal Policy	0.2370	1.0383	0.8003	0.4847
Inflation Targeting	0.2547	2.2958	1.9392	0.6253
Consumption Gap Targeting	0.2867	2.2416	2.0101	0.5338
Production Gap Targeting	0.2935	1.8600	1.5446	0.6152
Flexible Inflation Targeting (with Consumption)	0.2398	1.0458	0.8034	0.4939
Flexible Inflation Targeting (with Output)	0.2802	1.3976	1.1370	0.5458

Notes: The first two columns report the loss attributable to technology shocks and cost-push shocks generated in both sectors, respectively. The last two columns report the loss due to both shocks to either sector. The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

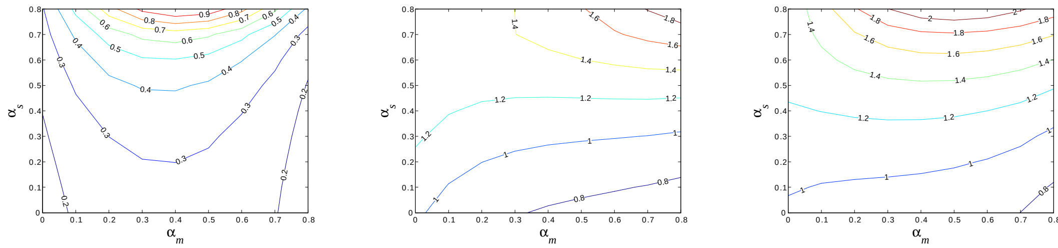
TABLE 3: ASYMMETRIC STICKINESS

	Tech. Shocks	Cost Push Shocks	Both Shocks
Optimal Policy	0.0593	0.7603	0.8159
Core Inflation Targeting	0.0593	2.1953	2.2476
Agg. Inflation Targeting	0.3508	1.0442	1.3554
Consumption Gap Targeting	0.0611	1.8125	1.8711
Production Gap Targeting	0.0596	1.7603	1.8185
Flex. Core Inflation Targeting (with Cons.)	0.0599	0.7607	0.8178
Flex. Agg. Inflation Targeting (with Cons.)	0.1917	0.7634	0.9326
Flex. Core Inflation Targeting (with Prod.)	0.0595	1.0918	1.1483
Flex. Agg. Inflation Targeting (with Prod.)	0.0966	1.0642	1.1598

Notes: We set the average duration of the price of services at 4 quarters ($\theta_s = 0.75$), whereas we reduce the duration of manufactured goods prices to 1.3 quarters ($\theta_m = 0.25$). The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

FIGURE 1: WELFARE UNDER OPTIMAL MONETARY POLICY FOR VARYING α_s AND α_m

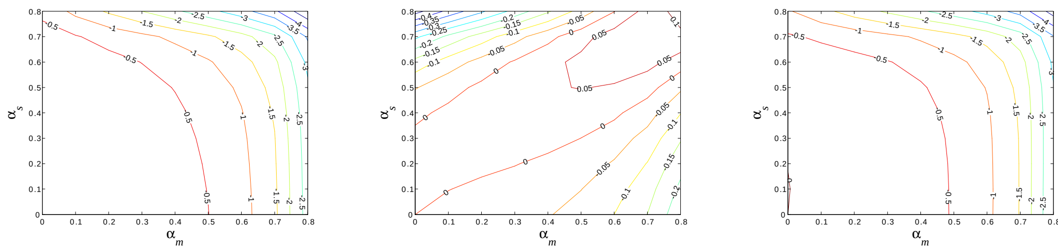
TECH. SHOCKS | COST PUSH SHOCKS | BOTH SHOCKS



Notes: The first panel of the figure reports the welfare loss defined over the subspace of the production parameters in the two sectors when only technology shocks buffet the model economy. The second panel reports analogous evidence under the assumption that cost-push shocks are the only source of exogenous perturbation. The third panel considers both sources of exogenous perturbation. The values of each contour line refer to the loss as a percentage of steady state aggregate consumption.

FIGURE 2: RELATIVE LOSS - FACTOR DEMAND LINKAGES VS. ROUNDABOUT PRODUCTION

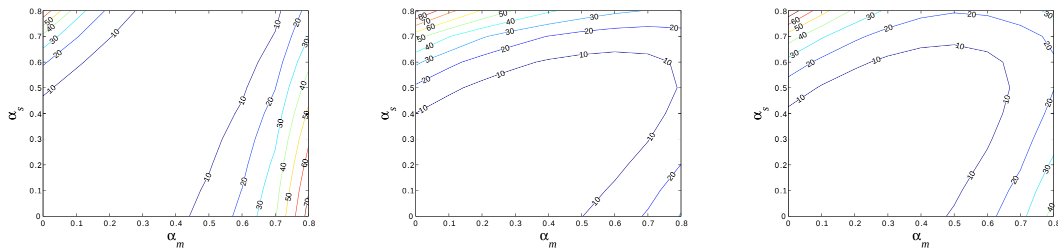
TECH. SHOCKS | COST PUSH SHOCKS | BOTH SHOCKS



Notes: We report the difference in the loss under the model with factor demand linkages and that under roundabout production ($\left|SW_0^{FDL}\right| - \left|SW_0^{Round}\right|$), over the subspace of the production parameters.

