

# Optimal Monetary Policy with Durable Consumption Goods and Factor Demand Linkages\*

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## Abstract

This paper deals with the implications of factor demand linkages for monetary policy design. We develop a general equilibrium model with two sectors that produce durable and non-durable goods, respectively. Part of the output of each sector serves as a production input in both sectors, according to an input-output matrix calibrated on the US economy. Strategic complementarities induced by factor demand linkages have a significant impact on the transmission of exogenous shocks and amplify the loss of social welfare under optimal monetary policy, compared to what is commonly observed in standard two-sector models. As the degree of interconnection between sectors increases, neglecting input-output interactions causes a large loss of social welfare. We show that a flexible inflation targeting regime, whereby the central bank balances aggregate inflation and value added variability, results in a welfare level close to that attained under the optimal policy.

*JEL classification:* E23; E32; E52

*Keywords:* Input-Output Interactions, Durable Goods, Optimal Monetary Policy

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# Introduction

This paper deals with the implications of factor demand linkages for monetary policy design. We build a dynamic stochastic general equilibrium (DSGE) model with two sectors that produce durable and non-durable goods. Goods produced in each sector serve either as a final consumption good, or as an intermediate production input in both sectors, according to an input-output matrix calibrated on the US economy.

Introducing factor demand linkages into otherwise standard general equilibrium models with durable and non-durable consumption goods is of key importance for two main reasons. First, it is well documented that standard sticky-price models incorporating sectoral heterogeneity in price stickiness - usually in the form of sticky non-durable goods prices and flexible durables prices - cannot generate positive co-movement between sectors following a monetary policy innovation (Barsky et al., 2007). As reported by various contributions (see, among others, Aoki, 2004; Erceg and Levin, 2006; Barsky et al., 2007), co-movement between non-durable and durable consumption is an inherent feature of the US economy that multi-sector DSGE models need to be able to replicate, and that raises a number of questions from the normative point of view. As shown by Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008), factor demand linkages turn out to be crucial to restore positive co-movement between non-durables and durables value added.<sup>1</sup>

Second, input-output interactions among sectors are empirically relevant<sup>2</sup> and are proven to act as a potent amplifier of shocks to the system. Horvath (1998, 2000) and Carvalho (2009) show that input materials can reinforce the effect of sectoral shocks, generating aggregate fluctuations and co-movement between different sectors of the economy. This is due to the emergence of strategic complementarities acting through the intermediate input channel.<sup>3</sup> These results emphasize the need to reconsider the policy implications drawn from otherwise standard multi-sector models in the light of the role played by inter-sectoral linkages in the transmission of macroeconomic fluctuations. We add to this literature, providing the first contribution exploring optimal monetary policy in a two-sector model with factor demand linkages.<sup>4</sup>

We first show that the central bank cannot attain the Pareto optimal allocation consistent with the full stabilization of output and inflation, even after removing sources of distortion in the labour market (imperfect labour mobility)<sup>5</sup> and in the goods market (monopolistic competition). In our variant model economy the occurrence of sectoral shocks of the same magnitude does not ensure that these offset each other. Bi-directional horizontal factor demand linkages impose a more restrictive set of conditions for the full stabilization of the model economy in the face of exogenous shocks, compared to models

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<sup>1</sup>An alternative line of enquiry is followed by Monacelli (2009), who stresses the role of financial market imperfections in generating co-movement. DiCecio (2009), instead, argues that rigidity in nominal wages is the key friction underlying sectoral co-movement.

<sup>2</sup>Bouakez, Cardia, and Ruge-Murcia (2008) report evidence on the empirical relevance of input-output interactions. Dale Jorgenson's data on input expenditures by US industries shows that materials (including energy) account for roughly 50 percent of outlays, while labor and capital account for 34% and 16%, respectively. The Input-Output accounts compiled by the Bureau of Labor Statistics (BLS) shows that 70 percent of the material-input expenditures by the durables sector goes into goods produced by the nondurables sector. The converse proportion is around 10 percent, which is much smaller but still not negligible.

<sup>3</sup>Kim and Kim (2006) show that a similar mechanism generates widespread co-movement of economic activity (e.g. in employment) across sectors. See also Hornstein and Praschnik (1997).

<sup>4</sup>Haug and Liu (2005) and Strum (2009) consider a two-sector model with a vertical input-output structure. Basu (1995) observes that, "Input-output studies certainly do not support the chain-of-production view. [...] This seems to lend credence to the view of 'roundabout' rather than 'in-line' production."

<sup>5</sup>Empirical evidence suggests that labor and capital are not perfectly mobile across sectors. Davis and Haltiwanger (2001) find limited labor mobility across sectors in response to monetary and oil shocks. Bouakez, Cardia, and Ruge-Murcia (2008) report evidence suggesting that perfect labor mobility across sectors, with its implication that sectoral nominal wages are the same (at the margin), is an imperfect characterization of the data.

where sectors of production are completely independent (Erceg and Levin, 2006), or where there is characterized by a vertical trading chain (Huang and Liu, 2005).

We then explore optimal monetary policy under the assumption that the policy maker can credibly commit to a policy rule derived from the minimization of a social welfare function. Following Rotemberg and Woodford (1998), we obtain a quadratic approximation to the representative household's utility function. We show that the welfare criterion assumed by the monetary authority balances, along with sectoral inflation variability, fluctuations in aggregate consumption (value added): this is a distinctive feature of our model, given that a non-trivial difference arises between consumption and production, due to factor demand linkages. Furthermore, approximating the social welfare associated with the durables sector involves the derivation of a term that reflects a strong preference to smooth the accumulation of the stock of durable goods. This constitutes a primary stabilization objective for the monetary authority, compared to the relative weight of other terms involved in the period loss.<sup>6</sup>

The introduction of factor demand linkages into an otherwise standard two-sector model with durable and non-durable goods amplifies the loss of social welfare, compared to the benchmark economy without input materials. Crucially, as the actual importance of factor demand linkages increases, the cost of misperceiving the correct production technology of each sector can be substantial. Furthermore, factor demand linkages alter the transmission of exogenous perturbations to the system. A distinctive feature of our model is that a technology shock to either sector also affects potential output in the other sector, even if preferences over different types of goods are separable. In addition, the presence of input materials implies that the relative price of non-durable goods does not only affect the marginal rate of substitution between durable and non-durable consumption, but also exerts a positive (negative) effect on the real marginal cost in the durable (non-durable) goods sector. A technology shock in the non-durable goods sector causes a rise in the relative price gap, thus inducing stronger inflationary pressures in the durable goods sector, compared to what is commonly observed in the baseline scenario without input materials. The amplification of the initial impulse due to the presence of strategic complementarities between sectors translates into strong inflationary pressures on the price of both consumption goods, whose stabilization represents a prohibitive task for the central bank. In the face of a cost push shock, instead, the intermediate input channel acts as an endogenous attenuator of the deflationary effect in the sector which is not hit by the shock. This works through the opposite impact that the relative price exerts on sectoral real marginal costs. Imperfect labour mobility magnifies this effect, as it implies that input materials become a primary mechanism of endogenous adjustment in the system.

We finally explore the welfare properties of the model under various alternative policy regimes. For each of them, we assume that the monetary authority selects an alternative policy objective, compared to the specification of the welfare criterion consistent with the model economy. Importantly, a "flexible" inflation targeting regime, whereby the central bank balances fluctuations in aggregate inflation with those in the real value added (or gross product), delivers a welfare loss close to that attained under an optimal monetary policy. The implementation of this regime allows the policy maker to partially smooth fluctuations in durables accumulation. Importantly, the central bank attains a welfare loss closer to that under the optimal policy when fluctuations in aggregate (or core) inflation are balanced with those in real value added, compared to the loss induced by controlling fluctuations in the gross product. This result emphasizes the distinction between consumption and production that naturally arises in frameworks with horizontally integrated sectors of production.

The remainder of the paper is laid out as follows: Section 1 introduces the theoretical setting; Section 2 reports the calibration of our model economy and shows how co-movement between durable and non-durable consumption emerges following a shock to an instrumental policy rule; Section 3 discusses the Pareto optimal outcome; Section 4 discusses the implementation of optimal monetary policy under

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<sup>6</sup>This feature of the social welfare function is not stressed by Erceg and Levin (2006), although their loss function is isomorphic to the one derived in this paper.

commitment, and compares the resulting loss of social welfare with that attainable under a number of alternative policy regimes. Last section concludes.

# 1 The Model

We develop a DSGE model with two sectors that produce durable and non-durable goods, respectively. The model economy is populated by a large number of infinitely-lived households. Each of them is endowed with one unit of time and derives utility from consumption of durable goods, non-durable goods and leisure. The two sectors of production are connected through factor demand linkages. Goods produced in each sector serve either as a final consumption good, or as an intermediate production input in both sectors. The size of the intermediate input flow from one sector to the other depends on the input-output structure of the economy.

## 1.1 Producers

Consider an economy that consists of two distinct sectors producing durable (sector  $d$ ) and non-durable goods (sector  $n$ ). Each sector is composed of a continuum of firms producing differentiated products. Let  $Y_t^n$  ( $Y_t^d$ ) denote gross output of the non-durable (durable) goods sector:

$$Y_t^n = \left[ \int_0^1 (Y_{jt}^n)^{\frac{\varepsilon_t^n - 1}{\varepsilon_t^n}} dj \right]^{\frac{\varepsilon_t^n}{\varepsilon_t^n - 1}} \quad Y_t^d = \left[ \int_0^1 (Y_{kt}^d)^{\frac{\varepsilon_t^d - 1}{\varepsilon_t^d}} dk \right]^{\frac{\varepsilon_t^d}{\varepsilon_t^d - 1}} \quad (1)$$

where  $\varepsilon_t^i$  ( $i = \{n, d\}$ ) denotes the time-varying elasticity of substitution between differentiated goods in the production composite. The composite products are produced in the "aggregator" sectors operating under perfect competition. It is possible to show that each aggregation sector has the following demand for goods produced by firm  $j$  in sector  $n$  and by firm  $k$  in sector  $d$ , respectively:

$$Y_{jt}^n = \left( \frac{P_{jt}^n}{P_t^n} \right)^{-\varepsilon_t^n} Y_t^n \quad Y_{kt}^d = \left( \frac{P_{kt}^d}{P_t^d} \right)^{-\varepsilon_t^d} Y_t^d, \quad (2)$$

where  $P_t^n$  and  $P_t^d$  are the prices of the composite good in the non-durable and in the durable goods sector respectively. From (1) and (2) the relationship between firm-specific and sector-specific prices is:

$$P_t^n = \left[ \int_0^1 (P_{jt}^n)^{1-\varepsilon_t^n} dj \right]^{\frac{1}{1-\varepsilon_t^n}} \quad P_t^d = \left[ \int_0^1 (P_{kt}^d)^{1-\varepsilon_t^d} dk \right]^{\frac{1}{1-\varepsilon_t^d}}. \quad (3)$$

In our model economy sectors are related by factor demand linkages. Therefore, the composite output of sector  $i = \{n, d\}$  serves partly as a consumption good and partly as an intermediate input in both sectors. The allocation of output produced in sector  $n$  requires that:

$$Y_t^n = C_t^n + M_t^{nn} + M_t^{nd}, \quad (4)$$

where  $C_t^n$  denotes the production of sector  $n$  consumed by households,  $M_t^{nn}$  represents the intermediate inputs produced in sector  $n$  and consumed in the production process of the firms in the same sector, while  $M_t^{nd}$  is the aggregate quantity of intermediate goods produced in sector  $n$  used in the production process of the firms in sector  $d$ . Symmetrically, the allocation of output in sector  $d$  requires that:

$$Y_t^d = C_t^d + M_t^{dn} + M_t^{dd}. \quad (5)$$

The production technology of a generic firm  $f$  in sector  $i$  is:

$$Y_{ft}^i = Z_t^i \left[ \frac{(M_{ft}^{ni})^{\gamma_{ni}} (M_{ft}^{di})^{\gamma_{di}}}{\gamma_{ni}^{\gamma_{ni}} \gamma_{di}^{\gamma_{di}}} \right]^{\alpha_i} (L_{ft}^i)^{1-\alpha_i}, \quad i = \{n, d\} \quad (6)$$

where  $Z_t^i$  ( $i = \{n, d\}$ ) is a sector specific productivity shock,  $L_{ft}^i$  denotes the number of hours worked in firm  $f$  in sector  $i$ ,  $M_{ft}^{ji}$  ( $j = \{n, d\}$ ) denotes material inputs produced in sector  $j$  and supplied to firm  $f$  in sector  $i$ . In the model we do not have capital and the input material supplied by the durable good producing sector fully depreciate within the period.<sup>7</sup> However, we allow the consumers to store durable goods from which they derive a flow of services. Material inputs are combined according to a CES aggregator:

$$M_{ft}^{ji} = \left( \int_0^1 (M_{kf,t}^{ji})^{(\varepsilon_t^j - 1)/\varepsilon_t^j} dk \right)^{\varepsilon_t^j / (\varepsilon_t^j - 1)}, \quad (7)$$

where  $\{M_{kf,t}^{ji}\}_{k \in [0,1]}$  is a sequence of intermediate inputs produced in sector  $j$  by firm  $k$ , that are employed in the production process of firm  $f$  in sector  $i$ .

The generic element  $\gamma_{ab}$  ( $a = \{n, d\}$ ;  $b = \{n, d\}$ ) of the  $(2 \times 2)$  input-output matrix,  $\Gamma$ , corresponds to the steady state share of total intermediate goods used in the production of sector  $b$  supplied by sector  $a$ . The input-output matrix is normalized, so that the elements of each column sum up to one, i.e.  $\sum_{i=\{n,d\}} \gamma_{ni} = 1$  (and analogously  $\sum_{i=\{n,d\}} \gamma_{di} = 1$ ).

Firms in both sectors are monopolistic competitors in the consumption goods markets and are price takers in the input markets. They set prices given the demand functions reported in (2). Firms are also assumed to be able to adjust their price with probability  $1 - \theta^i$  in each period. When they are able to do so, they set a price that maximizes expected profits according to the following problem:

$$\max_{P_{ft}^i} E_t \sum_{s=0}^{\infty} (\beta \theta^i)^s \Omega_{t+s} [P_{ft+s}^i (1 + \tau^i) - MC_{ft+s}^i] Y_{ft+s}^i, \quad i = \{n, d\} \quad (8)$$

where  $\Omega_{t+s}$  is the stochastic discount factor (consistent with the maximizing behavior of households, which is reported in the next subsection),  $\tau^i$  denotes lump sum subsidies to producers in sector  $i$ , while  $MC_{fs}^i$  denotes the marginal cost of production of the  $f^{th}$  firm in sector  $i$ . The optimal pricing choice, given the sequence  $\{P_t^n, P_t^d, Y_t^n, Y_t^d\}$ , reads as:

$$\bar{P}_{ft}^i = \frac{\varepsilon_t^i}{(\varepsilon_t^i - 1)(1 + \tau^i)} \frac{E_t \sum_{s=0}^{\infty} (\beta \theta^i)^s \Omega_{t+s} MC_{ft+s}^i Y_{ft+s}^i}{E_t \sum_{s=0}^{\infty} (\beta \theta^i)^s \Omega_{t+s} Y_{ft+s}^i}, \quad i = \{n, d\}. \quad (9)$$

In every period each firm solves a cost minimization problem in order to meet demand at its stated price. The first order conditions from this problem result in the following relationships:

$$MC_{ft}^i Y_{ft}^i = \frac{W_t^i L_{ft}^i}{1 - \alpha_i} = \frac{P_t^n M_{ft}^{ni}}{\alpha_i \gamma_{ni}} = \frac{P_t^d M_{ft}^{di}}{\alpha_i \gamma_{di}}, \quad i = \{n, d\}. \quad (10)$$

<sup>7</sup>In Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008) durables used as intermediate inputs are also modeled as fully depreciating goods on the production side. The BLS in the US publishes two different input-output tables: (i) the "input-use" table, which considers goods that fully depreciate in the same period they are produced, and are usually referred to as "materials" in the traditional KLEM setting; (ii) the "capital flow" table of the input-output accounts, that refers to input materials used as investment goods. Our  $\Gamma$  is calibrated according to the "Input-use" table. It is important to stress that what we usually define as durables producing sectors have non-zero entries in the input-use matrix.

It is useful to express the sectoral real marginal cost as a function of the relative price,  $Q_t = P_t^n/P_t^d$ , and of the sectoral real wage:

$$\frac{MC_t^n}{P_t^n} = \frac{\bar{\phi}^n \left( Q_t^{-\gamma_{dn}} \right)^{\alpha_n} (RW_t^n)^{1-\alpha_n}}{Z_t^n}, \quad (11)$$

where  $RW_t^n = W_t^n/P_t^n$  is the real wage in sector  $n$ , and  $\bar{\phi}^n$  is a convolution of the production parameters ( $\bar{\phi}^i = \alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i}$ ,  $i = \{n, d\}$ ). Analogously, for the durable goods sector:

$$\frac{MC_t^d}{P_t^d} = \frac{\bar{\phi}^d (Q_t^{\gamma_{nd}})^{\alpha_d} (RW_t^d)^{1-\alpha_d}}{Z_t^d}. \quad (12)$$

where  $RW_t^d = W_t^d/P_t^d$ .

The relative price of durable goods affects the real marginal cost of each sector in opposite directions and for each sector the magnitude of this effect is directly related to the role of the other sector as input supplier. Moreover, the variation over time of the elasticity of substitution among the differentiated goods produced in each sector imparts an idiosyncratic (cost-push) shocks to the supply schedule in each sector. Nevertheless, both sectors are affected by each of the sectoral cost-push shocks as those exert an indirect effect on the relative price through their influence on the sectoral rates of inflation.<sup>8</sup>

## 1.2 Consumers

Households derive income from working in the production sectors, investing in bonds, and from the stream of profits generated in the production sectors. Their preferences are defined over a composite of non-durable goods ( $C_t^n$ ), an "effective" stock of durable goods ( $\mathfrak{D}_t$ ), and labour ( $L_t$ ). They maximize the expected present discounted value of their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{H_t^{1-\sigma}}{1-\sigma} - \varrho \frac{L_t^{1+v}}{1+v} \right], \quad (13)$$

where

$$H_t = (C_t^n)^{\mu^n} \mathfrak{D}_t^{\mu^d}$$

and  $\beta$  is the discount factor,  $\mu^n$  and  $\mu^d (= 1 - \mu^n)$  denote the expenditure shares on non-durable and durable goods,  $\sigma$  is the inverse of the intertemporal elasticity of substitution,  $v$  is the inverse of the Frisch elasticity of labour supply. Durable goods are accumulated according to the following law of motion:

$$D_t = C_t^d + (1 - \delta) D_{t-1}, \quad (14)$$

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<sup>8</sup>Therefore idiosyncratic sectoral shocks are propagated to the whole economy and their effect is not limited to the sector where the shock has originated. Indeed, those shocks affect the relative price of goods, which itself affect the tilts demand of goods, both on the consumer (demand) side, but also on the supply side because of the factor demand linkages among sectors.

where  $\delta$  denotes the depreciation factor. The effective stock of durables scales the effect of a cost of adjustment:<sup>9</sup>

$$\mathfrak{D}_t = D_t - F(D_t, D_{t-1}). \quad (15)$$

Consequently, adjustment costs are implicitly included in the utility function. Specifically, we rely on a convex (quadratic) adjustment function, whose form is rather standard in the literature:<sup>10</sup>

$$F(D_t, D_{t-1}) = \frac{\Xi}{2} \frac{(D_t - D_{t-1})^2}{D}, \quad \Xi \geq 0, \quad (16)$$

where  $D$  denotes the steady state stock of durable consumption goods.

We assume that labour can be either supplied to sector  $n$  or  $d$  according to a CES aggregator:

$$L_t = \left[ \phi^{-\frac{1}{\lambda}} (L_t^n)^{\frac{1+\lambda}{\lambda}} + (1 - \phi)^{-\frac{1}{\lambda}} (L_t^d)^{\frac{1+\lambda}{\lambda}} \right]^{\frac{\lambda}{1+\lambda}}, \quad (17)$$

where  $\lambda$  denotes the elasticity of substitution in labour supply, and  $\phi$  is the steady state ratio of labour supply in the non-durable goods sector over total labour supply ( $\phi = L^n/L$ ). This functional form allows us to account for different degrees of labour mobility between sectors, depending on the value assumed by  $\lambda$ . For  $\lambda = 0$  labour is prevented from moving across sectors, whereas for  $\lambda \rightarrow \infty$  we allow for perfect labour mobility. Unlike Horvath (2000), this specification of the CES aggregator allows us to neutralize the impact of labour market frictions in the steady state.

The following sequence of (nominal) budget constraints applies:

$$\sum_{i=\{n,d\}} P_t^i C_t^i + B_t = R_{t-1} B_{t-1} + \sum_{i=\{n,d\}} W_t^i L_t^i + \sum_{i=\{n,d\}} \Psi_t^i - T_t, \quad (18)$$

where  $B_t$  denotes a one-period risk-free nominal bond remunerated at the gross risk-free rate  $R_t$ , and  $T_t$  denotes a lump-sum tax paid to the government. The term  $\Psi_t^n + \Psi_t^d$  captures the nominal flow of dividends from both sectors of production.

Maximizing (13) subject to (14), (15), (16), (17), and (18) leads to a set of first-order conditions

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<sup>9</sup>The inclusion of a cost of adjustment of the stock of durables should allow us to obtain results in line with the empirical evidence on the behavior of durable consumption over the business cycle. Adda and Cooper (2000) provide evidence on the discrete nature of durables purchases at the individual level. King and Thomas (2006) show how the partial adjustment mechanism helps to account for the aggregate effect of discrete and occasional microeconomic adjustment of a large number of heterogeneous consumers that face fixed costs of consumption adjustment.