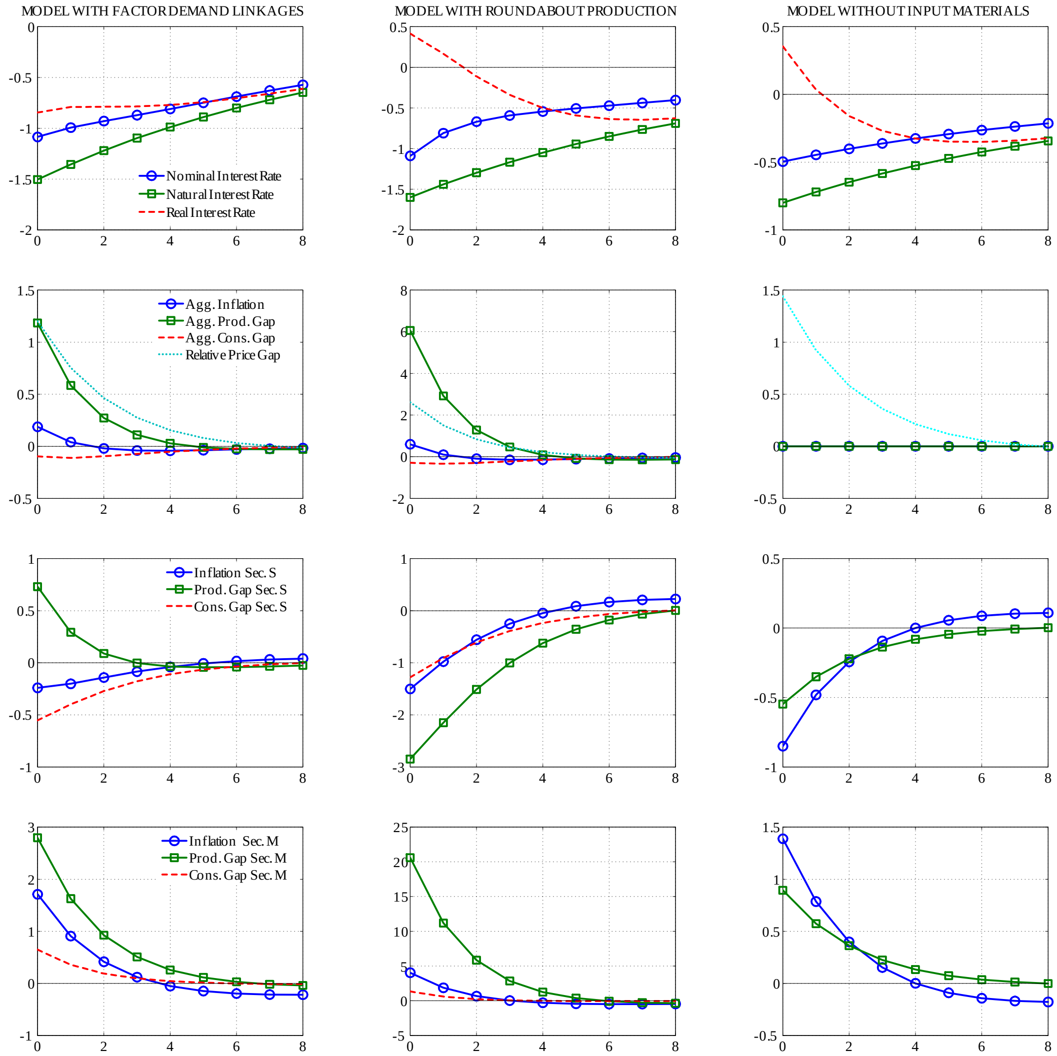
FIGURE 3: FACTOR DEMAND LINKAGES MISPERCEPTION

TECH. SHOCKS | COST PUSH SHOCKS | BOTH SHOCKS



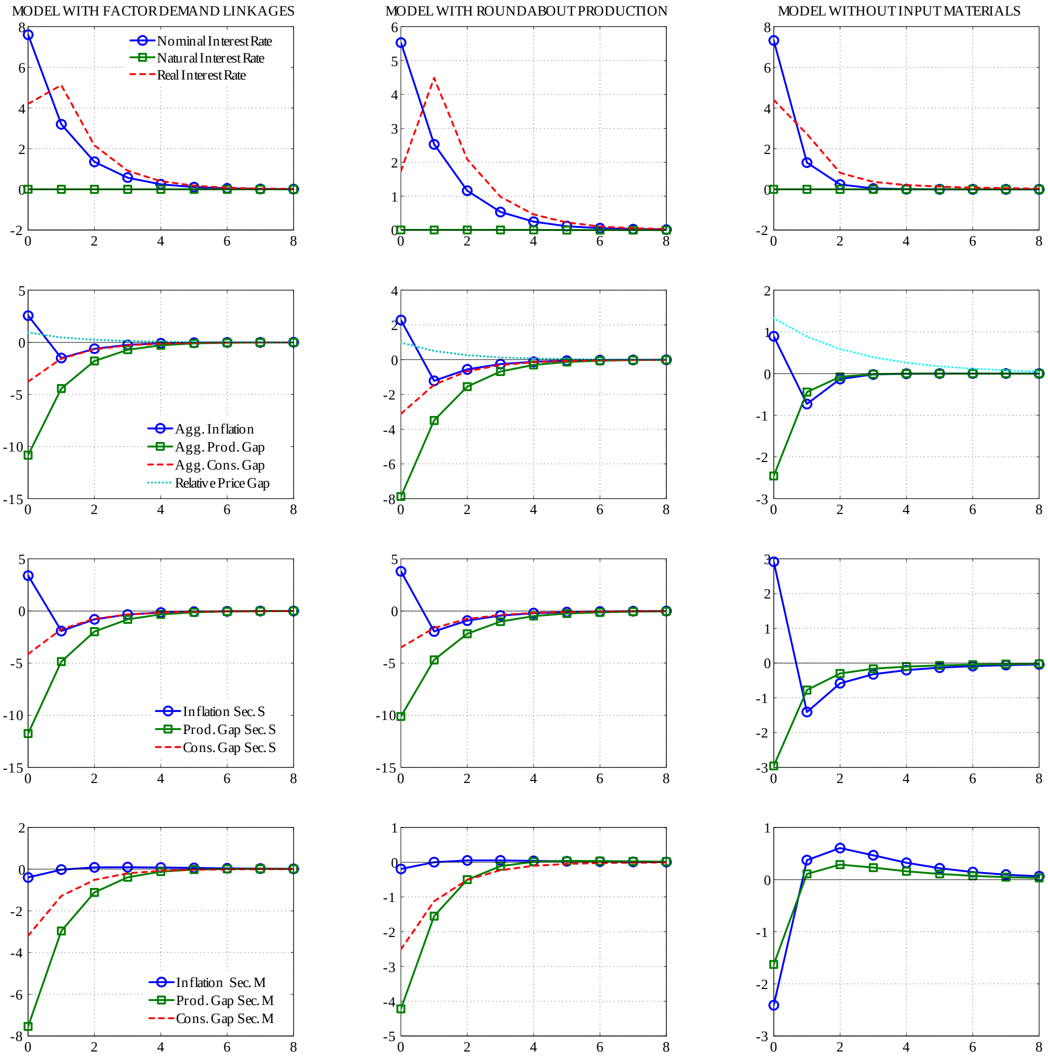
Notes: We report the percentage excess loss under a misperception of the input-output structure with respect to the loss under the correctly specified production structure of the economy, for different shock configurations.

FIGURE 4: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE SERVICES SECTOR



Notes: Equilibrium dynamics is reported under three different settings: (i) the model with factor demand linkages (under the calibration described in Section 2); (ii) the model under roundabout input-output structure ($\gamma_{ss}= \gamma_{mm}= 1$); (iii) the model with no input materials ($\alpha_s= \alpha_m= 0$). Inflation and interest rates are annualized. All variables except interest rates are reported in (percentage) deviation from their frictionless level.

FIGURE 5: IMPULSE RESPONSES TO A COST-PUSH SHOCK IN THE SERVICES SECTOR



Notes: See Figure 4.