<sup>10</sup>See, e.g., Bernanke (1985).

that can be re-arranged to obtain:

$$\mu^n H_t^{1-\sigma} (C_t^n)^{-1} = \beta R_t E_t \left[ \frac{\mu^n H_{t+1}^{1-\sigma} (C_{t+1}^n)^{-1}}{\Pi_{t+1}^n} \right], \quad (19a)$$

$$\begin{aligned} \frac{\mu^n H_t^{1-\sigma} P_t^d}{C_t^n P_t^n} &= E_t \left\{ \beta (1 - \delta) \mu^n \frac{H_{t+1}^{1-\sigma} P_{t+1}^d}{C_{t+1}^n P_{t+1}^n} + \right. \\ &\quad \left. + \frac{\mu^d H_t^{1-\sigma}}{\mathfrak{D}_t [1 - \frac{\Xi}{D} (D_t - D_{t-1})]^{-1}} + \beta \frac{\Xi}{D} \frac{\mu^d H_{t+1}^{1-\sigma}}{\mathfrak{D}_{t+1} (D_{t+1} - D_t)^{-1}} \right\}, \end{aligned} \quad (19b)$$

$$W_t^n \frac{\mu^n H_t^{1-\sigma} (C_t^n)^{-1}}{P_t^n} = \varrho \phi^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} (L_t^n)^{\frac{1}{\lambda}}, \quad (19c)$$

$$W_t^d \frac{\mu^n H_t^{1-\sigma} (C_t^n)^{-1}}{P_t^n} = \varrho (1 - \phi)^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}} (L_t^d)^{\frac{1}{\lambda}}. \quad (19d)$$

Thus, from (19c) and (19d):

$$\left( \frac{\phi}{1 - \phi} \right)^{-\frac{1}{\lambda}} \left( \frac{L_t^n}{L_t^d} \right)^{\frac{1}{\lambda}} = \frac{W_t^n}{W_t^d}. \quad (20)$$

As  $\lambda \rightarrow \infty$ , and perfect labour mobility is attained, sectoral nominal wages are equalized.

### 1.3 The Government and the Monetary Authority

We assume that the government serves two purposes in the economy. First, it delegates monetary policy to an independent central bank. We assume that the short-term nominal interest rate is used as the instrument of monetary policy, and that the policy maker is able to pre-commit to a time-invariant rule. We consider alternative specifications of the monetary policy rule, including both rules that can be regarded as reasonable characterizations of the recent historical experience (Section 2), and rules derived from an explicit optimization problem from the perspective of a benevolent central banker (Section 4).

The second task consists of taxing households and providing subsidies to firms to eliminate distortions arising from monopolistic competition in the markets for consumption goods. We assume that this task is pursued via lump-sum taxes that maintain a balanced fiscal budget.

### 1.4 Market Clearing and Aggregation

Total production reads as:

$$Y_t = Y_t^n + Y_t^d. \quad (21)$$

The allocation of output produced by each sector requires that sectoral gross product is partly sold on the market for consumption goods, while a proportion is sold on the markets for input materials. Therefore, (4) and (5) must hold jointly.

It is important to recognize that  $C_t^i$  and  $Y_t^i$  are not equivalent in our setting, given the presence of intermediate inputs. In particular,  $C_t^i$  can be interpreted as value added in the  $i^{\text{th}}$  sector, while  $Y_t^i$  is the sectoral gross output.<sup>11</sup> As sectoral gross output can be sold both on the intermediate goods market and on the final goods market, total production is typically greater than real value added. Thus, according to our model economy  $C_t^i$ , matches most closely the empirically relevant definition of real GDP.

<sup>11</sup>See also Nakamura and Steinsson (2008) on this distinction in multi-sector models with input materials.

## 2 Solution and Calibration

To solve the model, we log-linearize private sector's behavioral equations and resource constraints around the non-stochastic steady state and then take the deviation from their counterparts under flexible prices. The difference between log-variables under sticky prices and their linearized steady state is denoted by the symbol " $\hat{\cdot}$ ", while we use symbol " $\ast$ " to denote percent deviations of variables in the efficient equilibrium (i.e. flexible prices and constant elasticities of substitution) from the corresponding steady state value. Finally, we use symbol " $\tilde{\cdot}$ " to denote the difference between linearized variables under sticky prices and their counterparts in the efficient equilibrium.<sup>12</sup>

The model is calibrated at a quarterly frequency. We assume that the discount factor  $\beta = 0.993$ . We set  $\sigma = 1$ , a value in line with Ngai and Pissarides (2007) which implies separability in the utility derived from different consumption goods. The expenditure share on non-durable goods is  $\mu^n = 0.682$ . The inverse of the Frisch elasticity of labour supply ( $v$ ) is set to 3, while  $\lambda = 1$ , which reflects limited labour mobility.<sup>13</sup> The production parameters  $\alpha_n = \alpha_d = 0.6$ , while the entries of the two-sector input-use matrix are such that  $\gamma_{nn} = 0.899$  and  $\gamma_{nd} = 0.688$ ,<sup>14</sup> according to the calibration of the US economy used by Bouakez, Cardia, and Ruge-Murcia (2008).<sup>15</sup> These values imply a positive net flow of input materials from the non-durable goods sector to the durable goods sector. The depreciation rate of the stock of durables is assumed at 2.5%.<sup>16</sup> In the baseline calibration we assume that the degree of nominal rigidity is the same across sectors, with  $\theta_n = \theta_d = 0.75$ . We will also explore the implications of allowing for asymmetric degrees of price stickiness between the two sectors at different stages in the analysis. We assume that sectoral elasticities of substitution have a steady state value equal to 11. Finally, we set the cost of adjusting the stock of durable goods  $\Xi = 600$ , as in Erceg and Levin (2006).

As discussed above, the system features two sector-specific technology shocks,  $z_t^n$  and  $z_t^d$ . The cost-push shocks,  $\eta_t^n$  and  $\eta_t^d$ , are reduced form expressions for the time-varying cost-shift parameters in the sectoral New Keynesian Phillips curves. Exogenous variables are assumed to follow a first-order VAR:

$$\mathbf{v}_t = \mathbf{P}\mathbf{v}_{t-1} + \mathbf{e}_t,$$

where  $\mathbf{v}_t = [z_t^n \ z_t^d \ \eta_t^n \ \eta_t^d]'$ ,  $\mathbf{P}$  is a diagonal matrix whose eigenvalues lie within the unit circle, and  $\mathbf{e}_t$  is a vector of *iid* innovations with diagonal contemporaneous covariance matrix.<sup>17</sup>

We set the parameters describing the persistence and variance of the productivity growth stochastic process in both sectors so that  $\rho^{z^n} = \rho^{z^d} = 0.95$  and  $\sigma^{z^n} = \sigma^{z^d} = 0.02$ , respectively. These values are consistent with the empirical evidence showing that technology shocks are generally small, but highly persistent (see Cooley and Prescott, 1995; Huang and Liu, 2005). As to the cost push shocks, we follow Jensen (2002), Walsh (2003) and Strum (2009), and assume that these are purely transitory, with  $\sigma^{\eta^n} = \sigma^{\eta^d} = 0.02$ .

<sup>12</sup>Steady state conditions are reported in Appendix A. Appendix B reports the economy under flexible prices. The linearized system in extensive form is reported in Appendix C. Appendix D reports the matrices of parameters for the canonical form of the model.

<sup>13</sup>This value is in line with the calibration proposed by Horvath (2000).

<sup>14</sup>This implies that the marginal impact of changes in the relative price on the real sectoral marginal cost is in absolute value higher for the durable goods sector, as  $\alpha_n = \alpha_d$  and  $\gamma_{nd} > \gamma_{dn}$ .

<sup>15</sup>These shares were computed using the table "The Use of Commodities by Industries" for 1992 produced by the BLS. Sudo (2008) shows that the composition of the matrix is rather stable over time.

<sup>16</sup>As in Erceg and Levin (2006), this choice reflects that the durables sector in our model includes both consumer durables and residential investment, which have quarterly depreciation rates of about 5% and 0.75%, respectively, and that the expenditure share of consumer durables in the composite is about two-thirds.

<sup>17</sup>An alternative strategy could consist of assuming correlated innovations. This is not pursued, so as to avoid co-movement through an exogenously postulated mechanism.

## 2.1 Equilibrium Dynamics and Co-Movement in the Face of a Monetary Policy Shock

We briefly discuss the most significant dynamic features of the model before turning our attention to its equilibrium dynamics under optimal monetary policy. To close the model at this stage, it is necessary to specify how the monetary authority sets the nominal rate of interest. In the first instance we consider the following instrumental rule:<sup>18</sup>

$$\hat{r}_t = \rho^r \hat{r}_{t-1} + (1 - \rho^r) \chi_\pi \pi_t + \psi_t^r, \quad \chi_\pi > 0 \text{ and } 0 \leq \rho^r \leq 1 \quad (22)$$

where  $\psi_t^r$  is an *iid*(0, 1) monetary policy innovation. The interest rate smoothing parameter  $\rho^r$  is set to 0.7, while  $\chi_\pi = 1.5$ . We assume that the monetary authority responds to a convex combination of sector-specific rates of inflation. The corresponding price index is defined as:

$$\tilde{p}_t = \phi \tilde{p}_t^n + (1 - \phi) \tilde{p}_t^d, \quad (23)$$

where the weights associated to sectoral prices capture the relative size of each sector and equal the steady state ratio of labour supplied to the non-durable goods sector on the total labour force.

Figure 1 reports the effects of a one-standard-deviation shock to the monetary policy rule at  $t = 0$ , for various frequencies of adjustment in durables prices (the degree of stickiness in the non-durable sector is kept constant at four quarters). Factor demand linkages induce co-movement in durable and non-durable goods spending.<sup>19</sup> Durable and non-durable consumption decrease following the monetary contraction and gradually return to their equilibrium level thereafter. As expected, the first-round impact of the shock and the degree of persistence in the durable goods sector increase in the degree of nominal rigidity. When input materials are excluded ( $\alpha_n = \alpha_d = 0$ ) and prices in the durable goods sector are flexible, as reported in Figure 1, durable consumption increases, while non-durable consumption mirrors its path in the opposite direction. This negative co-movement is induced by the assumption of price flexibility in the durables sector, as opposed to the assumption of price stickiness in the non-durables sector, which implies that the relative price of durables decreases following the initial monetary contraction.<sup>20</sup> The economy reported in the right panel of Figure 1 behaves in line with the model developed by Barsky, House, and Kimball (2007), where the relative price only affects the marginal rate of substitution between different consumption goods. Notice that when input-output interactions are switched off, imperfect labour mobility is not enough to generate co-movement in the face of a monetary policy innovation. Thus, as discussed in Bouakez, Cardia, and Ruge-Murcia (2008), imperfect labour mobility and inter-sectoral linkages are mutually reinforcing elements in the transmission of monetary policy innovations.<sup>21</sup>

In accordance to the empirical evidence produced by Erceg and Levin (2006),<sup>22</sup> the sensitivity of durable spending to the monetary innovation is larger than that of non-durable spending, despite the

<sup>18</sup>See DiPace (2008) for a sensitivity analysis of different instrumental rules.

<sup>19</sup>Not only value added, but also sectoral productions co-move. This feature of the model is in line with the evidence discussed by Christiano and Fitzgerald (1998) and Rebelo (2005).

<sup>20</sup>Indeed Monacelli (2009) shows that a baseline two-sector New Keynesian model with asymmetric price stickiness actually cannot replicate such effects. Whenever consumption contracts in one sector following a monetary tightening, it would expand in the other one. Under perfect financial markets the shadow value of durables (i.e. the discounted stream of marginal utilities of durables) is roughly constant. As a result, durable spending is highly sensitive to variations in the user cost. If durable goods prices are flexible and non-durables prices are sticky, a monetary contraction lowers the relative price of durables and their user cost, leading to opposite effects in the two sectors.

<sup>21</sup>Sudo (2008) shows that in a model with perfect labour mobility factor demand linkages are enough to generate co-movement.

<sup>22</sup>See Barsky, House, and Kimball (2007), Erceg and Levin (2006), and Monacelli (2009).

introduction of an adjustment cost of the effective stock of durables. This effect is more pronounced as the degree of nominal rigidity in the durable goods sector increases. Two inherent features of this class of models are crucial to these results: (i) firstly, durables demand is for a stock, so that changes in the stock amplifies changes in the flow demand for newly produced goods; (ii) secondly, sectoral price rigidities mitigate the role that changes in the relative price (of durables) play in insulating the durables sector from exogenous perturbations. In Section 4 we show that these factors have clear-cut implications for optimal monetary policy design.

### 3 The Pareto Optimum

Removing sources of distortion in the labour market (imperfect labour mobility) and in the goods market (monopolistic competition) represents a desirable situation for a benevolent central banker. At this stage of the analysis we are interested in understanding whether, after removing these distortions, the monetary authority can attain a first best allocation where output gap and inflation in both sectors are jointly stabilized. As in Huang and Liu (2005), the answer to this question is negative for general parameter values and shock processes, but the structure of the model economy imposes a different explanation for this outcome. We consider a variant economy without cost-push shocks and perfect labour mobility. The following proposition formalizes our results.

**Proposition 1** *In the model with sticky prices and perfect labour mobility across sectors there exists no monetary policy that can attain the Pareto optimal allocation, unless the shock buffeting the non-durable goods sector equals the one buffeting the durable goods sector scaled by the factor  $(1 - \alpha_n) / (1 - \alpha_d)$ .*

**Proof.** *Suppose there were a monetary policy under which the equilibrium allocation under sticky prices would be Pareto optimal. Then, in such an equilibrium, the gaps would be completely closed for every period. That is,  $\widetilde{r}m\widetilde{c}_t^n = \widetilde{r}m\widetilde{c}_t^d = 0, \forall t$ . It follows from the pricing conditions that  $\pi_t^n = \pi_t^d = 0, \forall t$ . Recall that the relative price evolves as:*

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*.$$

*Since we also have that  $\Delta \widetilde{q}_t = 0$ , the equation above implies that  $\pi_t^n - \pi_t^d = \Delta q_t^*$ . It can be shown that (see Appendix E for the derivation of the relative price in the efficient equilibrium setting with perfect labour mobility):*

$$q_t^* = \frac{(1 - \alpha_d) z_t^n - (1 - \alpha_n) z_t^d}{1 + \varkappa},$$

where

$$\varkappa = \alpha_n \alpha_d (\gamma_{nn} + \gamma_{dd} - 1) - \alpha_n \gamma_{nn} - \alpha_d \gamma_{dd}.$$

*Therefore, it cannot be that  $\pi_t^n = \pi_t^d = 0$ , unless  $\Delta q_t^* = 0$ , which translates into:*

$$\frac{\Delta z_t^d}{\Delta z_t^n} = \frac{1 - \alpha_d}{1 - \alpha_n}.$$

■

Assuming that durable and non-durable goods are partly used as input materials in both sectors is crucial to these results. If, on top of this, we allow for imperfect labour mobility, this would further reduce the ability of the monetary authority to neutralize the real and nominal effects of the technology shocks. Although leading to the same fundamental conclusion of Huang and Liu (2005) that the Pareto optimal allocation cannot be attained when labour is required in the production of both goods, a

significant difference arises. Our model features the presence of a bi-directional input-output production structure ( $0 < \alpha_d, \alpha_n < 1$ ), whereas Huang and Liu (2005) envisage vertically integrated sectors of production, with the intermediate goods sector employing a constant returns to scale technology. In their case the Pareto optimum can be achieved as long as the sectoral shocks are equal or when (unrealistically) no labour is required in the production of final goods. In our case having the same shock in both sectors does not ensure that these wash out in the aggregate. Unless there is no labour employed in the production process, the only way to attain the Pareto optimum is when shocks buffeting the non-durable goods sector equal the one buffeting the durable goods sector, scaled by a factor that depends on the relative labour income share in the two sectors. Therefore, bi-directional horizontal factor demand linkages impose more restrictive conditions on the full stabilization of the model economy in the face of exogenous technology shocks, compared to models characterized by a vertical input-output structure.

## 4 Optimal Monetary Policy

As shown in the previous section, the central bank cannot attain the Pareto optimal allocation even after different sources of distortion in the labour and in the goods market are removed. Therefore, we turn our attention to policy strategies capable of attaining second best outcomes. We explore equilibrium dynamics under the assumption that the policy maker can commit to a rule derived from the minimization of his objective function. Optimal monetary policy consists of maximizing the conditional expectation of intertemporal household utility subject to private sector's behavioral equations and resource constraints, as discussed by Woodford (2003).<sup>23</sup> This policy regime constitutes a useful benchmark for assessing the performance of alternative policy strategies.

To evaluate social welfare we take a second-order Taylor approximation to the representative household's lifetime utility.<sup>24</sup> Our procedure follows the standard analysis of Rotemberg and Woodford (1998), adapted to account for the presence of factor demand linkages. The resulting intertemporal social loss function reads as:

$$\begin{aligned} SW_0 \approx & -\frac{U_H(H)H}{2}\Theta E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma-1}{\Theta} \left( \mu^n \tilde{c}_t^n + \mu^d \tilde{d}_t \right)^2 + S \left( \tilde{d}_t - \tilde{d}_{t-1} \right)^2 \right. \\ & \left. + \varsigma \left[ \varpi \left( \pi_t^n \right)^2 + (1-\varpi) \left( \pi_t^d \right)^2 \right] + (1+\nu) \left[ \omega \tilde{c}_t^n + (1-\omega) \tilde{c}_t^d \right]^2 \right\} \\ & + \text{t.i.p.} + O(\|\xi\|^3), \end{aligned} \tag{24}$$

<sup>23</sup>We pursue a “timeless perspective” approach, as in Woodford (1999). This involves ignoring the conditions that prevail at the regime's inception, thus imagining that the commitment to apply the rules deriving from the optimization problem had been made in the distant past. In this case, there is no dynamic inconsistency in terms of the central bank's own decision-making process. The system is solved for the evolution of the endogenous variables by relying on the common practice discussed, e.g., by Sims (2002).

<sup>24</sup>We assume that the shocks that hit the economy are not big enough to lead to paths of the endogenous variables distant from their steady state levels. This means that shocks do not drive the economy too far from its approximation point and, therefore, a linear quadratic approximation to the policy problem leads to reasonably accurate solutions. Appendix F reports the derivation of the quadratic welfare function.

where:

$$\begin{aligned}
S &= \mu^d \Theta^{-1} \Xi + (1 - \delta) (1 - \omega) \delta^{-2}, \\
\Theta &= \frac{\mu^n [1 - \beta (1 - \delta)] + \mu^d \delta}{1 - \beta (1 - \delta)}, \\
\varpi &= \phi \varepsilon^n (\kappa^n \varsigma)^{-1}, \\
\omega &= \frac{\mu^n [1 - \beta (1 - \delta)]}{\mu^n [1 - \beta (1 - \delta)] + \mu^d \delta}, \\
\varsigma &= \phi \varepsilon^n (\kappa^n)^{-1} + (1 - \phi) \varepsilon^d (\kappa^d)^{-1}, \\
\kappa^i &= \frac{(1 - \beta \theta^i) (1 - \theta^i)}{\theta^i}, \quad i = \{n, d\}
\end{aligned}$$

and t.i.p. collects the terms independent of policy stabilization, whereas  $O(\|\xi\|^3)$  summarizes all terms of third order or higher.

Function (24) reveals that the welfare criterion assumed by the central bank balances, along with sectoral inflation variability, fluctuations in aggregate consumption (or, equivalently, value added). This is a distinctive feature of our model, which differs from traditional multi-sector models where output gap variability is generally accounted for in the loss function (see, e.g., Aoki, 2001 and Woodford, 2003). The introduction of input materials is crucial to this result, as it implies a non-trivial distinction between output and consumption.

Approximating the terms of social welfare associated with the durables sector involves an additional term,  $S(\tilde{d}_t - \tilde{d}_{t-1})^2$ , which reflects a strong preference to smooth the accumulation of the stock of durable goods.<sup>25</sup> Quantitatively, this term constitutes a primary objective in this setting, compared to the weight of other components of the period loss. Notice that the second term in  $S$  is extremely high and increases in the degree of durability (i.e. decreases in  $\delta$ ). Even though (24) is isomorphic to the social welfare function derived by Erceg and Levin (2006), this feature is not stressed in their paper. The introduction of convex adjustment costs ( $\Xi > 0$ ) reinforces the relative importance of reducing fluctuations in durables accumulation. Moreover, assuming durables accumulation smoothing as a prominent objective helps to counteract the amplification effect of changes in the stock demand of durables on the flow demand of newly produced durable goods.

The remaining weights of the time-varying terms in (24) can be interpreted as follows: (i)  $\varsigma$  indexes the total degree of nominal stickiness in the economy and is inversely related to both  $\kappa^d$  and  $\kappa^n$ ; (ii)  $\varpi$  accounts for the relative degree of price stickiness in the non-durable goods sector; (iii)  $\omega$  represents the relative weight of non-durable consumption over total consumption when durable goods are reported as a flow. This is an inverse function of  $\Theta$ . In turn,  $\Theta$  depends upon the degree of durability of goods produced in sector  $d$ . For  $\delta = 0$  it reduces to  $\mu^n$ , whereas for  $\delta = 1$  it reduces to one. Therefore, as the degree of durability increases, the weight attached to the non-durable consumption gap increases with respect to that attached to the durable term.

Denote by  $\mathcal{W}_t^{VA}|_{\delta=1}$  the part of period loss which depends on the consumption gap variability under the assumption that the consumption goods produced in sector  $d$  fully depreciate: in this case households derive utility from consuming only non-durable goods, as  $\tilde{c}_t^d = \tilde{d}_t$ . Conversely,  $\mathcal{W}_t^{VA}|_{\delta=0}$  denotes the consumption-based component of deadweight loss under perfect durability. This implies that only non-durable consumption is accounted for in the quadratic term derived from labour disutility, as  $\omega = 1$ .

<sup>25</sup>Details on the linear approximation of this term are available in the technical appendix.

$$\mathcal{W}_t^{VA}|_{\delta=1} = -\frac{U_H(H)H}{2}(1+v)(\mu^n \tilde{c}_t^n + \mu^d \tilde{c}_t^d)^2 + \text{t.i.p.} + O(\|\xi\|^3), \quad (25)$$

$$\mathcal{W}_t^{VA}|_{\delta=0} = -\frac{U_H(H)H}{2}(1+v)\mu^n (\tilde{c}_t^n)^2 + \text{t.i.p.} + O(\|\xi\|^3). \quad (26)$$

Notice also that the relative importance of sector-specific inflation variability depends on the steady state ratio of labour supplied to the non-durable goods sector to the total labour force ( $\phi$ ). As  $\phi$  tends to zero, meaning that no labour is supplied to non-durable goods producing firms in the steady state, the relative weight of the terms pertaining to this sector vanishes. Figure 2 maps the evolution of  $\phi$  for different values of  $\alpha_n$  and  $\alpha_d$ . The contour map clearly shows that  $\phi$  decreases in the share of the intermediate input share in the non-durable sector,  $\alpha_n$ , and increases in that of the durable sector,  $\alpha_d$ . The elasticity of  $\phi$  to changes in the production function parameters is generally greater for  $\alpha_d$ , as compared to the marginal impact of changes in  $\alpha_n$ . When no input materials are employed in both sectors ( $\alpha_n = \alpha_d = 0$ ) the loss function reduces to the one obtained in traditional two-sector models with no factor demand linkages between sectors (e.g. Woodford, 2003).

How do factor demand linkages influence social welfare? The left panel of Figure 3 reports the social welfare loss defined over the subspace of the production parameters  $\alpha_n$  and  $\alpha_d$  when both technology and cost-push shocks buffet the model economy. The right panel of Figure 3 reports analogous evidence under the assumption that technology shocks are the only source of exogenous perturbation. The general pattern suggests that the loss of social welfare increases monotonically in the share of intermediate goods used in the non-durable goods sector, whereas the share of input materials in the durable goods sector exerts a negligible impact.

Yet, our main concern is not only with how factor demand linkages affect welfare loss, but also on how central banks' misperception about their importance generates excess loss with respect to the welfare criterion consistent with correctly specified model economy. To answer this question we implement the optimal policy under the assumption that the central bank neglects the relevance of factor demand linkages in the production process (i.e. act under the incorrect belief that  $\alpha_n = \alpha_d = 0$ ). Table 1 reports the (percentage) excess loss under misperception of the input-output structure with respect to the loss under the correctly specified production structure of the economy. As the actual weight of factor demand linkages increases, excess loss can be substantial. Notice also that the marginal impact of misperceiving  $\alpha_n$  is greater than that associated with  $\alpha_d$ , as non-durable consumption goods producers are the major suppliers of intermediate inputs in our calibrated model economy.

## 4.1 Impulse-Response Analysis under Optimal Monetary Policy

Figure 4 reports equilibrium dynamics of the model economy following a one-standard-deviation technology shock in the non-durable goods sector, while the effects of an analogous shock in the durable goods sector are reported in Figure 5. The graphs in the left panel report equilibrium dynamics with factor demand linkages between sectors, whereas on the right side we consider the model with no input materials ( $\alpha_n = \alpha_d = 0$ ). Inflation and interest rates are annualized. Symmetric nominal rigidity is assumed, with  $\theta_n = \theta_d = 0.75$ .<sup>26</sup>

Factor demand linkages amplify the effects of exogenous shocks to either sector, compared to the case without input materials. A technology shock in the non-durable goods sector makes production of these goods relatively cheaper, thus increasing their production and consumption. However, their price

<sup>26</sup>As in Strum (2009) we opt for this choice to prevent the central bank from focusing exclusively on the stickier sector in the formulation of its optimal policy, as predicted by Aoki (2001). In the next subsections we draw implications from the model under asymmetric degrees of nominal rigidity across sectors.

is prevented from reaching the level consistent with the situation of flexible prices. This determines a negative non-durable consumption gap. As to the response of the central bank, the real interest rate (on non-durables) initially rises in the model with no input materials, thus preventing output (and consumption) in the non-durable sector from rising as much as it would under flexible prices. Concurrently, real rates do not rise enough to prevent the output gap in the durables sector from rising too much. As discussed by Erceg and Levin (2006), keeping output at potential in the non-durable goods sector requires a "sharp and persistent fall" in the real interest rate. By contrast, a sharp rise in the policy instrument is required to keep output at potential in the durable goods sector. This is exactly what happens in our variant economy with no input materials. Conversely, in the model with factor demand linkages the real interest rate initially decreases, gradually converging to its equilibrium level thereafter. Keeping output at potential in the non-durable goods sector prevails over the alternative objective. This result is intimately related to the existence of factor demand linkages, which amplify the effect of the initial impulse on the output/consumption gap of non-durables. Moreover, when input materials are employed in the production process, durable consumption under flexible prices increases, thus helping to reduce durables consumption gap. This endogenous mechanism is not at work in the model without input materials, where durables consumption under flexible prices is not affected by the shock as a result of setting  $\sigma = 1$ , which implies separability of households' preferences in durable and non-durable consumption.

A distinctive feature of the model with factor demand linkages deserves attention. When input-output interactions take place between sectors, the relative price (of durables) does not only affect the marginal rate of substitution between durable and non-durable consumption. As reported in equations (11) and (12),  $Q_t$  exerts a positive (negative) effect on the real marginal cost in the durable (non-durable) goods sector. A technology shock in the non-durable goods sector causes an increase in the relative price gap, which implies a substitution away from non-durable to durable consumption goods. In addition, the intermediate input channel gives rise to higher inflationary pressures in the durable goods sector through its influence on real marginal cost. Conversely, the drop in the marginal cost of production of non-durables and the negative value added (output) gap translate into a persistent deflation in this sector. These factors, combined with the expansionary policy pursued by the central bank, imply that the inflationary pressures developed in the sector producing durable goods are in absolute value stronger than the deflationary effect experienced by non-durable goods prices. In turn, this determines rising inflationary pressures at the aggregate level.

Figure 6 reports equilibrium dynamics following a cost-push shock in the non-durable goods sector. A distinctive feature of the model with factor demand linkages is that a cost push shock to non-durable goods inflation drives the relative price gap up. In turn, this element partially counteracts the deflationary effect that is otherwise observed in the model without input-output interactions and that operates through the conventional demand channel. The overall contractionary effect is magnified by the presence of factor demand linkages. This leads the central bank to initially pursue a weakly contractionary policy, accompanied by a negative real rate of interest (on non-durables). Notice the marked discrepancy between the consumption and the production gap in each sector. Following the initial shock, a drop in the demand of both final and intermediate input goods takes place, which leads production to fall by more than consumption. By contrast, following a sectoral technology shock, sectoral production gaps (and value added gaps) co-move negatively under optimal monetary policy. Moreover, a technology shock generally induces consumption to fluctuate more than production. In fact, a drop in the consumption gap of the sector that experiences the positive technology shock translates into a drop in the demand gap of intermediate goods from the other sector. Thus, each sector experiences opposite demand effects on the markets for the consumption and intermediate goods. Importantly, imperfect labour mobility exacerbates this effect, increasing the wedge between consumption and production. When aggregate demand increases exogenously, as labour cannot flow across sectors without

frictions, firms need to increase intermediate inputs by more than they would under the assumption of perfect labour mobility to meet the increased demand. Consequently, fluctuations in production and consumption are wider in the presence of imperfect labour mobility.

Monetary policy is weakly contractionary in the face of a positive cost-push shock to durable goods inflation in the model with input materials (Figure 7), whereas in the variant economy without input materials we observe a substantial rise in the real interest rate (on non-durables). The optimal response in the baseline scenario is accompanied by a small first-round increase in the nominal rate of interest. From this period onwards the policy instrument falls below its steady state level, and gradually returns to equilibrium thereafter. This weakly contractionary monetary policy determines a positive non-durables consumption gap in the model with factor demand linkages. In addition, non-durables deflation is counteracted by the presence of factor demand linkages, compared to the alternative scenario without input materials. Thus, non-durables inflation is substantially stable, provided that the intermediate input channel acts as an endogenous stabilizer. Concurrently, this policy strategy keeps durable goods consumption closer to potential, compared to the alternative scenario with  $\alpha_n = \alpha_d = 0$ . This allows the central bank to place higher attention on controlling fluctuations in the consumption of the stock of durables. If there were no incentive to smooth durable goods accumulation - i.e.  $S = 0$  in (24) - the policy response would be quantitatively closer to that observed in traditional one-sector models or in our variant multi-sector economy without factor demand linkages (Figure 8).

## 4.2 Optimal Monetary Policy versus Alternative Policy Regimes

We now compare the performance of the optimal policy under commitment to a number of alternative policy regimes. These are formalized by means of simple loss functions, and are selected for their tractability. We use the second order welfare approximation (24) as a model-consistent metric. In each case we compute the unconditional expected welfare loss as the percentage of steady state aggregate consumption (and multiply the resulting term by 100).

We consider both strict and flexible inflation targeting regimes,<sup>27</sup> as well as consumption and output gap targeting.<sup>28</sup> Inflation targeting regimes alternatively consider core or aggregate inflation: in the first case the weights attached to sectoral inflations depend on the relative degree of price rigidity, as well as on the relative size of each sector, and the degree of substitutability among differentiated goods; in the second case the weights attached to sectoral inflations only depend on the relative size of each sector.<sup>29</sup> Flexible inflation targeting regimes balance fluctuations in core or aggregate inflation together with a term that penalizes fluctuations in aggregate consumption or gross production. Our focus on "flexible" regimes is relevant as we also consider the presence of sectoral cost-push shocks, which imply an explicit trade-off between the stabilization of output and inflation at the sectoral level. Finally, as the durable goods smoothing quadratic term in (24) constitutes a substantial fraction of the overall period loss, it seems relevant to consider a hybrid inflation targeting regime which combines a preference for durables accumulation smoothing with inflation stabilization.

Both the strict or the flexible inflation targeting stabilize the volatility of aggregate (or core) inflation and not explicitly the volatility of sectoral inflations separately. From a quantitative viewpoint, considering the square term of either aggregate or core inflation involves an additional term, compared to the case in which the central bank cares about fluctuations in sectoral inflations separately. This term is related to the covariance between sectoral inflations, and is not otherwise accounted for in the second-order welfare criterion derived above. More importantly, from a strategic viewpoint we are

<sup>27</sup>The analytic specification of each regime is reported in Appendix H.

<sup>28</sup>Consumption gap targeting can be alternatively interpreted as value added targeting, given that the two concepts are coincident in our setting.

<sup>29</sup>Notice that, when  $\alpha_n = \alpha_d$ , the weights are such that  $\phi = \frac{L^n}{L} = \frac{Y^n}{Y^n + Y^d}$ .

willing to understand whether the central bank can approximate the optimal outcome under commitment, without including the sectoral rates of inflation as "distinct" objectives. In principle, this should enable the monetary authority to provide the public with a more intelligible target, thus increasing the degree of accountability of its policy mandate. Svensson (1997) stresses the importance of assuming intermediate targets which are highly correlated with the goal, easy to control, and transparent, so as to enhance communication to the public. In this sense measures of overall inflation work better than sectoral inflations. Moreover, considering unique measures of aggregate or core inflation allows us to partly address the criticism advanced by Nelson (2002), who suggests that aggregate inflation targeting might not be an undesirable policy regime in this class of multi-sector models, as implied by our loss function.<sup>30</sup>

Table 2 reports the welfare loss under the optimal rule and various alternative policy regimes. The overall loss is disaggregated into that attributable to each weighted term in (24). To compare equilibrium paths under alternative regimes, we evaluate the associated loss by taking as a benchmark our second-order approximation to the representative household's utility function. Both technology and cost-push shocks are assumed to buffet the model economy. We assume that sectors are symmetric in the degree of nominal rigidity ( $\theta_n = \theta_d = 0.75$ ): in this case  $\pi_t^{core} = \pi_t^{agg}$ . A flexible inflation targeting regime, whereby fluctuations in consumption or production are balanced along with aggregate inflation, performs nearly as well as the optimal policy under commitment. The implementation of this hybrid regime allows the policy maker to partially smooth fluctuations in durables accumulation, by including a term that penalizes fluctuations in aggregate value added. Compared to the optimal policy, the central bank improves its performance in stabilizing sectoral inflations, partially counter-balancing the higher loss associated with the volatility in the accumulation of the stock of durables. Importantly, the central bank attains a welfare loss closer to that under optimal monetary policy when fluctuations in aggregate (or core) inflation are balanced with those in real value added, compared to the loss induced by controlling fluctuations in the gross product. The presence of factor demand linkages weakens the perfect correlation between consumption and production. Selecting consumption as a stabilization objective in the flexible inflation targeting regime allows the policy maker to attain a close-to-optimal performance. This is also supported by the second-order approximation to the representative consumer's utility, where consumption variability arises as a stabilization objective, and not production variability.

Table 3 reports the relative performance of alternative policy regimes under various sources of shocks to the economy. We consider both our benchmark model economy ( $\alpha_n = \alpha_d = 0.6$ ), and a model with no factor demand linkages ( $\alpha_n = \alpha_d = 0$ ). As observed at different stages of the analysis, deadweight loss is generally higher in the first case. Concurrently, the presence of cost-push shocks induces higher variability. Again, the flexible inflation targeting regime outperforms other alternative regimes. It is worth noticing that a regime based on strict inflation targeting displays a "competitive" performance only when technology shocks are the unique source of exogenous perturbation, as expected on a priori grounds. Moreover, notice that output gap targeting constantly outperforms consumption gap targeting. Given the non trivial distinction between production and consumption in the model with factor demand linkages, targeting consumption gap volatility enables the central bank to control only "part" of the variability in the marginal cost - and in turn its effect on inflation - compared to what obtained by targeting the production gap, which also accounts for the role of factor demand linkages. In turn, also sectoral inflation volatility benefits from this effect (see Table 2, columns 3 and 4).

Table 4 considers the case of asymmetric price stickiness, in the form of durables prices being more flexible than non-durables prices ( $\theta_n = 0.75, \theta_d = 0.25$ ).<sup>31</sup> This allows us to introduce a distinction be-

<sup>30</sup>Erceg and Levin (2006) explore similar issues, and show that the performance of the optimal policy is closely approximated by a rule that targets a weighted average of aggregate wage and price inflation.

<sup>31</sup>Bils and Klenow (2004) report a higher frequency of price adjustment for consumer durables than services. Erceg and Levin (2006) assume that non-durables and durables are equally sticky. By contrast, Barsky, House, and Kimball (2007)

tween core and aggregate inflation, as discussed earlier. We compare each regime under the assumption that either only sectoral technology shocks buffet the model economy or in the presence of sector-specific cost-push shocks.<sup>32</sup> Considering core inflation targeting as an alternative to aggregate inflation targeting is somewhat related to a long-standing debate on the information that the central bank can access when formulating its strategy. Woodford (2003) shows that in a two-sector model with no input materials optimal commitment policy is nearly replicated by an inflation targeting regime, whereby the weights attached to sectoral inflations depend on the relative degree of nominal stickiness.<sup>33</sup> This result can only be replicated in the presence of technology shocks, no matter whether factor demand linkages are at work. When only cost-push shocks buffet the model economy, or when both sources of exogenous perturbation are at work, core inflation targeting is by far outperformed by every alternative regimes. This results from the pronounced effect that cost-push shocks exert on consumption, and is further amplified in the presence of factor demand linkages. Focusing on the "stickier" sector under core inflation targeting implies that the central bank almost stabilizes inflation in the non-durables sector, at the cost of inducing high volatility on the remaining terms in the intertemporal welfare function (24), especially on aggregate consumption. Our parameterization implies a marked degree of asymmetry in terms of sectoral nominal stickiness ( $\varpi = 0.992$ ), which in turn leads the central bank to focus exclusively on non-durables inflation.

### 4.3 On the Role of Sectoral Asymmetries

We finally examine the role of asymmetry in the degree of competition and price stickiness between sectors for the optimal weighting of sectoral inflations and the resulting welfare properties of the model economy. Our exercise is performed by varying the sectoral Calvo parameters and the elasticities of substitution between goods produced within the same sector, under the assumption that their aggregate counterparts are fixed at the value consistent with the symmetric case.<sup>34</sup>

As the monetary policy authority operates in a timeless perspective, the resulting rule is optimal no matter the nature of the (additive) exogenous disturbances that buffet the model economy, as advocated by Giannoni and Woodford (2003). Thus, it is possible to analyze the effect of sector-specific technology and cost-push shocks separately. Results are reported in Figure 9. Specifically, the y-axis in each graph measures the stickiness gap and the x-axis reports the competitive gap, as defined in Lombardo (2006).<sup>35</sup> The exercise is performed both under the assumption that no input materials are employed, as well as under our baseline calibration of the input-output matrix, i.e.  $\alpha_n = \alpha_d = 0.6$ .

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and Carlstrom and Fuerst (2006) develop models where durables prices are flexible and non-durables prices are sticky. Studies in the housing literature also assume that house prices are flexible (for example, Aoki et al., 2004; Iacoviello, 2005).

<sup>32</sup>Notice that introducing both sources of exogenous perturbation would deliver a scenario similar to the case with only cost-push shocks, as these typically dominate the effect induced by technology shocks on equilibrium dynamics.

<sup>33</sup>Aoki (2001) shows that the welfare-theoretic loss function consistent with a multi-sector economy with heterogeneous degrees of price stickiness attaches higher weight on the inflation variability of sectors characterized by higher nominal stickiness. This provides a theoretical basis for seeking to stabilize an appropriately defined measure of "core" inflation rather than an equally weighted price index. Benigno (2004) relies on this reasoning to argue that a monetary union would maximize welfare by seeking to stabilize an index that does not weight the different countries' inflation rates strictly in proportion to the size of their economies.

<sup>34</sup>We set  $\bar{\theta} = 0.75$  and  $\bar{\varepsilon} = 11$ , as in the baseline calibration with symmetric sectoral nominal stickiness and elasticities of substitution in demand. Thus, we map the loss under an optimal monetary policy for different values of the "nominal rigidity gap" ( $\theta_n - \theta_d$ ) and the "competition gap" ( $\varepsilon_n - \varepsilon_d$ ), under the assumption that  $\phi\theta_n + (1 - \phi)\theta_d = \bar{\theta}$  and  $\phi\varepsilon_n + (1 - \phi)\varepsilon_d = \bar{\varepsilon}$ .

<sup>35</sup>Benigno (2004) explores the impact of asymmetry in the degree of nominal stickiness on the loss of social welfare in the context of a monetary union. Lombardo (2006) shows how asymmetries in the degree of competition can exacerbate or mitigate the effects of asymmetric price rigidity.

On average, input-output interactions increase deadweight loss for different degrees of asymmetry in either dimension (price stickiness and degree of competition). This result emerges both when only technology shocks are accounted for, as well as in the model with only cost-push shocks. In the model with cost push shocks and in the presence of factor demand linkages the loss is about 15% to 55% higher than that attainable without horizontal flows of input materials between the two sectors. When only technology shocks are considered the loss of social welfare is about 25% to 50% higher than that without input materials.

**Model with Technology Shocks** - We first consider technology shocks as the sole source of exogenous perturbation. Before turning to the analysis of asymmetry in our baseline model economy, it is instructive to provide a gradual overview of the effects of each distinctive feature of the model.<sup>36</sup> We start by assuming a two-sector economy with sectors of equal size, no durability in either of the goods produced, and no input-output linkages (i.e.  $\alpha_n = \alpha_d = 0$ ,  $\delta = 1$  and  $\mu^n = \mu^d = \omega = \phi$ ). The resulting loss function is concave over both dimensions of asymmetry. A hump-shaped pattern emerges over the subspace considered. Thus, as asymmetry in either dimension increases, overall unconditional variance decreases. This result is in line with Benigno (2004), Lombardo (2006)<sup>37</sup> and Woodford (2003), and is closely related to the existence of a trade-off in the stabilization of sectoral inflations. When sectors are the same size and have the same production characteristics, asymmetry in the degree of nominal stickiness and/or in the competition gap mitigates this trade-off, as the monetary authority can focus on keeping inflation under control in the sector with higher weight in the index of core inflation. In such circumstances a lower total deadweight loss derives from the combined effect of: (i) a higher effectiveness of the policy against fluctuations in the sector with higher price rigidity and/or higher elasticity of substitution in demand, and (ii) lower price rigidity and/or lower elasticity of substitution in demand in the other sector, which respectively imply lower inflation persistence and lower cross-sectional dispersion in prices. When factor demand linkages are introduced and sectors are of equal size ( $\alpha_n = \alpha_d = 0.6$ ,  $\delta = 1$  and  $\mu^n = \mu^d$ ) the loss is still symmetric over the "nominal rigidity gap", but decreases in the gap between the elasticity of substitution of the sector that represents the main supplier in the economy ( $n$ ) and the one of the other sector ( $d$ ). Due to the existence of factor demand linkages, inefficiencies from nominal rigidities in either sector are partially passed through into the other sector. This effect helps to counteract the influence of the "nominal rigidity gap" on social welfare. Moreover, if the competition gap is positive ( $\varepsilon_n > \varepsilon_d$ ), then the weight attached to non-durable goods inflation rises linearly in  $\varepsilon_n$ . In this sense the monetary authority has to give higher attention to inflation fluctuations in this sector (with respect to the symmetric case). In turn, provided that the non-durable goods sector is a net supplier of intermediate goods in the model economy, stabilizing  $\pi_t^n$  helps to reduce fluctuations in the marginal cost gap of the durable goods sector, which is also affected by the relative price gap. When durability is accounted for ( $\alpha_n = \alpha_d = 0.6$ ,  $\delta = 0.025$  and  $\mu^n = \mu^d$ ), an additional objective emerges in the central bank's welfare criterion, which reflects a rather strong preference for smoothing the accumulation of the stock of durables. The presence of this term amplifies the impact of nominal rigidities in the durable goods sector, thus leading the total deadweight loss to increase for  $\theta_d > \theta_n$  and  $\varepsilon_d > \varepsilon_n$ .

Eventually, under the baseline calibration of our model ( $\alpha_n = \alpha_d = 0.6$ ,  $\delta = 0.025$  and  $\mu^n = 0.682$ ), asymmetry in the degree of nominal stickiness exerts a stronger marginal impact on welfare. For a given level of asymmetry in sectoral competition, optimal monetary policy places greater weight on the

<sup>36</sup>For reasons of space we only describe the deadweight loss under different parameterizations, rather than reporting the corresponding graphs. These are available, upon request, from the authors.

<sup>37</sup>Lombardo (2006) shows that when prices are set in a staggered fashion, the amount of 'output dispersion' generated by a given deviation of prices from their average depends positively on the elasticity of substitution between goods (i.e. the degree of competition in our model). Therefore, the country with the largest degree of competition is the one that generates the greatest cost of inflation.