APPENDIX A: First Order Conditions from Households' Utility Maximization

Maximizing (12) subject to (14), and (13) leads to a set of first-order conditions that can be re-arranged to obtain:

$$H_t^{1-\sigma}\left(C_t^s\right)^{-1} = \beta R_t E_t\left[\frac{H_{t+1}^{1-\sigma}\left(C_{t+1}^s\right)^{-1}}{\Pi_{t+1}^s}\right], \tag{34a}$$

$$\frac{C_t^m}{C_t^s}H_t^{1-\sigma} = \frac{\mu_m P_t^s}{\mu_s P_t^m}$$

$$W_t^s\frac{\mu_s H_t^{1-\sigma}\left(C_t^s\right)^{-1}}{P_t^s} = \varrho\phi^{-\frac{1}{\lambda}}L_t^{v-\frac{1}{\lambda}}\left(L_t^s\right)^{\frac{1}{\lambda}}, \tag{34b}$$

$$W_t^m\frac{\mu_s H_t^{1-\sigma}\left(C_t^s\right)^{-1}}{P_t^s} = \varrho\left(1-\phi\right)^{-\frac{1}{\lambda}}L_t^{v-\frac{1}{\lambda}}\left(L_t^m\right)^{\frac{1}{\lambda}}. \tag{34c}$$

APPENDIX B: Some Useful Steady State Relationships

As in the competitive equilibrium real wage in each sector equals the marginal product of labor. Thus, we can derive the following relationship between the production in services and manufactured goods in the steady state:

$$\frac{Y^s}{Y^m} = \frac{(1-\alpha_m)\phi}{(1-\alpha_s)\left(1-\phi\right)}Q^{-1}.$$

Furthermore, the following relationship between services and manufactured goods consumption can be derived from the Euler conditions:

$$\frac{C^s}{C^m} = \frac{\mu_s}{\mu_m}Q^{-1}.$$

Moreover, the following shares of consumption and intermediate goods over total production are determined for the services sector:

$$\frac{C^s}{Y^s} = \frac{\left(1-\alpha_s\gamma_{ss}\right)\phi\left(1-\alpha_m\right)-\left(1-\alpha_s\right)\left(1-\phi\right)\alpha_m\gamma_{sm}}{\phi\left(1-\alpha_m\right)},$$

$$\frac{M^{ss}}{Y^s} = \alpha_s\gamma_{ss},$$

$$\frac{M^{sm}}{Y^s} = \frac{1-\alpha_s}{1-\alpha_m}\frac{1-\phi}{\phi}\alpha_m\gamma_{sm}.$$

Analogously, for the manufacturing sector:

$$
\begin{aligned}
\frac{C^m}{Y^m} &= \frac{(1-\alpha_m\gamma_{mm})(1-\phi)(1-\alpha_s)-(1-\alpha_m)\,\phi\alpha_s\gamma_{ms}}{(1-\phi)(1-\alpha_s)}, \\
\frac{M^{ms}}{Y^m} &= \frac{1-\alpha_m}{1-\alpha_s}\frac{\phi}{1-\phi}\alpha_s\gamma_{ms}, \\
\frac{M^{mm}}{Y^m} &= \alpha_m\gamma_{mm}.
\end{aligned}
$$

These conditions prove to be crucial in the second-order approximation of consumers' utility to eliminate the linear terms. Moreover, they allow us to derive the steady state ratio of labor supply in the services sector over the total labor supply (ϕ).

The Relative Price in the Steady State

We consider the steady state condition for the marginal cost in the services sector:

$$
\begin{aligned}
MC^s &= \Phi_s\left[(P^s)^{\gamma_{ss}}(P^m)^{\gamma_{ms}}\right]^{\alpha_s}(W^s)^{1-\alpha_s}, \\
\Phi_s &= \alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}.
\end{aligned}
$$

As in the steady state production subsidies neutralize distortions due to imperfect competition:

$$
\begin{aligned}
P^s &= MC^s \\
&= \Phi_s\left[(P^s)^{\gamma_{ss}}(P^m)^{\gamma_{ms}}\right]^{\alpha_s}(W^s)^{1-\alpha_s}.
\end{aligned}
$$

After some trivial manipulations it can be shown that:

$$
\Phi_s Q^{-\alpha_s\gamma_{ms}}(RW^s)^{1-\alpha_s} = 1.
$$

Analogously, for the manufacturing sector:

$$
\Phi_m Q^{\alpha_m\gamma_{sm}}(RW^m)^{1-\alpha_m} = 1.
$$

Using the fact that in steady state $W^s = W^m = W$:

$$
\begin{aligned}
\frac{RW^s}{RW^m}Q &= 1, \\
\frac{\left(\Phi_s^{-1}Q^{\alpha_s\gamma_{ms}}\right)^{\frac{1}{1-\alpha_s}}}{\left(\Phi_m^{-1}Q^{-\alpha_m\gamma_{sm}}\right)^{\frac{1}{1-\alpha_m}}}Q &= 1.
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
Q &= \left(\Phi_s^{1-\alpha_m}\Phi_m^{-(1-\alpha_s)}\right)^{\frac{1}{\varphi}}, \\
\varphi &= (1-\alpha_s)(1-\alpha_m)+\alpha_s\gamma_{ms}(1-\alpha_m)+\alpha_m\gamma_{sm}(1-\alpha_s).
\end{aligned}
$$

Notice that, when $\alpha_s = \alpha_m = 1$:

$$
Q = \Phi_s\Phi_m^{-1}
$$

as in the case considered by Huang and Liu (2005) and Strum (2009).

APPENDIX C: Equilibrium Dynamics in the Efficient Equilibrium

In this appendix we outline the solution method of the linear model under the efficient equilibrium. This is obtained when both prices are flexible and elasticities of substitution are constant. Let us start from the pricing rule under flexible prices:

$$\begin{aligned}
P_t^{s^*} &= \frac{\Theta^s}{1+\tau_s} MC_t^{s^*} \\
&= \frac{\Theta^s}{1+\tau_s} \frac{\overline{\phi}^s \left[\left(P_t^{s^*}\right)^{\gamma_{ss}} \left(P_t^{m^*}\right)^{\gamma_{ms}}\right]^{\alpha_s} \left(W_t^*\right)^{1-\alpha_s}}{Z_t^s}
\end{aligned}$$

$$\begin{aligned}
P_t^{m^*} &= \frac{\Theta^m}{1+\tau_m} MC_t^{m^*} \\
&= \frac{\Theta^m}{1+\tau_m} \frac{\overline{\phi}^m \left[\left(P_t^{s^*}\right)^{\gamma_{sm}} \left(P_t^{m^*}\right)^{\gamma_{mm}}\right]^{\alpha_m} \left(W_t^*\right)^{1-\alpha_m}}{Z_t^m}
\end{aligned}$$

where Θ^s and Θ^m denote the mark-up terms. In log-linear form the conditions above reduce to:

$$\begin{aligned}
(1-\alpha_s)\, rw_t^{s*} &= z_t^s + \alpha_s \gamma_{ms} q_t^* && (35) \\
(1-\alpha_m)\, rw_t^{m*} &= z_t^m - \alpha_m \gamma_{sm} q_t^* && (36)
\end{aligned}$$