"stickier" sector. The minimum loss is attained when the economy is characterized by the both the highest competitive gap and the highest asymmetry in the degree of nominal stickiness, and specifically when the non-durable goods sector is more competitive and "stickier". Notice that, given the relative size of the two sectors, in order to keep aggregate stickiness at a fixed level, a marginal increase in  $\theta_n$  is accompanied by a more than proportional decrease in  $\theta_d$ . Analogous observations apply to the "competition gap". Higher  $\theta_n$  and  $\varepsilon_n$  mean that the central bank penalizes inflationary pressures relatively more in the broader sector of our model.

**Model with Cost-Push Shocks** - In the remainder we only consider the baseline calibration reported in Section 2. The introduction of sectoral cost-push shocks introduces a trade-off between output and inflation stabilization. Total welfare decreases in the degree of asymmetry in nominal stickiness and competition between sectors. This can be explained intuitively. As it can be observed in (24), the Calvo parameters indexing the degree of nominal rigidity in either sector enter non-linearly in the component of loss associated with fluctuations in core inflation. By contrast, the elasticity of substitution between goods produced in either sector only affects the relative weight of core inflation with respect to other objectives,  $\varpi$ . The trade-off between inflation and consumption is such that inflation volatility induced by the sector-specific cost-push shocks is not completely stabilized by the monetary authority. Consequently, the contour map tracks the pattern of  $\varpi$  in the subspace defined over the asymmetry gaps. In particular,  $\varpi$  evolves convexically with respect to the nominal rigidity gap, with a minimum occurring at zero, and increases (decreases) for  $\theta_n > (<) \theta_d$ .

**Model with Technology and Cost-Push Shocks** - The analysis of the loss function in the presence of both sources of exogenous perturbation suggests that the monetary authority faces an easier task in the stabilization of the variability due to the technology shocks, compared to its performance in the presence of cost-push shocks. Moreover, provided that cost-push shocks produce a non-trivial trade-off between inflation and output stabilization, they have a predominant impact on welfare. This is clearly displayed in the last panel of Figure 9, where aggregate loss is generally convex with respect to the nominal rigidity gap, whereas the effect of the competition gap is negative (positive) when the nominal rigidity gap is positive (negative).

## 5 Conclusions

We have integrated an horizontal input-output production structure into a dynamic general equilibrium model where two sectors produce durable and non-durable goods. Part of the output produced in each sector is used as an intermediate input of production in both sectors, according to an input-output matrix calibrated on the US economy. This implies a non-trivial difference between consumption (or, equivalently, value added) and gross product, given the presence of input materials. The resulting input-output interactions have non-negligible implications for the formulation of policy aimed at reducing real and nominal fluctuations. The intermediate input channel generally amplifies the effects of exogenous perturbations to the system and alters the transmission mechanism, compared to standard multi-sector models. A key role is played by the relative price of non-durable goods, which acts not only as an allocative mechanism on the demand side, through its influence on the user cost of durable goods, but also on the supply side, through its effect on the sectoral real marginal costs of production.

The social welfare function derived through a second-order Taylor approximation to households' utility reveals that the policy maker is faced with the task of stabilizing fluctuations in sectoral inflation and aggregate value added. In addition, a term reflecting a strong preference for smoothing the accumulation of the stock of durable goods arises in the welfare criterion. This results as a primary objective of the central banker, whether convex costs are assumed for adjusting the effective stock of durable goods or not.

Strategic complementarities induced by factor demand linkages amplify welfare loss under optimal monetary policy, compared to what is commonly observed in otherwise standard two-sector models. As the degree of interconnection between sectors increases, neglecting input-output interactions causes an even greater loss of social welfare.

When factor demand linkages are explicitly accounted for, the relative price of non-durable goods not only influences the user cost of durables through the conventional demand channel, but also affects in opposite directions the real marginal cost of production in either sector through the intermediate input channel. For example, in the face of a sector-specific cost push shock the intermediate input channel acts as an endogenous stabilizer that attenuates the deflationary effect in the sector which is not hit by the shock.

Finally, we explore the welfare properties of the system under a wide set of alternative policy regimes. We assume that the monetary authority alternatively selects a "restricted" policy objective, compared to the specification of the welfare criterion consistent with the model economy. Importantly, a flexible inflation targeting regime, whereby the central bank balances fluctuations in aggregate or core inflation with those in value added delivers a welfare loss close to that attained under optimal monetary policy. This emphasizes the distinction between consumption and production that naturally arises in frameworks with horizontally integrated sectors of production.

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# TABLES

TABLE 1: FACTOR DEMAND LINKAGES MISPERCEPTION

TECH SHOCKS					COST PUSH SHOCKS					BOTH SHOCKS				
$\alpha_n/\alpha_d$	0	0.3	0.6	0.8	$\alpha_n/\alpha_d$	0	0.3	0.6	0.8	$\alpha_n/\alpha_d$	0	0.3	0.6	0.8
0	0	4.664	22.468	52.119	0	0	2.984	9.400	16.554	0	0	3.075	10.210	18.322
0.3	2.348	1.012	14.763	41.453	0.3	3.904	1.234	4.970	10.061	0.3	3.552	1.209	5.601	11.267
0.6	17.232	0.906	5.713	27.242	0.6	28.415	11.303	10.324	13.976	0.6	25.684	8.912	9.517	14.037
0.8	49.424	8.468	1.160	17.336	0.8	92.084	45.841	36.050	38.283	0.8	77.991	30.982	27.664	35.721

Note: We report the percentage excess loss under a misperception of the input-output structure with respect to the loss under the correctly specified production structure of the economy, for different shocks.

TABLE 2: WELFARE UNDER ALTERNATIVE POLICIES

	Loss Components				
	$\Delta\tilde{d}_t$	$\pi_t^n$	$\pi_t^d$	$x_t$	Total
Optimal Policy	0.0417	0.6477	0.2918	0.2274	1.2086
Inflation Targeting	0.5486	0.0785	0.3472	2.6662	3.6405
Inflation Targeting (with Durable Smoothing)	0.1937	0.2486	0.3043	1.0450	1.7916
Consumption Gap Targeting	0.0234	1.6396	0.4026	0.0000	2.0656
Production Gap Targeting	0.0133	1.5380	0.3721	0.0067	1.9300
Flexible Inflation Targeting (with Consumption)	0.0476	0.6360	0.2900	0.2377	1.2113
Flexible Inflation Targeting (with Output)	0.0121	1.1501	0.3234	0.0243	1.5099

Note:  $\Delta\tilde{d}_t$  refers to the durable smoothing objective, and  $x_t$  to aggregate consumption (value added). The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

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TABLE 3: WELFARE UNDER ALTERNATIVE POLICIES AND DIFFERENT PRODUCTION STRUCTURES

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MODEL WITH FACTOR DEMAND LINKAGES ( $\alpha_n = \alpha_d = 0.6$ )

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	Tech. Shocks	Cost Push Shocks	Dur. Sec. Shocks	Non-Dur. Sec. Shocks
Optimal Policy	0.1741	1.0645	0.2686	0.9658
Inflation Targeting	0.1768	3.5025	0.3891	3.3110
Inflation Targeting (with Dur. Smoot.)	0.1862	1.6418	0.2884	1.5459
Consumption Gap Targeting	0.2016	1.9235	0.2881	1.8400
Production Gap Targeting	0.1758	1.7903	0.2957	1.6749
Flex. Inflation Targeting (with Cons.)	0.1753	1.0654	0.2957	0.9665
Flex. Inflation Targeting (with Prod.)	0.1744	1.3664	0.2823	1.2581

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MODEL WITHOUT FACTOR DEMAND LINKAGES ( $\alpha_n = \alpha_d = 0$ )

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	Tech. Shocks	Cost Push Shocks	Dur. Sec. Shocks	Non-Dur. Sec. Shocks
Optimal Policy	0.1156	0.6897	0.2854	0.5152
Inflation Targeting	0.1166	0.8662	0.3028	0.6778
Inflation Targeting (with Dur. Smoot.)	0.1157	0.7890	0.2911	0.6111
Output Gap Targeting	0.1217	1.6498	0.4196	1.3670
Flex. Inflation Targeting	0.1163	0.6907	0.2856	0.5168

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Note: The first two columns report the loss attributable to technology shocks and cost-push shocks generated in both sectors, respectively. The last two columns report the loss due to both shocks in either sector. The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

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TABLE 4: ASYMMETRY IN PRICE STICKINESS

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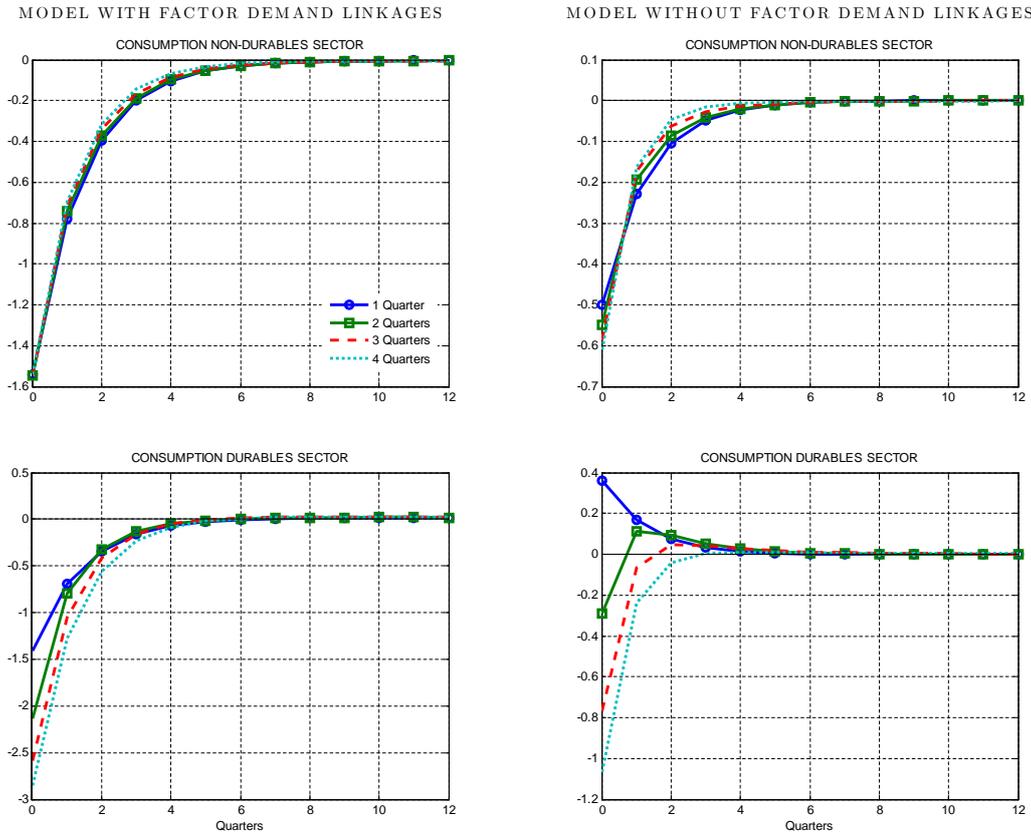
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	Tech. Shocks	Cost Push Shocks
Optimal Policy	0.0388	0.8648
Core Inflation Targeting	0.0391	3.6864
Agg. Inflation Targeting	0.2000	1.3171
Core Inflation Targeting (with Dur. Smoot.)	0.0389	2.0198
Agg. Inflation Targeting (with Dur. Smoot.)	0.1822	1.1883
Consumption Gap Targeting	0.0408	1.6711
Production Gap Targeting	0.0396	1.6575
Flex. Core Inflation Targeting (with Cons.)	0.0402	0.8654
Flex. Agg. Inflation Targeting (with Cons.)	0.0936	0.8757
Flex. Core Inflation Targeting (with Prod.)	0.0396	1.1855
Flex. Agg. Inflation Targeting (with Prod.)	0.0463	1.1517

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Note: We set the average duration of the non-durable goods' prices at 4 quarters, whereas we reduce the duration of durable goods prices to 1.3 quarters ( $\theta^d = 0.25$ ). The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by 100).

FIGURE 1: IMPULSE RESPONSES TO A MONETARY POLICY TIGHTENING



Note: We employ the following instrumental rule  $\hat{r}_t = \rho^r \hat{r}_{t-1} + (1 - \rho^r) \chi_\pi \pi_t + \psi_t^r$ , where  $\psi_t^r$  is an *iid*  $(0, 1)$  monetary policy innovation and the constant term involving the inflation target has been suppressed for simplicity. The interest rate smoothing parameter  $\rho^r$  is set to 0.7, while  $\chi_\pi = 1.5$ . We assume that the monetary authority responds to a convex combination of sector-specific rates of inflation.

FIGURE 2: LABOUR INCOME SHARE IN THE NON-DURABLE GOODS SECTOR ( $\phi$ )

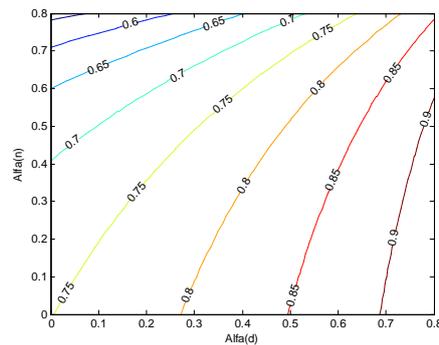
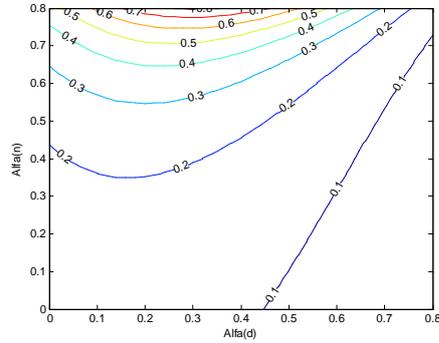
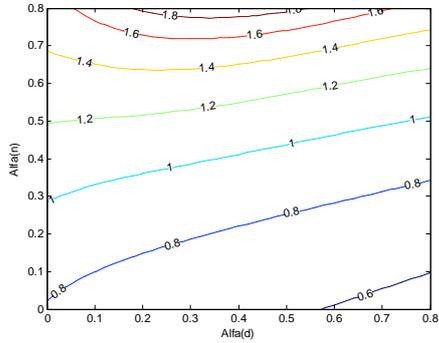
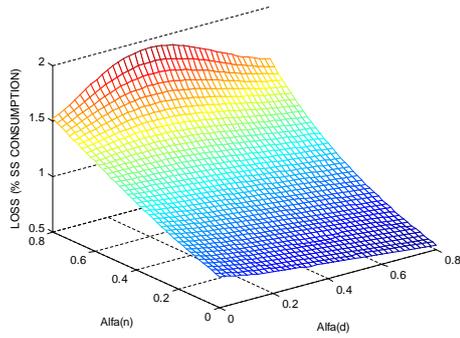


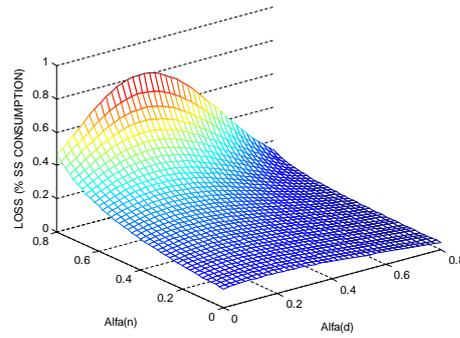
FIGURE 3: WELFARE UNDER OPTIMAL MONETARY POLICY FOR DIFFERENT  $\alpha_n$  AND  $\alpha_d$   
 TECHNOLOGY AND COST-PUSH SHOCKS      ONLY TECHNOLOGY SHOCKS



BOTH SHOCKS

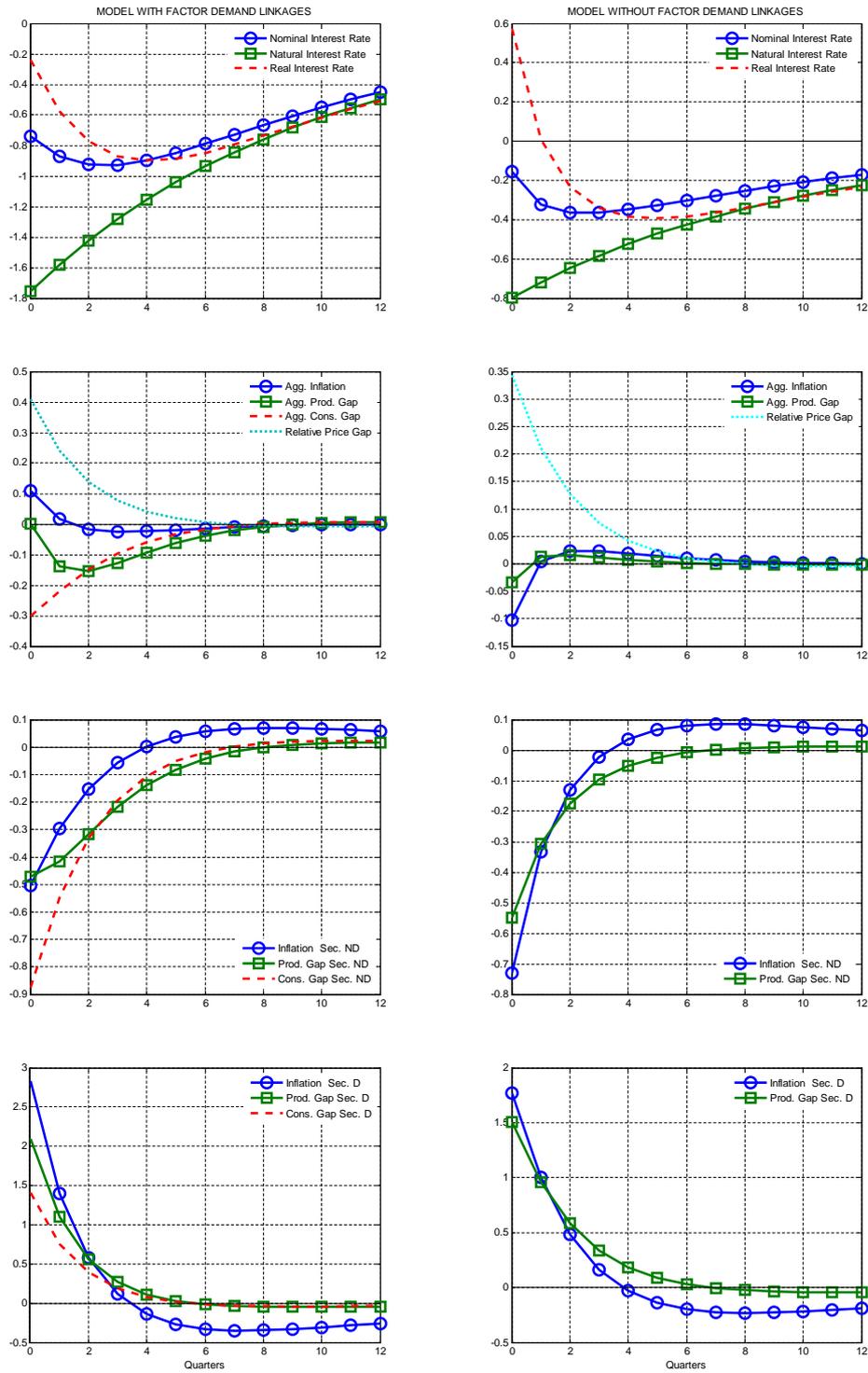


ONLY TECHNOLOGY SHOCKS



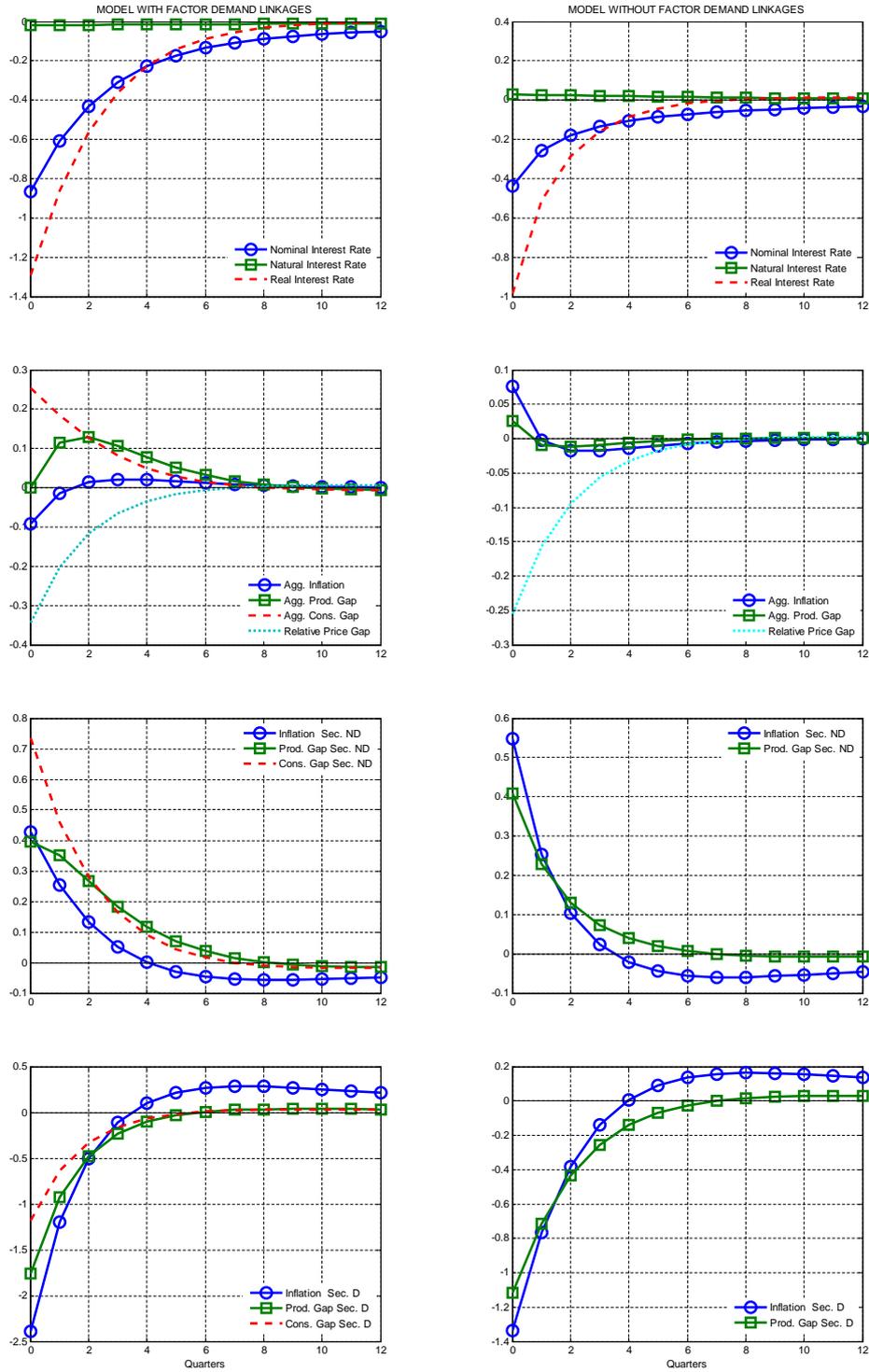
Note: The left panel of Figure 3 reports the welfare loss defined over the subspace of the production parameters in the two sectors when both technology and cost-push shocks buffet the model economy. The right panel reports analogous evidence under the assumption that technology shocks are the only source of exogenous perturbation.