We now recall some conditions under flexible prices from the linearized system:

$$\begin{aligned}
rw_t^{s*} &= -\gamma c_t^{s*} - (1-\sigma)\,\mu_m c_t^{m*} + \left[v\,(1-\phi) - \frac{1}{\lambda}\right] l_t^{m*} \\
&\quad + \left(\vartheta\phi + \frac{1}{\lambda}\right) l_t^{s*}, && (37) \\
l_t^{s*} &= \lambda \left(rw_t^{s*} - rw_t^{m*} + q_t^*\right) + l_t^{m*}, && (38) \\
y_t^{s*} &= \frac{C_t^s}{Y^s} c_t^{s*} + \frac{M^{ss}}{Y^s} m_t^{ss*} + \frac{M^{sm}}{Y^s} m_t^{sm*}, && (39) \\
y_t^{m*} &= \frac{C_t^m}{Y^m} c_t^{m*} + \frac{M^{ms}}{Y^m} m_t^{ms*} + \frac{M^{mm}}{Y^m} m_t^{mm*}, && (40) \\
0 &= rw_t^{s*} + l_t^{s*} - y_t^{s*}, && (41) \\
0 &= rw_t^{m*} + l_t^{m*} - y_t^{m*}, && (42) \\
0 &= m_t^{ss*} - y_t^{s*}, && (43) \\
0 &= m_t^{sm*} + q_t^* - y_t^{m*}, && (44) \\
0 &= m_t^{ms*} - q_t^* - y_t^{s*}, && (45) \\
0 &= m_t^{mm*} - y_t^{m*}, && (46)
\end{aligned}$$

where $\vartheta = \left(\upsilon - \frac{1}{\lambda}\right)$, $\gamma = (1-\sigma)\mu_s - 1$ and $\phi = \frac{L^s}{L}$. We substitute (35) and (36) into (40) and (41) respectively:

$$\begin{aligned} l_t^{s*} &= y_t^{s*} - \frac{1}{1-\alpha_s} z_t^s - \frac{\alpha_s \gamma_{ms}}{1-\alpha_s} q_t^*, && (47) \\ l_t^{m*} &= y_t^{m*} - \frac{1}{1-\alpha_m} z_t^m + \frac{\alpha_m \gamma_{sm}}{1-\alpha_m} q_t^*. && (48) \end{aligned}$$

We can use conditions (38), (39), and (43)-(46), to obtain:

$$y_t^{s*} = \frac{C^s}{Y^s} c_t^{s*} + \frac{M^{ss}}{Y^s} y_t^{s*} + \frac{M^{sm}}{Y^s} \left(y_t^{m*} - q_t^*\right)$$

and

$$y_t^{m*} = \frac{C^m}{Y^m} c_t^{m*} + \frac{M^{ms}}{Y^m} \left(q_t^* + y_t^{s*}\right) + \frac{M^{mm}}{Y^m} y_t^{m*}.$$

We can find a VAR solution to this system, so that we can express y_t^{s*} and y_t^{m*}as a function of c_t^{s*}, c_t^{m*} and q_t^* :

$$\mathbf{A} \begin{bmatrix} y_t^{s*} \\ y_t^{m*} \end{bmatrix} = \mathbf{B} \begin{bmatrix} c_t^{s*} \\ c_t^{m*} \end{bmatrix} + \mathbf{\Upsilon} q_t^*,$$

where

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 1 - \frac{M^{ss}}{Y^s} & -\frac{M^{sm}}{Y^s} \\ -\frac{M^{ms}}{Y^m} & 1 - \frac{M^{mm}}{Y^m} \end{bmatrix} = \begin{bmatrix} \frac{C^s}{Y^s} + \frac{M^{sm}}{Y^s} & -\frac{M^{sm}}{Y^s} \\ -\frac{M^{ms}}{Y^m} & \frac{C^m}{Y^m} + \frac{M^{ms}}{Y^m} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} \frac{C^s}{Y^s} & 0 \\ 0 & \frac{C^m}{Y^m} \end{bmatrix}, \\ \mathbf{\Upsilon} &= \begin{bmatrix} -\frac{M^{sm}}{Y^s} \\ \frac{M^{ms}}{Y^m} \end{bmatrix}. \end{aligned}$$

Thus, we obtain:

$$\begin{bmatrix} y_t^{s*} \\ y_t^{m*} \end{bmatrix} = \mathbf{A}^{-1}\mathbf{B} \begin{bmatrix} c_t^{s*} \\ c_t^{m*} \end{bmatrix} + \mathbf{A}^{-1}\mathbf{\Upsilon} q_t^*,$$

or equivalently:

$$\begin{aligned} y_t^{s*} &= \psi_1 c_t^{s*} + \psi_2 c_t^{m*} + \psi_5 q_t^*, \\ y_t^{m*} &= \psi_3 c_t^{s*} + \psi_4 c_t^{m*} + \psi_6 q_t^*. \end{aligned}$$

Clearly, interdependence among sectors reflects the presence of cross-industry flows of input materials that imply $\psi_2, \psi_3, \psi_5, \psi_6 \neq 0$ and $\psi_1, \psi_4 \neq 1$. By plugging these expressions into (47) and (48) we obtain:

$$\begin{aligned} l_t^{s*} &= \psi_1 c_t^{s*} + \psi_2 c_t^{m*} - \frac{1}{1-\alpha_s} z_t^s + \left(\psi_5 - \frac{\alpha_s \gamma_{ms}}{1-\alpha_s}\right) q_t^* && (49) \\ l_t^{m*} &= \psi_3 c_t^{s*} + \psi_4 c_t^{m*} - \frac{1}{1-\alpha_m} z_t^m + \left(\frac{\alpha_m \gamma_{sm}}{1-\alpha_m} + \psi_6\right) q_t^* && (50) \end{aligned}$$