FIGURE 4: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE NON-DURABLE GOODS SECTOR



Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 5: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE DURABLE GOODS SECTOR



Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 6: IMPULSE RESPONSES TO A COST PUSH SHOCK IN THE NON-DURABLE GOODS SECTOR

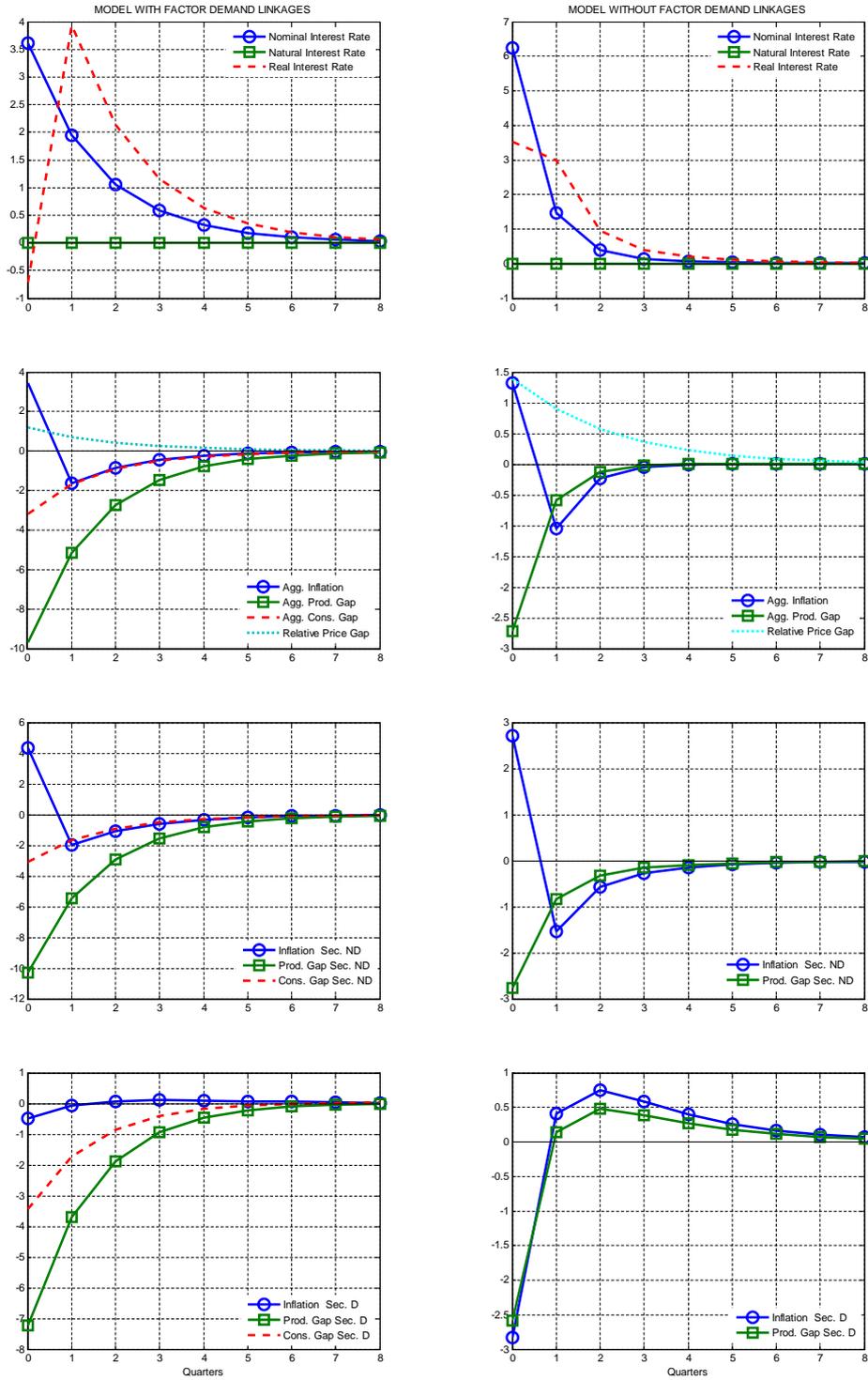


FIGURE 7: IMPULSE RESPONSES TO A COST PUSH SHOCK IN THE DURABLE GOODS SECTOR

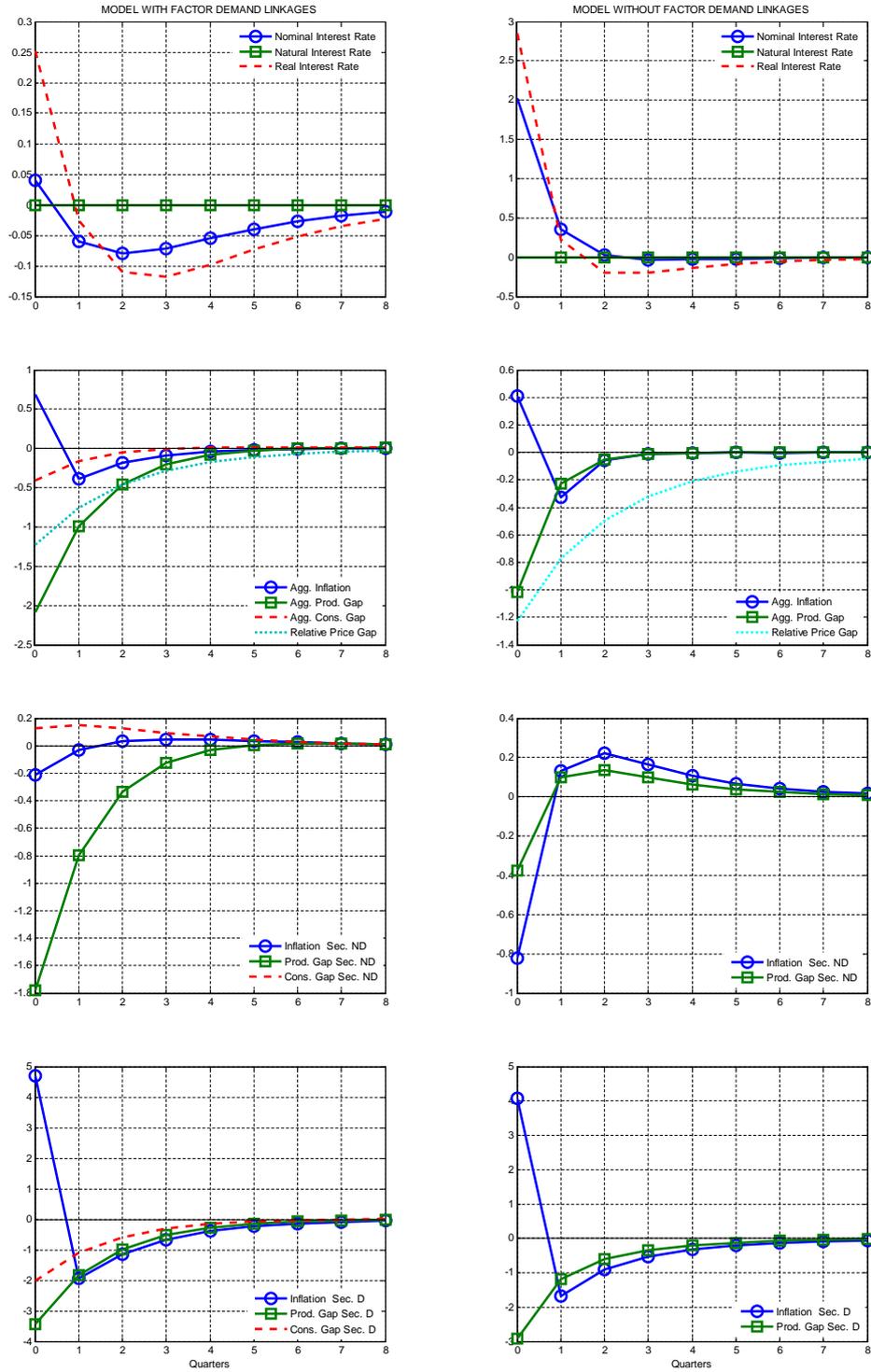


FIGURE 8: IMPULSE RESPONSES TO A COST PUSH SHOCK IN THE DURABLE GOODS SECTOR  
 Optimal Commitment Policy with no Durables Accumulation Smoothing ( $S = 0$ ).

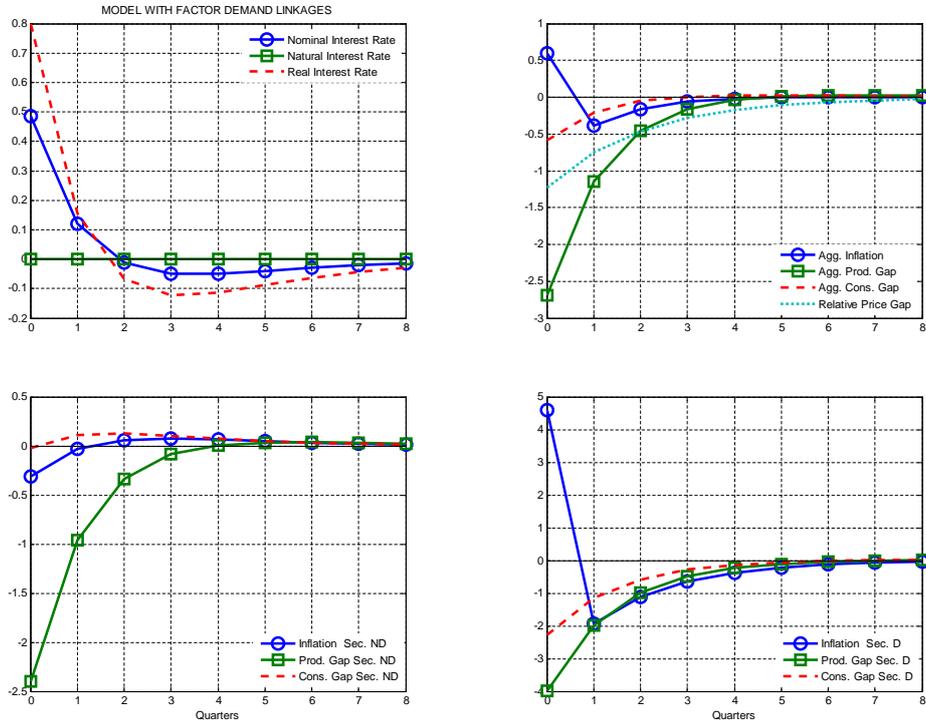
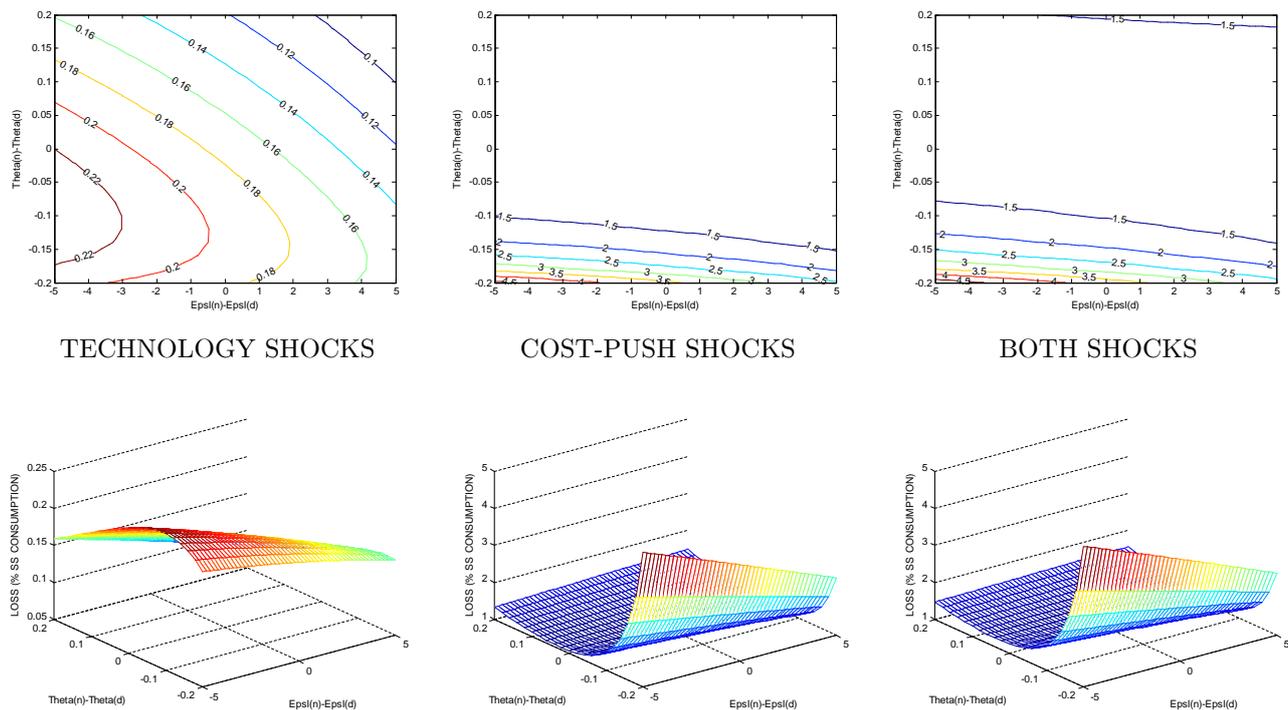


FIGURE 9: THE IMPACT OF ASYMMETRY ON WELFARE LOSS



Note: We set  $\bar{\theta} = 0.75$  and  $\bar{\varepsilon} = 11$ , as in the baseline calibration with symmetric sectoral nominal stickiness and elasticities of substitution in demand. Thus, we map the loss under optimal monetary policy for different values of the "nominal rigidity gap" ( $\theta_n - \theta_d$ ) and the "competition gap" ( $\varepsilon_n - \varepsilon_d$ ), under the assumption that  $\phi\theta_n + (1 - \phi)\theta_d = \bar{\theta}$  and  $\phi\varepsilon_n + (1 - \phi)\varepsilon_d = \bar{\varepsilon}$ .

## APPENDIX A: Some Useful Steady State Relationships

As in the competitive equilibrium real wage in each sector equals the marginal product of labour. Thus, we can derive the following relationship between the production in non-durable and durable goods in the steady state:

$$\frac{Y^n}{Y^d} = \frac{(1 - \alpha_d)\phi}{(1 - \alpha_n)(1 - \phi)} Q^{-1}.$$

Furthermore, the following relationship between durable and non-durable consumption can be derived from the Euler conditions:

$$\frac{C^n}{C^d} = (1 - \beta(1 - \delta)) \frac{\mu^n}{\mu^d} \frac{1}{\delta} Q^{-1}.$$

Moreover, the following shares of consumption and intermediate goods over total production are determined for the non-durable goods sector:

$$\begin{aligned} \frac{C^n}{Y^n} &= \frac{(1 - \alpha_n \gamma_{nn}) \phi (1 - \alpha_d) - (1 - \alpha_n) (1 - \phi) \alpha_d \gamma_{nd}}{\phi (1 - \alpha_d)}, \\ \frac{M^{nn}}{Y^n} &= \alpha_n \gamma_{nn}, \\ \frac{M^{nd}}{Y^n} &= \frac{(1 - \alpha_n)}{\phi} \frac{1 - \phi}{1 - \alpha_d} \alpha_d \gamma_{nd}. \end{aligned}$$

Analogously, for the durable goods sector:

$$\begin{aligned} \frac{C^d}{Y^d} &= \frac{(1 - \alpha_d \gamma_{dd}) (1 - \phi) (1 - \alpha_n) - (1 - \alpha_d) \phi \alpha_n \gamma_{dn}}{(1 - \phi) (1 - \alpha_n)}, \\ \frac{M^{dn}}{Y^d} &= \frac{1 - \alpha_d}{1 - \phi} \frac{\phi}{1 - \alpha_n} \alpha_n \gamma_{dn}, \\ \frac{M^{dd}}{Y^d} &= \alpha_d \gamma_{dd}. \end{aligned}$$

These conditions turn out to be crucial in the second order approximation of consumers' utility, in order to eliminate linear terms. Moreover, they allow us to derive the steady state ratio of labour supply in the non-durable goods sector over the total labour supply ( $\phi$ ).

### The Relative Price in the Steady State

We consider the steady state condition for the marginal cost in the non-durable goods sector:

$$\begin{aligned} MC^n &= \Phi_n [(P^n)^{\gamma_{nn}} (P^d)^{\gamma_{dn}}]^{\alpha_n} (W^n)^{1 - \alpha_n}, \\ \Phi_n &= \alpha_n^{\alpha_n} (1 - \alpha_n)^{1 - \alpha_n}. \end{aligned}$$

As in the steady state production subsidies neutralize distortions due to imperfect competition:

$$\begin{aligned} P^n &= MC^n \\ &= \Phi_n [(P^n)^{\gamma_{nn}} (P^d)^{\gamma_{dn}}]^{\alpha_n} (W^n)^{1 - \alpha_n}. \end{aligned}$$

After some trivial manipulations it can be shown that:

$$\Phi_n Q^{-\alpha_n \gamma_{dn}} (RW^n)^{1-\alpha_n} = 1.$$

Analogously, for the durables goods sector:

$$\Phi_d Q^{\alpha_d \gamma_{nd}} (RW^d)^{1-\alpha_d} = 1.$$

Using the fact that in steady state  $W^n = W^d = W$ :

$$\begin{aligned} \frac{RW^n}{RW^d} Q &= 1, \\ \frac{(\Phi_n^{-1} Q^{\alpha_n \gamma_{dn}})^{\frac{1}{1-\alpha_n}}}{(\Phi_d^{-1} Q^{-\alpha_d \gamma_{nd}})^{\frac{1}{1-\alpha_d}}} Q &= 1. \end{aligned}$$

Therefore:

$$\begin{aligned} Q &= \left( \Phi_n^{1-\alpha_d} \Phi_d^{-(1-\alpha_n)} \right)^{\frac{1}{\varphi}}, \\ \varphi &= (1-\alpha_n)(1-\alpha_d) + \alpha_n \gamma_{dn} (1-\alpha_d) + \alpha_d \gamma_{nd} (1-\alpha_n). \end{aligned}$$

Notice that, when  $\alpha_n = \alpha_d = 1$ :

$$Q = \Phi_n \Phi_d^{-1}$$

as in the case considered by Huang and Liu (2005) and Strum (2008).

## APPENDIX B: Equilibrium Dynamics in the Efficient Equilibrium

In this appendix we outline the solution method of the linear model under the efficient equilibrium. This is obtained when both prices are flexible and elasticities of substitution are constant. Let us start from the pricing rule under flexible prices:

$$\begin{aligned} P_t^{n*} &= \frac{\Theta^n}{1+\tau^n} MC_t^{n*} \\ &= \frac{\Theta^n}{1+\tau^n} \frac{\bar{\phi}^n [(P_t^{n*})^{\gamma_{nn}} (P_t^{d*})^{\gamma_{dn}}]^{\alpha_n} (W_t^*)^{1-\alpha_n}}{Z_t^n} \\ P_t^{d*} &= \frac{\Theta^d}{1+\tau^d} MC_t^{d*} \\ &= \frac{\Theta^d}{1+\tau^d} \frac{\bar{\phi}^d [(P_t^{n*})^{\gamma_{nd}} (P_t^{d*})^{\gamma_{dd}}]^{\alpha_d} (W_t^*)^{1-\alpha_d}}{Z_t^d} \end{aligned}$$

where  $\Theta^n$  and  $\Theta^d$  denote the mark-up terms. In log-linear form the conditions above reduce to:

$$(1-\alpha_n) r w_t^{n*} = z_t^n + \alpha_n \gamma_{dn} q_t^* \quad (27)$$

$$(1-\alpha_d) r w_t^{d*} = z_t^d - \alpha_d \gamma_{nd} q_t^* \quad (28)$$

We now recall some conditions under flexible prices from the linearized system:

$$c_t^{d*} = \frac{1}{\delta} d_t^* - \frac{1-\delta}{\delta} d_{t-1}^*, \quad (29)$$

$$rw_t^{n*} = -\gamma c_t^{n*} - (1-\sigma) \mu^d d_t^* + \left[ v(1-\phi) - \frac{1}{\lambda} \right] l_t^{d*} + \left( \vartheta \phi + \frac{1}{\lambda} \right) l_t^{n*}, \quad (30)$$

$$l_t^{n*} = \lambda (rw_t^{n*} - rw_t^{d*} + q_t^*) + l_t^{d*}, \quad (31)$$

$$y_t^{n*} = \frac{C^n}{Y^n} c_t^{n*} + \frac{M^{nn}}{Y^n} m_t^{nn*} + \frac{M^{nd}}{Y^n} m_t^{nd*}, \quad (32)$$

$$y_t^{d*} = \frac{C^d}{Y^d} x_t^* + \frac{M^{dn}}{Y^d} m_t^{dn*} + \frac{M^{dd}}{Y^d} m_t^{dd*}, \quad (33)$$

$$0 = rw_t^{n*} + l_t^{n*} - y_t^{n*}, \quad (34)$$

$$0 = rw_t^{d*} + l_t^{d*} - y_t^{d*}, \quad (35)$$

$$0 = m_t^{nn*} - y_t^{n*}, \quad (36)$$

$$0 = m_t^{nd*} + q_t^* - y_t^{d*}, \quad (37)$$

$$0 = m_t^{dn*} - q_t^* - y_t^{n*}, \quad (38)$$

$$0 = m_t^{dd*} - y_t^{d*}, \quad (39)$$

where  $\vartheta = (v - \frac{1}{\lambda})$ ,  $\gamma = (1-\sigma) \mu^n - 1$  and  $\phi = \frac{L^n}{L}$ . We substitute (27) and (28) into (33) and (34) respectively:

$$l_t^{n*} = y_t^{n*} - \frac{1}{1-\alpha_n} z_t^n - \frac{\alpha_n \gamma_{dn}}{1-\alpha_n} q_t^*, \quad (40)$$

$$l_t^{d*} = y_t^{d*} - \frac{1}{1-\alpha_d} z_t^d + \frac{\alpha_d \gamma_{nd}}{1-\alpha_d} q_t^*. \quad (41)$$

We can use conditions (31), (32), and (36)-(39), to get:

$$y_t^{n*} = \frac{C^n}{Y^n} c_t^{n*} + \frac{M^{nn}}{Y^n} y_t^{n*} + \frac{M^{nd}}{Y^n} (y_t^{d*} - q_t^*)$$

and

$$y_t^{d*} = \frac{C^d}{Y^d} c_t^{d*} + \frac{M^{dn}}{Y^d} (q_t^* + c_t^{g*}) + \frac{M^{dd}}{Y^d} y_t^{d*}.$$

We can find a VAR solution of this system, so that we can express  $y_t^{n*}$  and  $y_t^{d*}$  as a function of  $c_t^{n*}$ ,  $c_t^{d*}$  and  $q_t^*$ :

$$\mathbf{A} \begin{bmatrix} y_t^{n*} \\ y_t^{d*} \end{bmatrix} = \mathbf{B} \begin{bmatrix} c_t^{n*} \\ c_t^{d*} \end{bmatrix} + \mathbf{\Upsilon} q_t^*,$$

where

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} 1 - \frac{M^{nn}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & 1 - \frac{M^{dd}}{Y^d} \end{bmatrix} = \begin{bmatrix} \frac{C^n}{Y^n} + \frac{M^{nd}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & \frac{C^d}{Y^d} + \frac{M^{dn}}{Y^d} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} \frac{C^n}{Y^n} & 0 \\ 0 & \frac{C^d}{Y^d} \end{bmatrix}, \\ \Upsilon &= \begin{bmatrix} -\frac{M^{nd}}{Y^n} \\ \frac{M^{dn}}{Y^d} \end{bmatrix}.\end{aligned}$$

Thus, we obtain:

$$\begin{bmatrix} y_t^{n*} \\ y_t^{d*} \end{bmatrix} = \mathbf{A}^{-1}\mathbf{B} \begin{bmatrix} c_t^{n*} \\ c_t^{d*} \end{bmatrix} + \mathbf{A}^{-1}\Upsilon q_t^*,$$

or equivalently:

$$\begin{aligned}y_t^{n*} &= aq_t^* + bc_t^{n*} + ec_t^{d*}, \\ y_t^{d*} &= cq_t^* + dc_t^{d*} + fc_t^{n*}.\end{aligned}$$

These can be plugged into (40) and (41) to obtain:

$$l_t^{n*} = bc_t^{n*} + ec_t^{d*} - \frac{1}{1 - \alpha_n} z_t^n + \left( a - \frac{\alpha_n \gamma_{dn}}{1 - \alpha_n} \right) q_t^* \quad (42)$$

$$l_t^{d*} = dc_t^{d*} + fc_t^{n*} - \frac{1}{1 - \alpha_d} z_t^d + \left( \frac{\alpha_d \gamma_{nd}}{1 - \alpha_d} + c \right) q_t^* \quad (43)$$

Thus, we can substitute everything into (30) and (27):

$$\frac{1 + v\phi}{1 - \alpha_n} z_t^n + \xi_1 z_t^d = \xi_2 c_t^{n*} - (1 - \sigma) \mu^d d_t^* + \xi_3 c_t^{d*} + \xi_4 q_t^*, \quad (44)$$

where:

$$\begin{aligned}\xi_1 &= \frac{v(1 - \phi)\lambda - 1}{(1 - \alpha_n)\lambda}, \\ \xi_2 &= \frac{\lambda(v\phi b - \gamma) + f[v(1 - \phi)\lambda - 1]}{\lambda}, \\ \xi_3 &= \frac{\lambda v\phi e + d[v(1 - \phi)\lambda - 1]}{\lambda}, \\ \xi_4 &= -\frac{\alpha_n \gamma_{dn}(1 + v\phi)}{1 - \alpha_n} + \left[ \frac{v(1 - \phi)\lambda - 1}{(1 - \alpha_d)\lambda} \left( \frac{\alpha_d \gamma_{nd}}{1 - \alpha_d} + c \right) + av\phi \right].\end{aligned}$$

In turn, we can plug (42), (43), (27) and (28) into (30):

$$\xi_5 q_t^* = \frac{1}{1 - \alpha_n} z_t^n - \frac{1}{1 - \alpha_d} z_t^d - \frac{k}{(1 + \lambda)} (c_t^{n*} - c_t^{d*}) \quad (45)$$

where

$$\xi_5 = \frac{a - c - \lambda}{1 + \lambda} - \left[ \frac{\alpha_n \gamma_{dn}}{(1 - \alpha_n)} + \frac{\alpha_d \gamma_{nd}}{(1 - \alpha_d)} \right].$$

Conditions (44) and (45), together with the Euler conditions for the durable and the non-durable goods, and the law of accumulation for durable goods, allow us to determine a system of linear difference equations. Equilibrium dynamics under flexible prices can be reported in canonical form as:

$$E_t [\mathbf{F}_0 \kappa_{t+1}^* + \mathbf{F}_1 \kappa_t^* + \mathbf{F}_2 \kappa_{t-1}^* + \mathbf{C}_1 \varpi_t] = 0 \quad (46)$$

$$\begin{aligned} \kappa_t^* &= [c_t^{n*} \quad c_t^{d*} \quad d_t^* \quad q_t^* \quad rr_t^*]' \\ \varpi_t &= [z_t^n \quad z_t^d]' \end{aligned}$$

and

$$\mathbf{F}_0 = \begin{bmatrix} & & \mathbf{0}_{2 \times 5} & & \\ & -\gamma & 0 & -(1-\sigma)\mu^d & \mathbf{0}_{1 \times 2} \\ \beta(1-\delta)[(1-\sigma)\mu^n - 1] & 0 & F_0^{(4,3)} & 0 & -\beta(1-\delta) \\ & & \mathbf{0}_{1 \times 5} & & \end{bmatrix},$$

$$F_0^{(4,3)} = \beta \{ (1-\delta)(1-\sigma)\mu^d + \Xi [1 - \beta(1-\delta)] \}$$

$$\mathbf{F}_1 = \begin{bmatrix} F_1^{(1,1)} & F_1^{(1,2)} & (1-\sigma)\mu^d & F_1^{(1,4)} & 0 & \\ \frac{k}{(1+\lambda)} & -\frac{k}{(1+\lambda)} & 0 & F_1^{(2,4)} & 0 & \\ \gamma & 0 & (1-\sigma)\mu^d & 0 & -1 & \\ [1 - \beta(1-\delta)(1-\sigma)\mu^n] & 0 & F_1^{(4,3)} & 1 & 0 & \\ 0 & 1 & -\frac{1}{\delta} & \mathbf{0}_{1 \times 2} & & \end{bmatrix},$$

$$F_1^{(1,1)} = -\left(\vartheta\phi + \frac{1}{\lambda}\right)b + \gamma - \vartheta(1-\phi)f,$$

$$F_1^{(1,2)} = -\vartheta(1-\phi)d - \left(\vartheta\phi + \frac{1}{\lambda}\right)e,$$

$$F_1^{(1,4)} = -\frac{\alpha_n \gamma d_n}{1-\alpha_n} \left[1 + \left(\vartheta\phi + \frac{1}{\lambda}\right)\right] + \vartheta(1-\phi) \left(\frac{\alpha_d \gamma n d}{1-\alpha_d} + c\right) + a \left(\vartheta\phi + \frac{1}{\lambda}\right),$$

$$F_1^{(2,4)} = \frac{a-c-\lambda}{1+\lambda} - \frac{\alpha_n \gamma d_n}{(1-\alpha_n)} - \frac{\alpha_d \gamma n d}{(1-\alpha_d)},$$

$$F_1^{(4,3)} = -\{1 + \beta(1-\delta)[(1-\sigma)\mu^d - 1]\} - \Xi(1+\beta)[1 - \beta(1-\delta)]$$

$$\mathbf{F}_2 = \begin{bmatrix} & & \mathbf{0}_{3 \times 5} & & \\ \mathbf{0}_{1 \times 2} & \Xi [1 - \beta(1-\delta)] & & & \mathbf{0}_{1 \times 2} \\ & \mathbf{0}_{1 \times 2} & \frac{1-\delta}{\delta} & & \mathbf{0}_{1 \times 2} \end{bmatrix},$$

$$\mathbf{C}_1 = \begin{bmatrix} \frac{1+\vartheta\phi+\frac{1}{\lambda}}{1-\alpha_n} & \frac{\vartheta(1-\phi)}{1-\alpha_d} \\ -\frac{1}{1-\alpha_n} & \frac{1}{1-\alpha_d} \\ \mathbf{0}_{3 \times 2} & \end{bmatrix}.$$

Moreover, (46) can be re-written to include all the shocks  $\mathbf{v}_t$  as

$$E_t [\mathbf{F}_0 \kappa_{t+1}^* + \mathbf{F}_1 \kappa_t^* + \mathbf{F}_2 \kappa_{t-1}^* + \bar{\mathbf{C}}_1 \mathbf{v}_t] = 0 \quad (47)$$

with

$$\bar{\mathbf{C}}_1 = [ \mathbf{C}_1 \quad \mathbf{0}_{5 \times 2} ].$$

## APPENDIX C1: Log-linear Economy

Here we report the log-linear economy in extensive form:

$$\begin{aligned} \tilde{c}_t^n &= \frac{1}{\gamma} (\hat{r}_t - E_t \pi_{t+1}^n - r r_t^*) + E_t \tilde{c}_{t+1}^n + \frac{(1-\sigma)\mu^d}{\gamma} E_t \Delta \tilde{d}_{t+1}, \\ \tilde{c}_t^n &= \frac{1}{\mu^n (1-\sigma)} \left\{ [1 - \mu^d (1-\sigma)] \tilde{d}_t + \frac{1}{1-\beta(1-\delta)} [(\mu^n (1-\sigma) - 1) \tilde{c}_t + \mu^d (1-\sigma) \tilde{d}_t - \tilde{q}_t] + \right. \\ &\quad \left. - \frac{(1-\delta)\beta}{[1-\beta(1-\delta)]} [(\mu^n (1-\sigma) - 1) \tilde{c}_{t+1}^n + \mu^d (1-\sigma) \tilde{d}_{t+1} - \tilde{q}_{t+1}] + \right. \\ &\quad \left. + \Xi (\tilde{d}_t - \tilde{d}_{t-1}) - \beta \Xi (\tilde{d}_{t+1} - \tilde{d}_t) \right\} \\ \tilde{c}_t^d &= \frac{1}{\delta} \tilde{d}_t - \frac{1-\delta}{\delta} \tilde{d}_{t-1}, \\ \tilde{r}w_t^n &= -\gamma \tilde{c}_t^n - (1-\sigma) \mu^d \tilde{d}_t + \vartheta (1-\phi) \tilde{l}_t^d + \left( \vartheta \phi + \frac{1}{\lambda} \right) \tilde{l}_t^n, \\ \tilde{l}_t^n &= \lambda (\tilde{r}w_t^n - \tilde{r}w_t^d + \tilde{q}_t) + \tilde{l}_t^d, \\ \pi_t^n &= \beta E_t \pi_{t+1}^n + \frac{(1-\beta\theta^n)(1-\theta^n)}{\theta^n} \tilde{r}m\tilde{c}_t^n + \eta_t^n, \\ \pi_t^d &= \beta E_t \pi_{t+1}^d + \frac{(1-\beta\theta^d)(1-\theta^d)}{\theta^d} \tilde{r}m\tilde{c}_t^d + \eta_t^d, \\ \tilde{y}_t^n &= \alpha_n \gamma_{nn} \tilde{m}_t^{nn} + \alpha_n \gamma_{dn} \tilde{m}_t^{dn} + (1-\alpha_n) \tilde{l}_t^n, \\ \tilde{y}_t^d &= \alpha_d \gamma_{nd} \tilde{m}_t^{nd} + \alpha_d \gamma_{dd} \tilde{m}_t^{dd} + (1-\alpha_d) \tilde{l}_t^d, \\ \tilde{y}_t^n &= \frac{C^n}{Y^n} \tilde{c}_t^n + \frac{M^{nn}}{Y^n} \tilde{m}_t^{nn} + \frac{M^{nd}}{Y^n} \tilde{m}_t^{nd}, \\ \tilde{y}_t^d &= \frac{C^d}{Y^d} \tilde{c}_t^d + \frac{M^{dn}}{Y^d} \tilde{m}_t^{dn} + \frac{M^{dd}}{Y^d} \tilde{m}_t^{dd}, \\ \tilde{r}m\tilde{c}_t^n &= \tilde{r}w_t^n + \tilde{l}_t^n - \tilde{y}_t^n, \\ \tilde{r}m\tilde{c}_t^d &= \tilde{r}w_t^d + \tilde{l}_t^d - \tilde{y}_t^d, \\ \tilde{r}m\tilde{c}_t^n &= \tilde{m}_t^{nn} - \tilde{y}_t^n, \\ \tilde{r}m\tilde{c}_t^d &= \tilde{m}_t^{nd} + \tilde{q}_t - \tilde{y}_t^d, \\ \tilde{r}m\tilde{c}_t^n &= \tilde{m}_t^{dn} - \tilde{q}_t - \tilde{y}_t^n, \\ \tilde{r}m\tilde{c}_t^d &= \tilde{m}_t^{dd} - \tilde{y}_t^d, \\ \tilde{q}_t &= \tilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*. \end{aligned}$$

where  $\gamma = (1-\sigma)\mu^n - 1$ .

## APPENDIX C2: The Model in Canonical Form

The log-linear system in canonical form reads as:

$$E_t \left[ \mathbf{A}_0 \tilde{\boldsymbol{\xi}}_{t+1} + \mathbf{A}_1 \tilde{\boldsymbol{\xi}}_t + \mathbf{A}_2 \tilde{\boldsymbol{\xi}}_{t-1} + \mathbf{A}_3 \hat{r}_t + \mathbf{A}_4 \boldsymbol{\kappa}_t^* + \mathbf{A}_5 \boldsymbol{\kappa}_{t-1}^* + \mathbf{B}_1 \mathbf{v}_t \right] = 0, \quad (48)$$

$$\mathbf{v}_t = \mathbf{P} \mathbf{v}_{t-1} + \mathbf{e}_t, \quad (49)$$

where

$$\begin{aligned} \tilde{\boldsymbol{\xi}}_t &= \left[ \tilde{c}_t^n \quad \tilde{y}_t^n \quad \tilde{c}_t^d \quad \tilde{y}_t^d \quad \tilde{d}_t \quad \tilde{r}w_t^n \quad \tilde{r}w_t^d \quad \pi_t^n \quad \pi_t^d \quad \tilde{q}_t \right]', \\ \boldsymbol{\kappa}_t^* &= \left[ c_t^{n*} \quad c_t^{d*} \quad d_t^* \quad q_t^* \quad rr_t^* \right]', \\ \mathbf{v}_t &= \left[ z_t^n \quad z_t^d \quad \eta_t^n \quad \eta_t^d \right]'. \end{aligned}$$

The matrices of parameters are:

$$\mathbf{A}_0 = \begin{bmatrix} -\gamma & 0_{1 \times 3} & -\frac{\mu^d}{(1-\sigma)^{-1}} & 0_{1 \times 2} & 1 & 0_{1 \times 2} \\ (1-\delta)\beta\gamma & 0_{1 \times 3} & A_0^{(2,5)} & 0_{1 \times 4} & -\beta(1-\delta) \\ & & 0_{3 \times 10} \\ & & 0_{1 \times 7} & -\beta & 0_{1 \times 2} \\ & & 0_{1 \times 8} & -\beta & 0 \\ & & & & 0_{3 \times 10} \end{bmatrix}$$

$$A_0^{(2,5)} = \beta \{ (1-\delta)(1-\sigma)\mu^d + \Xi [1 - \beta(1-\delta)] \}$$

$$\mathbf{A}_1 = \begin{bmatrix} & & & \gamma & 0_{1 \times 3} & \frac{\mu^d}{(1-\sigma)^{-1}} & 0_{1 \times 5} \\ & & & \epsilon & 0_{1 \times 3} & A_1^{(2,5)} & 0_{1 \times 4} & 1 \\ & & & & 0_{1 \times 2} & 1 & 0 & -\frac{1}{\delta} & 0_{1 \times 5} \\ \gamma & \frac{1-\vartheta\phi\lambda}{\lambda} & 0 & \frac{\vartheta}{(\phi-1)^{-1}} & \frac{\mu^d}{(1-\sigma)^{-1}} & \frac{\lambda+(1+\vartheta\phi\lambda)\alpha_n}{\lambda} & \frac{\vartheta\alpha_d}{(1-\phi)^{-1}} & 0_{1 \times 2} & A_1^{(4,10)} \\ 0 & -1 & 0 & 1 & 0 & (\alpha_n + \lambda) & -(\alpha_d + \lambda) & 0_{1 \times 2} & (\alpha_n\gamma_{dn} + \alpha_d\gamma_{nd} + \lambda) \\ & & & & 0_{1 \times 5} & -\kappa^n(1-\alpha_n) & 0 & 1 & 0 & \kappa^n\alpha_n\gamma_{dn} \\ & & & & & 0_{1 \times 6} & -\kappa^d(1-\alpha_d) & 0 & 1 & -\kappa^d\alpha_d\gamma_{nd} \\ -\frac{C^n}{Y^n} & \left(1 - \frac{M^{nn}}{Y^n}\right) & 0 & -\frac{M^{nd}}{Y^n} & 0 & -\frac{(1-\alpha_n)M^{nn}}{Y^n} & -\frac{(1-\alpha_d)M^{nd}}{Y^n} & 0_{1 \times 2} & A_1^{(8,10)} \\ 0 & -\frac{M^{dn}}{Y^d} & -\frac{C^d}{Y^d} & \left(1 - \frac{M^{dd}}{Y^d}\right) & 0 & -\frac{(1-\alpha_n)M^{dn}}{Y^d} & -\frac{(1-\alpha_d)M^{dd}}{Y^d} & 0_{1 \times 2} & A_1^{(9,10)} \\ & & & & 0_{1 \times 7} & -1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned}
A_1^{(2,5)} &= -\{1 + \beta(1 - \delta)[(1 - \sigma)\mu^d - 1]\} - \Xi(1 + \beta)[1 - \beta(1 - \delta)] \\
A_1^{(4,10)} &= -\vartheta(1 - \phi)\alpha_d\gamma_{nd} + \left(\vartheta\phi + \frac{1}{\lambda}\right)\alpha_n\gamma_{dn} \\
A_1^{(8,10)} &= -\frac{M^{nd}}{Y^n}(\alpha_d\gamma_{nd} - 1) + \frac{M^{nn}}{Y^n}\alpha_n\gamma_{dn} \\
A_1^{(9,10)} &= -\frac{M^{dn}}{Y^d}(1 - \alpha_n\gamma_{dn}) - \frac{M^{dd}}{Y^d}\alpha_d\gamma_{nd} \\
\kappa^n &= \frac{(1 - \beta\theta^n)(1 - \theta^n)}{\theta^n} \\
\kappa^d &= \frac{(1 - \beta\theta^d)(1 - \theta^d)}{\theta^d} \\
\epsilon &= [1 - \beta(1 - \delta)(1 - \sigma)\mu^n]
\end{aligned}$$

$$\mathbf{A}_2 = \begin{bmatrix} & 0_{2 \times 10} & \\ 0_{1 \times 4} & \frac{1-\delta}{\delta} & 0_{1 \times 5} \\ & 0_{6 \times 10} & \\ & 0_{1 \times 9} & -1 \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} -1 \\ 0_{9 \times 1} \end{bmatrix}$$

$$\mathbf{A}_4 = \begin{bmatrix} 0_{1 \times 4} & 1 \\ 0_{8 \times 5} \\ 0_{1 \times 3} & 1 & 0 \end{bmatrix}$$

$$\mathbf{A}_5 = \begin{bmatrix} 0_{9 \times 5} \\ 0_{1 \times 3} & -1 & 0 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0_{5 \times 4} \\ 0_{1 \times 2} & -1 & 0 \\ 0_{1 \times 3} & -1 \\ 0_{3 \times 4} \end{bmatrix}$$

## Closing the Model with a Policy Rule

Assume that the central bank sets the interest rate according to a simple rule of the form:

$$r_t = \zeta \xi_t$$

For instance, we can assume that:

$$\begin{aligned}
r_t &= \chi_\pi [\omega\pi_t^n + (1 - \omega)\pi_t^d], \\
\zeta &= [ \mathbf{0}_{1 \times 7} \quad \chi_\pi\omega \quad \chi_\pi(1 - \omega) \quad 0 ].
\end{aligned}$$

Define:

$$\bar{\xi}_t = [ \xi_t' \quad r_t ]'.$$

Thus (48) can be rewritten as:

$$E_t [\bar{\mathbf{A}}_0 \bar{\xi}_{t+1} + \bar{\mathbf{A}}_1 \bar{\xi}_t + \bar{\mathbf{A}}_2 \bar{\xi}_{t-1} + \bar{\mathbf{A}}_4 \kappa_t^* + \bar{\mathbf{A}}_5 \kappa_{t-1}^* + \bar{\mathbf{B}}_1 \mathbf{v}_t] = 0$$

$$\begin{aligned}
\bar{\mathbf{A}}_0 &= \begin{bmatrix} \mathbf{A}_0 & \mathbf{0}_{10 \times 1} \\ & \mathbf{0}_{1 \times 11} \end{bmatrix} & \bar{\mathbf{A}}_1 &= \begin{bmatrix} \mathbf{A}_1 & \mathbf{0}_{10 \times 1} \\ -\zeta & 1 \end{bmatrix} \\
\bar{\mathbf{A}}_2 &= \begin{bmatrix} \mathbf{A}_2 & \mathbf{0}_{10 \times 1} \\ & \mathbf{0}_{1 \times 11} \end{bmatrix} & \bar{\mathbf{A}}_4 &= \begin{bmatrix} \mathbf{A}_4 \\ \mathbf{0}_{1 \times 5} \end{bmatrix} \\
\bar{\mathbf{A}}_5 &= \begin{bmatrix} \mathbf{A}_5 \\ \mathbf{0}_{1 \times 5} \end{bmatrix} & \bar{\mathbf{B}}_1 &= \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{0}_{1 \times 4} \end{bmatrix}
\end{aligned}$$