Thus, we can substitute everything into (37) and (35):

$$\frac{1+v\phi}{1-\alpha_s}z_t^s+\xi_1 z_t^m=\xi_2 c_t^{s*}-(1-\sigma)\,\mu_m c_t^{m*}+\xi_3 c_t^{m*}+\xi_4 q_t^*, \tag{51}$$

where:

$$\begin{aligned}
\xi_1 &= \frac{v(1-\phi)\lambda-1}{(1-\alpha_s)\,\lambda},\\
\xi_2 &= \frac{\lambda\,(v\phi\psi_1-\gamma)+\psi_3\,[v(1-\phi)\lambda-1]}{\lambda},\\
\xi_3 &= \frac{\lambda v\phi\psi_2+\psi_4\,[v(1-\phi)\lambda-1]}{\lambda},\\
\xi_4 &= -\frac{\alpha_s\gamma_{ms}\,(1+v\phi)}{1-\alpha_s}+\left[\frac{v(1-\phi)\lambda-1}{(1-\alpha_m)\,\lambda}\left(\frac{\alpha_m\gamma_{sm}}{1-\alpha_m}+\psi_6\right)+\psi_5 v\phi\right].
\end{aligned}$$

In turn, we can plug (49), (50), (35) and (36) into (37):[35]

$$\xi_5 q_t^*=\frac{1}{1-\alpha_s}z_t^s-\frac{1}{1-\alpha_m}z_t^m-\frac{\psi}{(1+\lambda)}\left(c_t^{s*}-c_t^{m*}\right) \tag{52}$$

where

$$\xi_5=\frac{\psi_5-\psi_6-\lambda}{1+\lambda}-\left[\frac{\alpha_s\gamma_{ms}}{(1-\alpha_s)}+\frac{\alpha_m\gamma_{sm}}{(1-\alpha_m)}\right].$$

Conditions (51) and (52), together with the Euler conditions for the services and manufactured goods, allow us to determine a system of linear difference equations from which we derive equilibrium dynamics under flexible prices.

APPENDIX D: Relative Price in the Efficient Equilibrium with Perfect labor Mobility

We now define the efficient equilibrium in the model with no frictions in both the consumption goods and the labor market. On the labor market this condition, obtained for $\lambda\to\infty$, ensures that nominal salaries are equalized across sectors of the economy:

$$W_t^{s^*}=W_t^{m^*}=W_t^*. \tag{53}$$

Moreover, given the production subsidies that eliminate sectoral distortions due to monopolistic competition:

$$P_t^{s^*}=MC_t^{s^*}\qquad\qquad P_t^{m^*}=MC_t^{m^*}. \tag{54}$$

Conditions (53) and (54) imply that:

$$P_t^{s^*}=\left(\overline{\phi}^s\right)^{\frac{1}{1-\alpha_s\gamma_{ss}}}\left(P_t^{m^*}\right)^{\frac{\alpha_s\gamma_{ms}}{1-\alpha_s\gamma_{ss}}}\left(W_t^*\right)^{\frac{1-\alpha_s}{1-\alpha_s\gamma_{ss}}}\left(Z_t^s\right)^{-\frac{1}{1-\alpha_s\gamma_{ss}}}, \tag{55}$$

[35] It can be shown that $\psi_1-\psi_3=-(\psi_2-\psi_4)=\left(\frac{M^{ms}}{C^m}+\frac{M^{sm}}{C^s}+1\right)^{-1}=\psi<1$.

$$P_t^{m^*} = \left(\overline{\phi}^m\right)^{\frac{1}{1-\alpha_m\gamma_{mm}}} \left(P_t^{s^*}\right)^{\frac{\alpha_m\gamma_{sm}}{1-\alpha_m\gamma_{mm}}} \left(W_t^*\right)^{\frac{1-\alpha_m}{1-\alpha_m\gamma_{mm}}} \left(Z_t^m\right)^{-\frac{1}{1-\alpha_m\gamma_{mm}}}. \tag{56}$$

We then substitute (55) into (56) to eliminate W_t^*:

$$\left(P_t^{s^*}\right)^{\vartheta_s} = \Upsilon^{(1-\alpha_s\gamma_{ss})(1-\alpha_m)} \left(P_t^{m^*}\right)^{\vartheta_m} \left(Z_t^s\right)^{-(1-\alpha_m)} \left(Z_t^m\right)^{(1-\alpha_s)}$$

where

$$\Upsilon = \left(\overline{\phi}^s\right)^{\frac{1}{1-\alpha_s\gamma_{ss}}} \left(\overline{\phi}^m\right)^{-\frac{1}{1-\alpha_m}\frac{1-\alpha_s}{1-\alpha_s\gamma_{ss}}}$$

and

$$\vartheta_s = \vartheta_m = (1-\alpha_m)(1-\alpha_s\gamma_{ss}) + (\alpha_m\gamma_{sm})(1-\alpha_s).$$

Thus, after some trivial algebra we can show that the relative price reads as:

$$\begin{aligned} Q_t^* &= \frac{P_t^{s^*}}{P_t^{m^*}} = \Upsilon\left[\left(Z_t^s\right)^{-(1-\alpha_m)} \left(Z_t^m\right)^{1-\alpha_s}\right]^{\frac{1}{\varkappa+1}} \\ &= \Upsilon\left[\left(Z_t^s\right)^{-(1-\alpha_m)} \left(Z_t^m\right)^{1-\alpha_s}\right]^{\frac{1}{\varkappa+1}}. \end{aligned}$$

where

$$\varkappa = \alpha_s\alpha_m(\gamma_{ss} + \gamma_{mm} - 1) - \alpha_s\gamma_{ss} - \alpha_m\gamma_{mm}.$$

Proof of Proposition 1

Suppose there were a monetary policy under which the equilibrium allocation under sticky prices would be Pareto optimal. Then, in such an equilibrium, the gaps would be completely closed for every period. That is, $\widetilde{rmc}_t^s = \widetilde{rmc}_t^m = 0$, $\forall t$. It follows from the pricing conditions that $\pi_t^s = \pi_t^m = 0$, $\forall t$. Recall that the relative price evolves as:

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^s - \pi_t^m - \Delta q_t^*.$$

Since we also have that $\Delta\widetilde{q}_t = 0$, the equation above implies that $\pi_t^s - \pi_t^m = \Delta q_t^*$. From the analysis above:

$$q_t^* = \frac{(1-\alpha_m)\, z_t^s - (1-\alpha_s)\, z_t^m}{1+\varkappa},$$

and, therefore, it cannot be that $\pi_t^s = \pi_t^m = 0$, unless $\Delta q_t^* = 0$, which translates into:

$$\frac{\Delta z_t^m}{\Delta z_t^s} = \frac{1-\alpha_m}{1-\alpha_s}.$$

APPENDIX E: Second-order Approximation of the Utility Function

Following Rotemberg and Woodford (1998), we derive a well-defined welfare function from the utility function of the representative household:

$$\mathcal{W}_t = U\left(C_t^s, C_t^m\right) - V\left(L_t\right).$$

We start from a second-order approximation of the utility from consumption of services and manufactured goods:

$$\begin{aligned} U\left(C_t^s, C_t^m\right) \approx\; & U\left(C^s, C^m\right) + U_{C^s}\left(C^s, C^m\right)\left(C_t^s - C^s\right) + \frac{1}{2}U_{C^sC^s}\left(C^s, C^m\right)\left(C_t^s - C^s\right)^2 \\ & + U_{C^m}\left(C^s, C^m\right)\left(C_t^m - C^m\right) + \frac{1}{2}U_{C^mC^m}\left(C^s, C^m\right)\left(C_t^m - C^m\right)^2 + \\ & + U_{C^sC^m}\left(C^s, C^m\right)\left(C_t^s - C^s\right)\left(C_t^m - C^m\right) + O\left(\|\xi\|^3\right), \end{aligned} \tag{57}$$

where $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher. Notice that:

$$\begin{aligned} U_{C^m}\left(C^s, C^m\right) &= \left(\mu_m C_t^s / \mu_s C^m\right) U_{C^s}\left(C^s, C^m\right), \\ U_{C^sC^s}\left(C^s, C^m\right) &= \left[\mu_s\left(1-\sigma\right) - 1\right]\left(C^s\right)^{-1} U_{C^s}\left(C^s, C^m\right), \\ U_{C^mC^m}\left(C^s, C^m\right) &= \left[\mu_m\left(1-\sigma\right) - 1\right]\left(\mu_m C^s / \mu_s C^m\right) U_{C^s}\left(C^s, C^m\right), \\ U_{C^sC^m}\left(C_t^s, C^m\right) &= \mu_m\left(1-\sigma\right)\left(C^m\right)^{-1} U_{C^s}\left(C^s, C^m\right). \end{aligned}$$

As $\frac{C_t^s - C^s}{C^s} = \widehat{c}_t^s + \frac{1}{2}\left(\widehat{c}_t^s\right)^2$, where $\widehat{c}_t^s = \log\left(\frac{C_t^s}{C^s}\right)$ is the log-deviation from steady state under sticky prices, we obtain:

$$\begin{aligned} U\left(C_t^s, C_t^m\right) \approx\; & U\left(C^s, C^m\right) + U_{C^s}\left(C^s, C^m\right) C^s \left[\widehat{c}_t^s + \frac{1}{2}\left(\widehat{c}_t^s\right)^2\right] + \\ & + \frac{1}{2}\left[\mu_s\left(1-\sigma\right) - 1\right] U_{C^s}\left(C^s, C^m\right) C^s \left[\widehat{c}_t^s + \frac{1}{2}\left(\widehat{c}_t^s\right)^2\right]^2 + \\ & + U_{C^m}\left(C^s, C^m\right) C^m \left[\widehat{c}_t^m + \frac{1}{2}\left(\widehat{c}_t^m\right)^2\right] + \\ & + \frac{1}{2}\left[\mu_m\left(1-\sigma\right) - 1\right] U_{C^m}\left(C^s, C^m\right) C^m \left[\widehat{c}_t^m + \frac{1}{2}\left(\widehat{c}_t^m\right)^2\right]^2 + \\ & + \mu_m\left(1-\sigma\right) U_{C^s}\left(C^s, C^m\right) C^s \left[\widehat{c}_t^s + \frac{1}{2}\left(\widehat{c}_t^s\right)^2\right]\left[\widehat{c}_t^m + \frac{1}{2}\left(\widehat{c}_t^m\right)^2\right] + \text{t.i.p.} + O\left(\|\xi\|^3\right), \end{aligned}$$

where t.i.p. collects terms independent of policy stabilization.

The next step is to derive a second-order approximation for labor disutility. Recall that:

$$\widehat{l}_t = \phi \widehat{l}_t^s + \left(1-\phi\right)\widehat{l}_t^m.$$

Therefore the second-order approximation reads:

$$\begin{aligned} V\left(L_t\right) &\approx V_L\left(L\right) L\left[\phi \widehat{l}_t^s+(1-\phi) \widehat{l}_t^m+\frac{1+v}{2}\left(\phi \widehat{l}_t^s+(1-\phi) \widehat{l}_t^m\right)^2\right]+ \\ & +\text{t.i.p.}+O\left(\|\xi\|^3\right). \end{aligned}$$

After these preliminary steps, we need to find an expression for $\widehat{l}_t^s$ and $\widehat{l}_t^m$. Given the definition of the marginal costs, in equilibrium we get:

$$\begin{aligned} L_t^s &= \frac{\left(1-\alpha_s\right) MC_t^s}{W_t^s} \int_0^1 Y_{jt}^s dj=\frac{\left(1-\alpha_s\right) \overline{\phi}^s}{Z_t^s}\left(\frac{Q_t^{-\gamma_{ms}}}{RW_t^s}\right)^{\alpha_s} Y_t^s \int_0^1\left(\frac{P_{jt}^s}{P_t^s}\right)^{-\varepsilon_t^s} dj, \\ L_t^m &= \frac{\left(1-\alpha_m\right) MC_t^m}{W_t^m} \int_0^1 Y_{kt}^m dk=\frac{\left(1-\alpha_m\right) \overline{\phi}^m}{Z_t^m}\left(\frac{Q_t^{\gamma_{sm}}}{RW_t^m}\right)^{\alpha_m} Y_t^m \int_0^1\left(\frac{P_{kt}^m}{P_t^m}\right)^{-\varepsilon_t^m} dk. \end{aligned}$$

Thus, we can report the linear approximation of the expressions above:

$$\begin{aligned} \widehat{l}_t^s &= -\alpha_s \gamma_{ms} \widehat{q}_t-\alpha_s \widehat{rw}_t^s-z_t^s+\widehat{y}_t^s+S_{st}, \\ \widehat{l}_t^m &= \alpha_m \gamma_{sm} \widehat{q}_t-\alpha_m \widehat{rw}_t^m-z_t^m+\widehat{y}_t^m+S_{mt}, \end{aligned}$$

where:

$$S_{st}=\log \left[\int_0^1\left(\frac{P_{jt}^s}{P_t^s}\right)^{-\varepsilon_t^s} dj\right] \qquad S_{mt}=\log \left[\int_0^1\left(\frac{P_{kt}^m}{P_t^m}\right)^{-\varepsilon_t^m} dk\right]$$

If we set $\widehat{p}_{jt}^s$ to be the log-deviation of $\frac{P_{jt}^s}{P_t^s}$ from its steady state, which means that a second-order Taylor expansion of $\int_0^1\left(\frac{P_{jt}^s}{P_t^s}\right)^{-\varepsilon_t^s} dj$ reads as:

$$\begin{aligned} \int_0^1\left(\frac{P_{jt}^s}{P_t^s}\right)^{-\varepsilon_t^s} dj &\approx \int_0^1\left[1-\varepsilon^s \widehat{p}_{jt}^s-\varepsilon^s \widehat{p}_{jt}^s \widehat{\varepsilon}_t^s+\frac{1}{2}\left(\varepsilon^s\right)^2\left(\widehat{p}_{jt}^s\right)^2\right] dj+O\left(\|\xi\|^3\right) \\ &= 1-\varepsilon^s \mathbf{E}_i \widehat{p}_{jt}^s-\varepsilon^s \mathbf{E}_i \widehat{p}_{jt}^s \widehat{\varepsilon}_t^s+\frac{1}{2}\left(\varepsilon^s\right)^2 \mathbf{E}_i\left(\widehat{p}_{jt}^s\right)^2+O\left(\|\xi\|^3\right), \end{aligned}$$

where $\mathbf{E}_i \widehat{p}_{jt}^s \equiv \int_0^1 \widehat{p}_{jt}^s dj$ and $\mathbf{E}_i\left(\widehat{p}_{jt}^s\right)^2 \equiv \int_0^1\left(\widehat{p}_{jt}^s\right)^2 dj$. At this stage, we need an expression for $\mathbf{E}_i \widehat{p}_{jt}^s$. Let us start from

$$P_t^s=\left[\int_0^1\left(P_{jt}^s\right)^{1-\varepsilon_t^s} dj\right]^{\frac{1}{1-\varepsilon_t^s}},$$

which can be re-arranged as:

$$1 \equiv \int_0^1 \left(\frac{P_{jt}^s}{P_t^s} \right)^{1-\varepsilon_t^s} dj.$$

Following the procedure above, it can be shown that:

$$\left(\frac{P_{jt}^s}{P_t^s} \right)^{1-\varepsilon_t^s} \approx 1 + (1-\varepsilon^s)\,\widehat{p}_{jt}^s - \varepsilon^s \widehat{p}_{jt}^s \widehat{\varepsilon}_t^s + \frac{1}{2}(1-\varepsilon^s)^2 \left(\widehat{p}_{jt}^s\right)^2 + O\left(\|\xi\|^3\right).$$

Substituting this into the preceding equations yields:

$$0 = \int_0^1 \left[(1-\varepsilon^s)\,\widehat{p}_{jt}^s - \varepsilon^s \widehat{p}_{jt}^s \widehat{\varepsilon}_t^s + \frac{1}{2}(1-\varepsilon^s)^2 \left(\widehat{p}_{jt}^s\right)^2 \right] dj + O\left(\|\xi\|^3\right),$$

which reduces to:

$$\mathbf{E}_i \widehat{p}_{jt}^s = \frac{\varepsilon^s - 1}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^s\right)^2 + O\left(\|\xi\|^3\right).$$

Thus:

$$\int_0^1 \left(\frac{P_{jt}^s}{P_t^s} \right)^{-\varepsilon_t^s} dj = 1 + \frac{\varepsilon^s}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^s\right)^2 + O\left(\|\xi\|^3\right).$$

Now, notice that:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^s\right)^2 = \mathbf{E}_i \left[\left(p_{jt}^s\right)^2 - 2p_{jt}^s p_t^s + \left(p_t^s\right)^2 \right] + O\left(\|\xi\|^3\right),$$

where lower case letters denote the log-value of the capital letters. Here we can use a first-order approximation of $p_t^s = \int_0^1 p_{jt}^s dj$, as this term is multiplied by other first-order terms each time it appears. With this, we have a second-order approximation:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^s\right)^2 \equiv var_j p_{jt}^s.$$

Therefore, the second-order approximation can be represented as:

$$S_{st} = \frac{\varepsilon^s}{2} var_j p_{jt}^s + O\left(\|\xi\|^3\right).$$

Analogous steps for the manufacturing sector lead us to:

$$S_{mt} = \frac{\varepsilon^m}{2} var_k p_{kt}^m + O\left(\|\xi\|^3\right).$$

Following Woodford (2003, Ch. 6, Proposition 6.3), we can obtain a correspondence between cross-sectional price dispersions in the two sectors and their inflation rates:

$$\begin{aligned} var_j p_{jt}^s &= \theta_s var_j p_{jt-1}^s + \frac{\theta_s}{1-\theta_s}\left(\pi_t^s\right)^2 + O\left(\|\xi\|^3\right), \\ var_k p_{kt}^m &= \theta_m var_k p_{kt-1}^m + \frac{\theta_m}{1-\theta_m}\left(\pi_t^m\right)^2 + O\left(\|\xi\|^3\right). \end{aligned}$$

Iterating these expressions forward leads to:

$$\sum_{t=0}^{\infty} \beta^t var_j p_{jt}^s = \left(\kappa_s\right)^{-1} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^s\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{58}$$

$$\sum_{t=0}^{\infty} \beta^t var_k p_{kt}^m = \left(\kappa_m\right)^{-1} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^m\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right). \tag{59}$$

After these preliminary steps, we can write $\mathcal{W}_t$ as:

$$\begin{aligned} \mathcal{W}_t \approx\ & U_{C^s}\left(C^s, C^m\right) C^s \left\{ \widehat{c}_t^s + \frac{1}{2}\left[\mu_s\left(1-\sigma\right)\right]\left(\widehat{c}_t^s\right)^2 + \left(\mu_m/\mu_s\right)\widehat{c}_t^m + \right. \\ & \left. + \frac{1}{2}\left[\mu_m\left(1-\sigma\right)\right]\left(\mu_m/\mu_s\right)\left(\widehat{c}_t^m\right)^2 + \mu_m\left(1-\sigma\right)\widehat{c}_t^s\widehat{c}_t^m \right\} + \\ & -V_L\left(L\right) L \left\{ \phi\widehat{l}_t^s + \left(1-\phi\right)\widehat{l}_t^m + \frac{1+v}{2}\left(\phi\widehat{l}_t^s + \left(1-\phi\right)\widehat{l}_t^m\right)^2 \right\} + \\ & + \text{t.i.p.} + O\left(\|\xi\|^3\right). \end{aligned}$$

We now consider the linear terms in $\mathcal{W}_t$, which are collected under $\mathcal{LW}_t$:

$$\begin{aligned} \mathcal{LW}_t =\ & \frac{U_{C^s}\left(C^s, C^m\right) C^s}{\mu_s}\left\{\mu_s\widehat{c}_t^s + \mu_m\widehat{c}_t^m\right\} + \\ & -V_L\left(L\right) L \left\{\phi\left(-\alpha_s\gamma_{ms}\widehat{q}_t - \alpha_s\widehat{rw}_t^s + \widehat{y}_t^s\right) + \right. \\ & \left. + \left(1-\phi\right)\left(\alpha_m\gamma_{sm}\widehat{q}_t - \alpha_m\widehat{rw}_t^m + \widehat{y}_t^m\right)\right\} + \\ & + \text{t.i.p.} + O\left(\|\xi\|^2\right). \end{aligned}$$