Following Sims (2002), we cast the model in the following form:

$$\mathbf{\Gamma}_0 \mathbf{y}_t = \mathbf{c} + \mathbf{\Gamma}_1 \mathbf{y}_{t-1} + \mathbf{\Psi} \mathbf{z}_t + \mathbf{\Pi} \boldsymbol{\eta}_t \quad (50)$$

where  $\mathbf{c}$  is a vector of constant terms,  $\mathbf{z}_t$  is an exogenously evolving, possibly serially correlated, random disturbance, and  $\boldsymbol{\eta}_t$  is an expectational error, such that  $E_t(\boldsymbol{\eta}_{t+1}) = 0, \forall t$ . Moreover:

$$\begin{aligned}
\mathbf{y}_t &= \begin{bmatrix} \mathbf{v}_t \\ \bar{\boldsymbol{\xi}}_t \\ E_t \bar{\boldsymbol{\xi}}_{t+1} \\ \kappa_t^* \\ E_t \kappa_{t+1}^* \end{bmatrix}, & \mathbf{\Psi} &= \begin{bmatrix} \boldsymbol{\Sigma} \\ \mathbf{0}_{11 \times 4} \\ \mathbf{0}_{11 \times 4} \\ \mathbf{0}_{5 \times 4} \\ \mathbf{0}_{5 \times 4} \end{bmatrix}, & \mathbf{\Pi} &= \begin{bmatrix} \mathbf{0}_{4 \times 16} \\ \mathbf{0}_{11 \times 16} \\ \mathbf{I}_{11} & \mathbf{0}_{11 \times 5} \\ \mathbf{0}_{5 \times 16} \\ \mathbf{0}_{5 \times 11} & \mathbf{I}_5 \end{bmatrix}, \\
\mathbf{\Gamma}_0 &= \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 5} & \mathbf{0}_{4 \times 5} \\ \bar{\mathbf{B}}_1 & \bar{\mathbf{A}}_1 & \bar{\mathbf{A}}_0 & \bar{\mathbf{A}}_4 & \mathbf{0}_{11 \times 5} \\ \mathbf{0}_{11 \times 4} & \mathbf{I}_{11} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} \\ \bar{\mathbf{C}}_1 & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} & \mathbf{F}_1 & \mathbf{F}_0 \\ \mathbf{0}_{5 \times 4} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} & \mathbf{I}_5 & \mathbf{0}_{5 \times 5} \end{bmatrix}, \\
\mathbf{\Gamma}_1 &= \begin{bmatrix} \mathbf{P} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 5} & \mathbf{0}_{4 \times 5} \\ \mathbf{0}_{11 \times 4} & -\bar{\mathbf{A}}_2 & \mathbf{0}_{11 \times 11} & -\bar{\mathbf{A}}_5 & \mathbf{0}_{11 \times 5} \\ \mathbf{0}_{11 \times 4} & \mathbf{0}_{11 \times 11} & \mathbf{I}_{11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} \\ \mathbf{0}_{5 \times 4} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} & -\mathbf{F}_2 & \mathbf{0}_{5 \times 5} \\ \mathbf{0}_{5 \times 4} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 5} & \mathbf{I}_5 \end{bmatrix}, \\
\boldsymbol{\Sigma} &= \text{diag} \left[ \sigma^{z^n} \quad \sigma^{z^d} \quad \sigma^{\eta^n} \quad \sigma^{\eta^d} \right].
\end{aligned}$$

The  $\boldsymbol{\eta}_t$  terms are not given exogenously. Instead, they are determined as part of the model solution. Models with more lags, or with lagged expectations, or with expectations of more distant future values, can be accommodated in this framework by expanding the  $\mathbf{y}$  vector.

## APPENDIX D: Relative Price in the Efficient Equilibrium with Perfect Labour Mobility

We now define the efficient equilibrium in the model with no frictions in both the goods and the labour market. On the labour market this condition, obtained for  $\lambda \rightarrow \infty$ , ensures that nominal salaries are equalized across sectors of the economy:

$$W_t^{n*} = W_t^{d*} = W_t^*. \quad (51)$$

Moreover, given the production subsidies that eliminate sectoral distortions due to monopolistic competition:

$$P_t^{n*} = MC_t^{n*} \quad P_t^{d*} = MC_t^{d*}. \quad (52)$$

Conditions (51) and (52) imply that:

$$P_t^{n*} = \left(\bar{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(P_t^{d*}\right)^{\frac{\alpha_n\gamma_{dn}}{1-\alpha_n\gamma_{nn}}} \left(W_t^*\right)^{\frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}} \left(Z_t^n\right)^{-\frac{1}{1-\alpha_n\gamma_{nn}}}, \quad (53)$$

$$P_t^{d*} = \left(\bar{\phi}^d\right)^{\frac{1}{1-\alpha_d\gamma_{dd}}} \left(P_t^{n*}\right)^{\frac{\alpha_d\gamma_{nd}}{1-\alpha_d\gamma_{dd}}} \left(W_t^*\right)^{\frac{1-\alpha_d}{1-\alpha_d\gamma_{dd}}} \left(Z_t^d\right)^{-\frac{1}{1-\alpha_d\gamma_{dd}}}. \quad (54)$$

We then substitute (53) into (54) to eliminate  $W_t^*$ :

$$\left(P_t^{n*}\right)^{\vartheta_n} = \Upsilon^{(1-\alpha_n\gamma_{nn})(1-\alpha_d)} \left(P_t^{d*}\right)^{\vartheta_d} \left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{(1-\alpha_n)}$$

where

$$\Upsilon = \left(\bar{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(\bar{\phi}^d\right)^{-\frac{1}{1-\alpha_d} \frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}}$$

and

$$\vartheta_n = \vartheta_d = (1 - \alpha_d) (1 - \alpha_n \gamma_{nn}) + (\alpha_d \gamma_{nd}) (1 - \alpha_n).$$

Thus, after some trivial algebra we can show that the relative price reads as:

$$\begin{aligned} Q_t^* &= \frac{P_t^{n*}}{P_t^{d*}} = \Upsilon \left[ \left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{1-\alpha_n} \right]^{\frac{1}{\varkappa+1}} \\ &= \Upsilon \left[ \left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{1-\alpha_n} \right]^{\frac{1}{\varkappa+1}}. \end{aligned}$$

where

$$\varkappa = \alpha_n \alpha_d (\gamma_{nn} + \gamma_{dd} - 1) - \alpha_n \gamma_{nn} - \alpha_d \gamma_{dd}.$$

## APPENDIX E1: Second Order Approximation of the Utility Function

Following Rotemberg and Woodford (1998), we derive a well-defined welfare function from the utility function of the representative household:

$$\mathcal{W}_t = U(C_t^n, D_t) - V(L_t).$$

We start from a second order approximation of the utility from consumption of durable and non-durable goods:

$$U(C_t^n, D_t) \approx U(C^n, D) + U_{C^n}(C^n, D)(C_t^n - C^n) + \frac{1}{2} U_{C^n C^n}(C^n, D)(C_t^n - C^n)^2 \quad (55)$$

$$\begin{aligned} &+ U_D(C^n, D)(D_t - D) + \frac{1}{2} U_{DD}(C^n, D)(D_t - D)^2 + \frac{1}{2} \Xi U_D(C^n, D)(D_t - D_{t-1})^2 \\ &+ U_{C^n D}(C^n, D)(C_t^n - C^n)(D_t - D) + O(\|\xi\|^3), \end{aligned} \quad (56)$$

where  $O(\|\xi\|^3)$  summarizes all terms of third order or higher. Notice that:

$$\begin{aligned} U_D(C^n, D) &= (\mu^d C^n / \mu^n D) U_{C^n}(C^n, D), \\ U_{C^n C^n}(C^n, D) &= [\mu^n (1 - \sigma) - 1] (C^n)^{-1} U_{C^n}(C^n, D), \\ U_{DD}(C^n, D) &= [\mu^d (1 - \sigma) - 1] (\mu^d C^n / \mu^n D) U_{C^n}(C^n, D), \\ U_{C^n D}(C^n, D) &= \mu^d (1 - \sigma) D^{-1} U_{C^n}(C^n, D). \end{aligned}$$

As  $\frac{C_t^n - C^n}{C^n} = \widehat{c}_t^n + \frac{1}{2} (\widehat{c}_t^n)^2$ , where  $\widehat{c}_t^n = \log\left(\frac{C_t^n}{C^n}\right)$  is the log-deviation from steady state under sticky prices, we obtain:

$$\begin{aligned} U(C_t^n, D_t) &\approx U(C^n, D) + U_{C^n}(C^n, D) C^n \left[ \widehat{c}_t^n + \frac{1}{2} (\widehat{c}_t^n)^2 \right] + \\ &+ \frac{1}{2} [\mu^n (1 - \sigma) - 1] U_{C^n C^n}(C^n, D) C^n \left[ \widehat{c}_t^n + \frac{1}{2} (\widehat{c}_t^n)^2 \right]^2 + \\ &+ U_D(C^n, D) D \left( \widehat{d}_t + \frac{1}{2} \widehat{d}_t^2 \right) + \frac{1}{2} [\mu^d (1 - \sigma) - 1] U_{DD}(C^n, D) D \left( \widehat{d}_t + \frac{1}{2} \widehat{d}_t^2 \right)^2 + \\ &+ \frac{1}{2} \Xi U_{C^n D}(C^n, D) D \left( \widehat{d}_t - \widehat{d}_{t-1} \right)^2 + \\ &+ \mu^d (1 - \sigma) U_{C^n D}(C^n, D) C^n \left[ \widehat{c}_t^n + \frac{1}{2} (\widehat{c}_t^n)^2 \right] \left( \widehat{d}_t + \frac{1}{2} \widehat{d}_t^2 \right) + \text{t.i.p.} + O(\|\xi\|^3), \end{aligned}$$

where t.i.p. collects terms independent of policy stabilization.

Next, we introduce a second order approximation to the transition law for the stock of durables. This will substitute out the linear term for durables in the expression above (see Erceg and Levin, 2006). The law of motion reads as:

$$D_t = (1 - \delta) D_{t-1} + X_t.$$

For a general function  $F(Y, X)$  the second order Taylor approximation can be written as:

$$\begin{aligned} F(Y, X) &\approx F_Y(Y, X) Y y + F_X(Y, X) X x + \frac{1}{2} (F_{XX}(Y, X) X^2) x^2 \\ &+ \frac{1}{2} (F_{YY}(Y, X) Y^2) y^2 + F_{YX}(Y, X) Y X x y. \end{aligned}$$

Now, we can rewrite the accumulation equation as:

$$F(D_{t-1}, C_t^d) = \log[(1 - \delta) D_{t-1} + C_t^d].$$

Therefore:

$$\begin{aligned}
F_D &= \frac{(1-\delta)}{(1-\delta)D + C^d} = \frac{(1-\delta)}{(1-\delta)D + \delta D} = \frac{(1-\delta)}{D}, \\
F_{C^d} &= \frac{1}{(1-\delta)D + C^d} = \frac{1}{D}, \\
F_{DD} &= -\frac{(1-\delta)^2}{[(1-\delta)D + C^d]^2} = -\frac{(1-\delta)^2}{D^2}, \\
F_{C^d C^d} &= -\frac{1}{[(1-\delta)D + C^d]^2} = -\frac{1}{D^2}, \\
F_{DC^d} &= -\frac{1-\delta}{[(1-\delta)D + C^d]^2} = -\frac{1-\delta}{D^2}.
\end{aligned}$$

Considering that in the steady state  $C^d = \delta D$ :

$$\begin{aligned}
\hat{d}_t &\approx \frac{(1-\delta)}{D} D \hat{d}_{t-1} + \frac{1}{D} \delta D \hat{c}_t^d + \\
&+ \frac{1}{2} \left[ \frac{(1-\delta)}{D} D - \frac{(1-\delta)^2}{D^2} D^2 \right] \hat{d}_{t-1}^2 + \\
&+ \frac{1}{2} \left( \frac{1}{D} D - \frac{1}{D^2} D^2 \right) (\hat{c}_t^d)^2 - \frac{1-\delta}{D^2} \hat{d}_{t-1} \hat{x}_t \\
&\approx (1-\delta) \hat{d}_{t-1} + \delta \hat{c}_t^d + \frac{(1-\delta)\delta}{2} \hat{d}_{t-1}^2 + \frac{(1-\delta)\delta}{2} (\hat{c}_t^d)^2 - \frac{(1-\delta)\delta}{2} \hat{c}_t^d \hat{d}_{t-1} \\
&\approx (1-\delta) \hat{d}_{t-1} + \delta \hat{c}_t^d + \frac{(1-\delta)\delta}{2} (\hat{d}_{t-1} - \hat{c}_t^d)^2.
\end{aligned}$$

Thus:

$$\hat{d}_t \approx (1-\delta) \hat{d}_{t-1} + \delta \hat{c}_t^d + \psi_t, \quad (57)$$

where:

$$\begin{aligned}
\hat{\psi}_t &= \frac{(1-\delta)\delta}{2} (\hat{c}_t^d - \hat{d}_{t-1})^2 \\
&= \frac{(1-\delta)}{2\delta} (\hat{d}_t - \hat{d}_{t-1})^2.
\end{aligned}$$

Now, let us iterate backward (57), to obtain:

$$\sum_{t=0}^{\infty} \beta^t \hat{d}_t = \frac{1}{1-\beta(1-\delta)} d_0 + \sum_{t=0}^{\infty} \beta^t \left[ \frac{\delta}{1-\beta(1-\delta)} \hat{c}_t^d + \frac{1}{1-\beta(1-\delta)} \hat{\psi}_t \right].$$

In turn, the term on the RHS will replace the one on the LHS into the intertemporal loss function.

The next step is to derive a second-order approximation for from labour disutility. Recall that:

$$\hat{l}_t = \phi \hat{l}_t^n + (1-\phi) \hat{l}_t^d.$$

Therefore the second order approximation reads:

$$V(L_t) \approx V_L(L) L \left[ \phi \widehat{l}_t^n + (1 - \phi) \widehat{l}_t^d + \frac{\phi(1 + 2\nu\phi)}{2} (\widehat{l}_t^n)^2 + \frac{(1 - \phi)[1 + 2\nu(1 - \phi)]}{2} (\widehat{l}_t^d)^2 \right] + \text{t.i.p.} + O(\|\xi\|^3).$$

After these preliminary steps, we need to find an expression for  $\widehat{l}_t^n$  and  $\widehat{l}_t^d$ . Given the definition of the marginal cost, in equilibrium we get:

$$\begin{aligned} L_t^n &= \frac{(1 - \alpha_n) MC_t^n}{W_t^n} \int_0^1 Y_{jt}^n dj = \frac{(1 - \alpha_n) \bar{\phi}^n}{Z_t^n} \left( \frac{Q_t^{-\gamma_{dn}}}{RW_t^n} \right)^{\alpha_n} Y_t^n \int_0^1 \left( \frac{P_{jt}^n}{P_t^n} \right)^{-\varepsilon_t^n} dj, \\ L_t^d &= \frac{(1 - \alpha_d) MC_t^d}{W_t^d} \int_0^1 Y_{kt}^d dk = \frac{(1 - \alpha_d) \bar{\phi}^d}{Z_t^d} \left( \frac{Q_t^{\gamma_{nd}}}{RW_t^d} \right)^{\alpha_d} Y_t^d \int_0^1 \left( \frac{P_{kt}^d}{P_t^d} \right)^{-\varepsilon_t^d} dk. \end{aligned}$$

Thus, we can report the linear approximation of the expressions above:

$$\begin{aligned} \widehat{l}_t^n &= -\alpha_n \gamma_{dn} \widehat{q}_t - \alpha_n \widehat{r} \widehat{w}_t^n - z_t^n + \widehat{y}_t^n + S_{nt}, \\ \widehat{l}_t^d &= \alpha_d \gamma_{nd} \widehat{q}_t - \alpha_d \widehat{r} \widehat{w}_t^d - z_t^d + \widehat{y}_t^d + S_{dt}, \end{aligned}$$

where:

$$S_{nt} = \log \left[ \int_0^1 \left( \frac{P_{jt}^n}{P_t^n} \right)^{-\varepsilon_t^n} dj \right] \quad S_{dt} = \log \left[ \int_0^1 \left( \frac{P_{kt}^d}{P_t^d} \right)^{-\varepsilon_t^d} dk \right] \quad (58)$$

If we set  $\widehat{p}_{jt}^n$  to be the log-deviation of  $\frac{P_{jt}^n}{P_t^n}$  from its steady state, which means that a second-order Taylor

expansion of  $\int_0^1 \left( \frac{P_{jt}^n}{P_t^n} \right)^{-\varepsilon_t^n} dj$  reads as:

$$\begin{aligned} \int_0^1 \left( \frac{P_{jt}^n}{P_t^n} \right)^{-\varepsilon_t^n} dj &\approx \int_0^1 \left[ 1 - \varepsilon^n \widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2} (\varepsilon^n)^2 (\widehat{p}_{jt}^n)^2 \right] dj + O(\|\xi\|^3) \\ &= 1 - \varepsilon^n \mathbf{E}_i \widehat{p}_{jt}^n - \varepsilon^n \mathbf{E}_i \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2} (\varepsilon^n)^2 \mathbf{E}_i (\widehat{p}_{jt}^n)^2 + O(\|\xi\|^3), \end{aligned}$$

where  $\mathbf{E}_i \widehat{p}_{jt}^n \equiv \int_0^1 \widehat{p}_{jt}^n dj$  and  $\mathbf{E}_i (\widehat{p}_{jt}^n)^2 \equiv \int_0^1 (\widehat{p}_{jt}^n)^2 dj$ . At this stage, we need an expression for  $\mathbf{E}_i \widehat{p}_{jt}^n$ . Let us start from

$$P_t^n = \left[ \int_0^1 (P_{jt}^n)^{1-\varepsilon_t^n} dj \right]^{\frac{1}{1-\varepsilon_t^n}},$$

which can be re-arranged as:

$$1 \equiv \int_0^1 \left( \frac{P_{jt}^n}{P_t^n} \right)^{1-\varepsilon_t^n} dj.$$

Following the procedure above, it can be shown that:

$$\left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} \approx 1 + (1 - \varepsilon^n)\widehat{p}_{jt}^n - \varepsilon^n\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n + \frac{1}{2}(1 - \varepsilon^n)^2(\widehat{p}_{jt}^n)^2 + O(\|\xi\|^3).$$

Substituting this into the preceding equations yields:

$$0 = \int_0^1 \left[ (1 - \varepsilon^n)\widehat{p}_{jt}^n - \varepsilon^n\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n + \frac{1}{2}(1 - \varepsilon^n)^2(\widehat{p}_{jt}^n)^2 \right] dj + O(\|\xi\|^3),$$

which reduces to:

$$\mathbf{E}_i \widehat{p}_{jt}^n = \frac{\varepsilon^n - 1}{2} \mathbf{E}_i (\widehat{p}_{jt}^n)^2 + O(\|\xi\|^3).$$

Thus:

$$\int_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj = 1 + \frac{\varepsilon^n}{2} \mathbf{E}_i (\widehat{p}_{jt}^n)^2 + O(\|\xi\|^3).$$

Now, notice that:

$$\mathbf{E}_i (\widehat{p}_{jt}^n)^2 = \mathbf{E}_i \left[ (p_{jt}^n)^2 - 2p_{jt}^n p_t^n + (p_t^n)^2 \right] + O(\|\xi\|^3),$$

where lower case letters denote the log-value of the capital letters. Here we can use a first-order approximation of  $p_t^n = \int_0^1 p_{jt}^n dj$ , as this term is multiplied by other first-order terms each time it appears.