We substitute for the real wage from marginal cost expressions to get:

$$\begin{aligned} \mathcal{LW}_t =\ & \frac{U_{C^s}\left(C^s, C^m\right) C^s}{\mu_s}\left\{\mu_s\widehat{c}_t^s + \mu_m\widehat{c}_t^m\right\} + \\ & -V_L\left(L\right) L\phi\left(\frac{1}{1-\alpha_s}\widehat{y}_t^s - \frac{\alpha_s\gamma_{ss}}{1-\alpha_s}\widehat{m}_t^{ss} - \frac{\alpha_s\gamma_{ms}}{1-\alpha_s}\widehat{m}_t^{ms}\right) + \\ & -V_L\left(L\right) L\left(1-\phi\right)\left(\frac{1}{1-\alpha_m}\widehat{y}_t^m - \frac{\alpha_m\gamma_{sm}}{1-\alpha_m}\widehat{m}_t^{sm} - \frac{\alpha_m\gamma_{mm}}{1-\alpha_m}\widehat{m}_t^{mm}\right) + \\ & + \text{t.i.p.} + O\left(\|\xi\|^2\right). \end{aligned} \tag{60}$$

Notice that the following linear approximations for the market clearing conditions hold:

$$\begin{aligned} \widehat{y}_t^s &= \frac{1-\alpha_s}{\phi}\mu_s\widehat{c}_t^s + \alpha_s\gamma_{ss}\widehat{m}_t^{ss} + \frac{\left(1-\alpha_s\right)\left(1-\phi\right)}{\phi\left(1-\alpha_m\right)}\alpha_m\gamma_{sm}\widehat{m}_t^{sm}, \\ \widehat{y}_t^m &= \frac{\mu_m\left(1-\alpha_m\right)}{\left(1-\phi\right)}\widehat{c}_t^m + \frac{\left(1-\alpha_m\right)\phi}{\left(1-\phi\right)\left(1-\alpha_s\right)}\alpha_s\gamma_{ms}\widehat{m}_t^{ms} + \alpha_m\gamma_{mm}\widehat{m}_t^{mm}. \end{aligned}$$

It can be shown that, in the steady state, the following relationships hold:

$$V_{L^s}(L^s)\,L^s = \phi V_L(L)\,L \qquad\qquad V_{L^m}(L^m)\,L^m = (1-\phi)\,V_L L(L)$$

Moreover, the presence of production subsidies allows us to express the steady state marginal rate of substitution between labor supply and the consumption of services (or manufactured goods) as:

$$\begin{aligned}\frac{-V_{L^s}(L^s)}{U_{C^s}(C^s)} &= \frac{Y^s(1-\alpha_s)}{L^s},\\ \frac{-V_{L^m}(L^m)}{U_{C^s}(C^s)} &= \frac{Y^m(1-\alpha_m)}{L^m Q}.\end{aligned}$$

It is now convenient to express the marginal utility from services consumption in terms of the marginal utility derived from total consumption:

$$U_{C^s}(C^s) = \mu_s U_H(H)\,H.$$

Therefore, we can re-write (60) as:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{L}\mathcal{W}_t &= U_H(H)\,H\sum_{t=0}^{\infty}\beta^t\Big\{\left(\mu_s\widehat{c}_t^s+\mu_m\widehat{c}_t^m\right)+\\ &\quad -\mu_s\left(\frac{C^s}{Y^s}\right)^{-1}(1-\alpha_s)\left[-\alpha_s\gamma_{ms}\widehat{q}_t-\alpha_s\widehat{rw}_t^s-z_t^s+\widehat{y}_t^s\right]+\\ &\quad -\mu_s\left(\frac{C^s}{Y^m}\right)^{-1}(1-\alpha_m)\,Q^{-1}\left[\alpha_m\gamma_{sm}\widehat{q}_t-\alpha_m\widehat{rw}_t^m-z_t^m+\widehat{y}_t^m\right]\Big\}+\\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^2\right).\end{aligned}$$

It is now possible to show, given the linearized market clearing conditions in the two sectors, that $\sum_{t=0}^{\infty}\beta^t\mathcal{L}\mathcal{W}_t=0$. The linear term in $\mathcal{W}_t$ can therefore be dropped. Thus we are left only with second-order terms:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{W}_t &\approx U_H(H)\,H\sum_{t=0}^{\infty}\beta^t\Big\{\frac{1-\sigma}{2}\left(\mu_s\widehat{c}_t^s+\mu_m\widehat{c}_t^m\right)^2++\\ &\quad -\frac{1}{2}\left[\phi\varepsilon^s(\kappa_s)^{-1}(\pi_t^s)^2+(1-\phi)\,\varepsilon^m(\kappa_m)^{-1}(\pi_t^m)^2\right]+\\ &\quad -\left(\frac{1+\nu}{2}\right)\left(\mu_s\widehat{c}_t^s+\mu_m\widehat{c}_t^m\right)^2\Big\}+\text{t.i.p.}+O\left(\|\xi\|^3\right),\\ &\approx U_H(H)\,H\sum_{t=0}^{\infty}\beta^t\Big\{-\left(\frac{\nu+\sigma}{2}\right)\left(\mu_s\widehat{c}_t^s+\mu_m\widehat{c}_t^m\right)^2+\\ &\quad -\frac{1}{2}\left[\phi\varepsilon^s(\kappa_s)^{-1}(\pi_t^s)^2+(1-\phi)\,\varepsilon^m(\kappa_m)^{-1}(\pi_t^m)^2\right]+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

We next consider the deviation of social welfare from its Pareto-optimal level:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\widetilde{\mathcal{W}}_t &= \sum_{t=0}^{\infty}\beta^t\left(\mathcal{W}_t-\mathcal{W}_t^*\right)\approx\\&-\frac{U_H\left(H\right)H}{2}\sum_{t=0}^{\infty}\beta^t\left\{\left(\nu+\sigma\right)\left(\mu_s\widetilde{c}_t^s+\mu_m\widetilde{c}_t^m\right)^2+\right.\\&\left.+\varsigma\left[\varpi\left(\pi_t^s\right)^2+\left(1-\varpi\right)\left(\pi_t^m\right)^2\right]\right\}+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where the following notation has been introduced:

$$\begin{aligned}\varpi &= \frac{\phi\varepsilon^s\left(\kappa_s\right)^{-1}}{\varsigma},\\ \varsigma &= \phi\frac{\varepsilon^s}{\kappa_s}+\left(1-\phi\right)\frac{\varepsilon^m}{\kappa_m}.\end{aligned}$$