With this, we have a second order approximation:

$$\mathbf{E}_i (\widehat{p}_{jt}^n)^2 \equiv \text{var}_j p_{jt}^n.$$

Combining all of this the second-order approximation can be represented as:

$$S_{nt} = \frac{\varepsilon^n}{2} \text{var}_j p_{jt}^n + O(\|\xi\|^3).$$

Analogous steps in the sector producing durable goods lead us to:

$$S_{dt} = \frac{\varepsilon^d}{2} \text{var}_k p_{kt}^d + O(\|\xi\|^3).$$

Following Woodford (2003, Ch. 6, Proposition 6.3), we can obtain a correspondence between cross-sectional price dispersions in the two sectors and their inflation rates:

$$\begin{aligned} \text{var}_j p_{jt}^n &= \theta^n \text{var}_j p_{jt-1}^n + \frac{\theta^n}{1 - \theta^n} (\pi_t^n)^2 + O(\|\xi\|^3), \\ \text{var}_k p_{kt}^d &= \theta^d \text{var}_k p_{kt-1}^d + \frac{\theta^d}{1 - \theta^d} (\pi_t^d)^2 + O(\|\xi\|^3). \end{aligned}$$

Iterating these expressions forward leads to:

$$\sum_{t=0}^{\infty} \beta^t \text{var}_j p_{jt}^n = (\kappa^n)^{-1} \sum_{t=0}^{\infty} \beta^t (\pi_t^n)^2 + \text{t.i.p.} + O(\|\xi\|^3), \quad (59)$$

$$\sum_{t=0}^{\infty} \beta^t \text{var}_k p_{kt}^d = (\kappa^d)^{-1} \sum_{t=0}^{\infty} \beta^t (\pi_t^d)^2 + \text{t.i.p.} + O(\|\xi\|^3), \quad (60)$$

where

$$\begin{aligned} \kappa^n &= \frac{(1 - \beta\theta^n)(1 - \theta^n)}{\theta^n}, \\ \kappa^d &= \frac{(1 - \beta\theta^d)(1 - \theta^d)}{\theta^d}. \end{aligned}$$

After these preliminary steps, we can write  $\mathcal{W}_t$  as:

$$\begin{aligned} \mathcal{W}_t &\approx U_{C^n}(C^n, D) C^n \left\{ \widehat{c}_t^n + \frac{1}{2} [\mu^n (1 - \sigma)] (\widehat{c}_t^n)^2 + (\mu^d / \mu^n) \widehat{d}_t + \right. \\ &\quad \left. + \frac{1}{2} [\mu^d (1 - \sigma)] (\mu^d / \mu^n) \widehat{d}_t^2 + \mu^d (1 - \sigma) \widehat{c}_t^n \widehat{d}_t + \frac{1}{2} \Xi (\mu^d / \mu^n) (\widehat{d}_t - \widehat{d}_{t-1})^2 \right\} + \\ &\quad - V_L(L) L \left\{ \phi \widehat{l}_t^n + (1 - \phi) \widehat{l}_t^d + \right. \\ &\quad \left. + \left( \frac{1 + \nu}{2} \right) \left[ \phi^2 (\widehat{l}_t^n)^2 + (1 - \phi) (\widehat{l}_t^d)^2 + 2\phi(1 - \phi) \widehat{l}_t^n \widehat{l}_t^d \right] \right\} + \\ &\quad + \text{t.i.p.} + O(\|\xi\|^3). \end{aligned}$$

We now consider the linear terms in  $\mathcal{W}_t$ , which are collected under  $\mathcal{LW}_t$ :

$$\begin{aligned} \mathcal{LW}_t &= \frac{U_{C^n}(C^n, D) C^n}{\mu^n} \left\{ \mu^n \widehat{c}_t^n + \mu^d \widehat{d}_t \right\} + \\ &\quad - \left\{ V_L(L) L \phi (-\alpha_n \gamma_{dn} \widehat{q}_t - \alpha_n \widehat{r} \widehat{w}_t^n + \widehat{y}_t^n) + \right. \\ &\quad \left. + (1 - \phi) (\alpha_d \gamma_{nd} \widehat{q}_t - \alpha_d \widehat{r} \widehat{w}_t^d + \widehat{y}_t^d) \right\} + \\ &\quad + \text{t.i.p.} + O(\|\xi\|^2). \end{aligned}$$

We substitute for the real wage from marginal cost expressions to get:

$$\begin{aligned} \mathcal{LW}_t &= \frac{U_{C^n}(C^n, D) C^n}{\mu^n} \left\{ \mu^n \widehat{c}_t^n + \mu^d \widehat{d}_t \right\} + \\ &\quad - V_L(L) L \phi \left( \frac{1}{1 - \alpha_n} \widehat{y}_t^n - \frac{\alpha_n \gamma_{nn}}{1 - \alpha_n} \widehat{m}_t^{nn} - \frac{\alpha_n \gamma_{dn}}{1 - \alpha_n} \widehat{m}_t^{dn} \right) + \\ &\quad - V_L(L) L (1 - \phi) \left( \frac{1}{1 - \alpha_d} \widehat{y}_t^d - \frac{\alpha_d \gamma_{nd}}{1 - \alpha_d} \widehat{m}_t^{nd} - \frac{\alpha_d \gamma_{dd}}{1 - \alpha_d} \widehat{m}_t^{dd} \right) + \\ &\quad + \text{t.i.p.} + O(\|\xi\|^2). \end{aligned}$$

After substituting the second order approximation for the accumulation equation of durables we get:

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \mathcal{LW}_t &= U_{C^n}(C^n, D) C^n \sum_{t=0}^{\infty} \beta^t \left\{ \widehat{c}_t + \frac{\delta}{1 - \beta(1 - \delta)} \frac{\mu^d}{\mu^n} \widehat{c}_t^d \right\} + \\
&- V_L(L) L \sum_{t=0}^{\infty} \beta^t \left\{ \phi \left( \frac{1}{1 - \alpha_n} \widehat{y}_t^n - \frac{\alpha_n \gamma_{nn}}{1 - \alpha_n} \widehat{m}_t^{nn} - \frac{\alpha_n \gamma_{dn}}{1 - \alpha_n} \widehat{m}_t^{dn} \right) + \right. \\
&+ (1 - \phi) \left( \frac{1}{1 - \alpha_d} \widehat{y}_t^d - \frac{\alpha_d \gamma_{nd}}{1 - \alpha_d} \widehat{m}_t^{nd} - \frac{\alpha_d \gamma_{dd}}{1 - \alpha_d} \widehat{m}_t^{dd} \right) \left. \right\} + \\
&+ \text{t.i.p.} + O(\|\xi\|^2).
\end{aligned} \tag{61}$$

Notice that the following linear approximations for the market clearing conditions hold:

$$\begin{aligned}
\widehat{y}_t^n &= \frac{1 - \alpha_n}{\phi} \mu^n \widehat{c}_t^n + \alpha_n \gamma_{nn} \widehat{m}_t^{nn} + \frac{(1 - \alpha_n)(1 - \phi)}{\phi(1 - \alpha_d)} \alpha_d \gamma_{nd} \widehat{m}_t^{nd}, \\
\widehat{y}_t^d &= \frac{\delta \mu^d (1 - \alpha_d)}{(1 - \phi)[1 - \beta(1 - \delta)]} \widehat{c}_t^d + \frac{(1 - \alpha_d)\phi}{(1 - \phi)(1 - \alpha_n)} \alpha_n \gamma_{dn} \widehat{m}_t^{dn} + \alpha_d \gamma_{dd} \widehat{m}_t^{dd}.
\end{aligned}$$

It can be shown that, in the steady state, the following relationships hold:

$$V_{L^n}(L^n) L^n = \phi V_L(L) L \qquad V_{L^d}(L^d) L^d = (1 - \phi) V_L(L) L$$

Moreover, the presence of production subsidies allows us to express the steady state marginal rate of substitution between labour supply and consumption of non-durable goods as:

$$\begin{aligned}
\frac{-V_{L^n}(L^n)}{U_{C^n}(C^n)} &= \frac{Y^n(1 - \alpha_n)}{L^n}, \\
\frac{-V_{L^d}(L^d)}{U_{C^n}(C^n)} &= \frac{Y^d(1 - \alpha_d)}{L^d Q}.
\end{aligned}$$

It is now convenient to express the marginal utility from non-durable consumption in terms of the marginal utility derived from total consumption:

$$U_{C^n}(C^n) = U_H(H) H \mu^n.$$

Therefore, we can re-write (61) as:

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \mathcal{LW}_t &= U_H(H) H \sum_{t=0}^{\infty} \beta^t \left\{ \left( \mu^n \widehat{c}_t^n + \frac{\delta \mu^d}{1 - \beta(1 - \delta)} \widehat{c}_t^d \right) + \right. \\
&- \mu^n \left( \frac{C^n}{Y^n} \right)^{-1} (1 - \alpha_n) [-\alpha_n \gamma_{dn} \widehat{q}_t - \alpha_n \widehat{r} w_t^n - z_t^n + \widehat{y}_t^n] + \\
&- \mu^n \left( \frac{C^m}{Y^d} \right)^{-1} (1 - \alpha_d) Q^{-1} [\alpha_d \gamma_{nd} \widehat{q}_t - \alpha_d \widehat{r} w_t^d - z_t^d + \widehat{y}_t^d] \left. \right\} + \\
&+ \text{t.i.p.} + O(\|\xi\|^2).
\end{aligned}$$

It is now possible to show, given the linearized market clearing conditions in the two sectors, that  $\sum_{t=0}^{\infty} \beta^t \mathcal{LW}_t = 0$ . Therefore, the linear term in  $\mathcal{W}_t$  can be dropped. Thus we are left only with second

order terms:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \mathcal{W}_t &\approx U_H(H) H \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\sigma}{2} \left( \mu^n \widehat{c}_t^n + \mu^d \widehat{d}_t \right)^2 + \frac{1}{1-\beta(1-\delta)} \mu^d \widehat{\psi}_t + \frac{\mu^d}{2} \Xi \left( \widehat{d}_t - \widehat{d}_{t-1} \right)^2 + \right. \\ &\quad - \frac{\Theta}{2} \left[ \phi \varepsilon^n (\kappa^n)^{-1} (\pi_t^n)^2 + (1-\phi) \varepsilon^d (\kappa^d)^{-1} (\pi_t^d)^2 \right] + \\ &\quad \left. - \left( \frac{1+\nu}{2} \right) \Theta^{-1} \left[ \mu^n \widehat{c}_t^n + \frac{\delta \mu^d}{1-\beta(1-\delta)} \widehat{c}_t^d \right]^2 \right\} + \\ &\quad + \text{t.i.p.} + O(\|\xi\|^3), \end{aligned}$$

where

$$\Theta = \left( \frac{C^n}{Y^n} \right)^{-1} \frac{(1-\alpha_n) \mu^n}{\phi} = \frac{\mu^n [1-\beta(1-\delta)] + \mu^d \delta}{1-\beta(1-\delta)}.$$

We next consider the deviation of social welfare from its Pareto-optimal level:

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t \widetilde{\mathcal{W}}_t &= \sum_{t=0}^{\infty} \beta^t (\mathcal{W}_t - \mathcal{W}_t^*) \approx \\ &\quad - \frac{U_H(H) H}{2} \Theta \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma-1}{\Theta} \left( \mu^n \widetilde{c}_t^n + \mu^d \widetilde{d}_t \right)^2 + \right. \\ &\quad + [\mu^d \Theta^{-1} \Xi + (1-\delta)(1-\omega) \delta^{-2}] \left( \widetilde{d}_t - \widetilde{d}_{t-1} \right)^2 + \\ &\quad \left. + \varsigma \left[ \varpi (\pi_t^n)^2 + (1-\varpi) (\pi_t^d)^2 \right] + (1+\nu) [\omega \widetilde{c}_t^n + (1-\omega) \widetilde{c}_t^d]^2 \right\} + \text{t.i.p.} + O(\|\xi\|^3), \end{aligned}$$

where the following notation has been introduced:

$$\begin{aligned} \omega &= \frac{\mu^n [1-\beta(1-\delta)]}{\mu^n [1-\beta(1-\delta)] + \mu^d \delta}, \\ \varpi &= \frac{\phi \varepsilon^n (\kappa^n)^{-1}}{\varsigma}, \\ \varsigma &= \phi \frac{\varepsilon^n}{\kappa^n} + (1-\phi) \frac{\varepsilon^d}{\kappa^d}. \end{aligned}$$

## APPENDIX E2: General Model Setup for Optimal Monetary Policy Analysis

The welfare function can be written in matrix form as:

$$S\mathcal{W}_0 = -\frac{U_H(H) H}{2} E_0 \sum_{t=0}^{\infty} \beta^t \{ \mathbf{X}'_t \mathbf{W} \mathbf{X}_t + \widehat{r}_t R \widehat{r}_t \}, \quad (62)$$

where the state vector is augmented to include the past stock of durables:

$$\begin{aligned} \mathbf{X}_t &= \left[ \boldsymbol{\xi}'_t \quad \widetilde{d}_{t-1} \right]', \\ \widetilde{\boldsymbol{\xi}}_t &= \left[ \widetilde{c}_t^n \quad \widetilde{y}_t^n \quad \widetilde{c}_t^d \quad \widetilde{y}_t^d \quad \widetilde{d}_t \quad \widetilde{r}w_t^n \quad \widetilde{r}w_t^d \quad \pi_t^n \quad \pi_t^d \quad \widetilde{q}_t \right]'. \end{aligned}$$

Notice that  $R = 0$  in (62). Moreover, we define

$$\mathbf{W} = \mathbf{K}'\overline{\mathbf{Q}}\mathbf{K}$$

where  $\overline{\mathbf{Q}}$  is a diagonal matrix and  $\mathbf{K}$  reports the weighting coefficients included in the loss function:

$$\mathbf{K} = \begin{bmatrix} \mu^n & \mathbf{0}_{1 \times 3} & \mu^d & \mathbf{0}_{1 \times 6} \\ \mathbf{0}_{1 \times 2} & -1 & \mathbf{0}_{1 \times 7} & 1 \\ \mathbf{0}_{1 \times 7} & 1 & \mathbf{0}_{1 \times 2} & 0 \\ \mathbf{0}_{1 \times 8} & 1 & 0 & 0 \\ \mu^n & \mathbf{0}_{1 \times 3} & \frac{\delta}{1-\beta(1-\delta)}\mu^d & \mathbf{0}_{1 \times 6} \end{bmatrix},$$

$$\overline{\mathbf{Q}} = \text{diag} \left[ (\sigma - 1), \left[ \mu^d \Xi + \Theta (1 - \delta) (1 - \omega) \delta^{-2} \right], \Theta \varepsilon^n \phi (\kappa^n)^{-1}, \Theta (1 - \phi) \varepsilon^d (\kappa^d)^{-1}, \Theta (1 + \nu) \right]. \quad \blacksquare$$

To tackle the optimization problem we follow Dennis (2007) and consider the system under its structural (singular) form:

$$\begin{aligned} S\mathcal{W}_0 &= E_0 \sum_{t=0}^{\infty} \beta^t \{ \mathbf{X}'_t \mathbf{W} \mathbf{X}_t + \\ &\quad + 2\boldsymbol{\lambda}'_t [ \mathbf{G}_0 \mathbf{X}_{t+1} + \mathbf{G}_1 \mathbf{X}_t + \mathbf{G}_2 \mathbf{X}_{t-1} + \mathbf{G}_3 r_t + \mathbf{G}_4 \boldsymbol{\kappa}_t^* + \mathbf{G}_5 \boldsymbol{\kappa}_{t-1}^* + \mathbf{N}_1 \mathbf{v}_t ] \}. \end{aligned}$$

For  $t > t_0$  the following conditions hold:

$$\begin{aligned} \frac{\partial S\mathcal{W}_0}{\partial \boldsymbol{\lambda}_t} &= E_t [ \mathbf{G}_0 \mathbf{X}_{t+1} + \mathbf{G}_1 \mathbf{X}_t + \mathbf{G}_2 \mathbf{X}_{t-1} + \mathbf{G}_3 r_t + \mathbf{G}_4 \boldsymbol{\kappa}_t^* + \mathbf{G}_5 \boldsymbol{\kappa}_{t-1}^* + \mathbf{N}_1 \mathbf{v}_t ] = \mathbf{0}, \\ \frac{\partial S\mathcal{W}_0}{\partial \mathbf{X}_t} &= \mathbf{W} \mathbf{X}_t + \mathbf{G}'_1 \boldsymbol{\lambda}_t + \beta^{-1} \mathbf{G}'_0 \boldsymbol{\lambda}_{t-1} + \beta \mathbf{G}'_2 E_t (\boldsymbol{\lambda}_{t+1}) = \mathbf{0}, \\ \frac{\partial S\mathcal{W}_0}{\partial r_t} &= \mathbf{G}'_3 \boldsymbol{\lambda}_t = 0. \end{aligned}$$

Notice that the last condition suggests that the multiplier associated with the IS curve,  $\lambda_t^{IS}$ , is equal zero. The matrices of the system now read as:

$$\begin{aligned} \mathbf{G}_0 &= \begin{bmatrix} \mathbf{A}_0 & \mathbf{0}_{10 \times 1} \\ \mathbf{0}_{1 \times 10} & 0 \end{bmatrix}, & \mathbf{G}_1 &= \begin{bmatrix} \mathbf{A}_1 & \mathbf{0}_{10 \times 1} \\ \mathbf{0}_{1 \times 10} & 1 \end{bmatrix}, \\ \mathbf{G}_2 &= \begin{bmatrix} \mathbf{A}_2 & \mathbf{0}_{10 \times 1} \\ \mathbf{b} & 0 \end{bmatrix}, & \mathbf{b} &= [ \mathbf{0}_{1 \times 4} \quad -1 \quad \mathbf{0}_{1 \times 5} ], \\ \mathbf{G}_3 &= \begin{bmatrix} \mathbf{A}_3 \\ 0 \end{bmatrix}, & \mathbf{G}_4 &= \begin{bmatrix} \mathbf{A}_4 \\ \mathbf{0}_{1 \times 5} \end{bmatrix}, \\ \mathbf{G}_5 &= \begin{bmatrix} \mathbf{A}_5 \\ \mathbf{0}_{1 \times 5} \end{bmatrix}, & \mathbf{N}_1 &= \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{0}_{1 \times 5} \end{bmatrix}. \end{aligned}$$

Following Sims, under the commitment optimal policy the economy evolves according to:

$$\Gamma_0 \mathbf{y}_t = \Gamma_1 \mathbf{y}_{t-1} + \mathbf{c} + \Psi \mathbf{z}_t + \Pi \boldsymbol{\eta}_t \quad t = 1, \dots, T \quad (63)$$

where  $\mathbf{c}$  is a vector of constants,  $\mathbf{z}_t$  is an exogenously evolving, possibly serially correlated, random disturbance, and  $\boldsymbol{\eta}_t$  is an expectational error, satisfying  $E_t(\boldsymbol{\eta}_{t+1}) = 0$ , all  $t$ . The  $\boldsymbol{\eta}_t$  terms are not given exogenously, but instead are treated as determined as part of the model solution. Models with

more lags, or with lagged expectations, or with expectations of more distant future values, can be accommodated in this framework by expanding the  $\mathbf{y}$  vector. Specifically, the whole model can be cast in the form above by writing:

$$\mathbf{y}_t = \begin{bmatrix} \mathbf{v}_t \\ \mathbf{X}_t \\ r_t \\ E_t \mathbf{X}_{t+1} \\ \kappa_t^* \\ E_t \kappa_{t+1}^* \\ \lambda_t \\ E_t \lambda_{t+1} \end{bmatrix}, \quad \mathbf{\Pi} = \begin{bmatrix} \mathbf{0}_{15 \times 27} \\ \mathbf{I}_{11} & \mathbf{0}_{11 \times 16} \\ \mathbf{0}_{5 \times 27} \\ \mathbf{0}_{5 \times 11} & \mathbf{I}_5 & \mathbf{0}_{5 \times 11} \\ \mathbf{0}_{11 \times 27} \\ \mathbf{0}_{11 \times 16} & \mathbf{I}_{11} \\ \mathbf{0}_{1 \times 27} \end{bmatrix}, \quad \mathbf{\Psi} = \begin{bmatrix} \mathbf{\Sigma} \\ \mathbf{0}_{55 \times 4} \end{bmatrix},$$

$$\mathbf{\Gamma}_0 = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 1} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 5} & \mathbf{0}_{4 \times 5} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 11} \\ \mathbf{N}_1 & \mathbf{G}_1 & \mathbf{G}_3 & \mathbf{G}_0 & \mathbf{G}_4 & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 11} \\ \mathbf{0}_{11 \times 4} & \mathbf{I}_{11} & \mathbf{0}_{11 \times 1} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 11} \\ \overline{\mathbf{C}}_1 & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 1} & \mathbf{0}_{5 \times 11} & \mathbf{F}_1 & \mathbf{F}_0 & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} \\ \mathbf{0}_{5 \times 4} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 1} & \mathbf{0}_{5 \times 11} & \mathbf{I}_5 & \mathbf{0}_{5 \times 5} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} \\ \mathbf{0}_{11 \times 4} & \mathbf{W} & \mathbf{0}_{11 \times 1} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} & \mathbf{G}'_1 & \beta \mathbf{G}'_2 \\ \mathbf{0}_{11 \times 4} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 1} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} & \mathbf{I}_{11} & \mathbf{0}_{11 \times 11} \\ \mathbf{0}_{1 \times 4} & \mathbf{0}_{1 \times 11} & \mathbf{0}_{1 \times 1} & \mathbf{0}_{1 \times 11} & \mathbf{0}_{1 \times 5} & \mathbf{0}_{1 \times 5} & \mathbf{G}'_3 & \mathbf{0}_{1 \times 11} \end{bmatrix},$$

$$\mathbf{\Gamma}_1 = \begin{bmatrix} \mathbf{P} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 1} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 5} & \mathbf{0}_{4 \times 5} & \mathbf{0}_{4 \times 11} & \mathbf{0}_{4 \times 11} \\ \mathbf{0}_{11 \times 4} & -\mathbf{G}_2 & \mathbf{0}_{11 \times 1} & \mathbf{0}_{11 \times 11} & -\mathbf{G}_5 & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 11} \\ \mathbf{0}_{11 \times 4} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 1} & \mathbf{I}_{11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 11} \\ \mathbf{0}_{5 \times 4} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 1} & \mathbf{0}_{5 \times 11} & -\mathbf{F}_2 & \mathbf{0}_{5 \times 5} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} \\ \mathbf{0}_{5 \times 4} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 1} & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 5} & \mathbf{I}_5 & \mathbf{0}_{5 \times 11} & \mathbf{0}_{5 \times 11} \\ \mathbf{0}_{11 \times 4} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 1} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} & -\beta^{-1} \mathbf{G}'_0 & \mathbf{0}_{11 \times 11} \\ \mathbf{0}_{11 \times 4} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 1} & \mathbf{0}_{11 \times 11} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 5} & \mathbf{0}_{11 \times 11} & \mathbf{I}_{11} \\ \mathbf{0}_{1 \times 4} & \mathbf{0}_{1 \times 11} & \mathbf{0}_{1 \times 1} & \mathbf{0}_{1 \times 11} & \mathbf{0}_{1 \times 5} & \mathbf{0}_{1 \times 5} & \mathbf{0}_{1 \times 11} & \mathbf{0}_{1 \times 11} \end{bmatrix},$$

where  $\mathbf{\Sigma} = \text{diag} [ \sigma^{z^n} \quad \sigma^{z^d} \quad \sigma^{\eta^n} \quad \sigma^{\eta^d} ]$ .

## APPENDIX F: Alternative Policy Regimes

The following alternative period loss functions are considered in Section 4.3:

$$\begin{aligned}
 \text{Strict inflation targeting} & : \widetilde{\mathcal{W}}_t^{IT} = (\pi_t^{IT})^2, \\
 \text{Gap targeting} & : \widetilde{\mathcal{W}}_t^{GT} = (\widetilde{x}_t^{GT})^2, \\
 \text{Flexible inflation targeting} & : \widetilde{\mathcal{W}}_t^{FIT} = \widetilde{\mathcal{W}}_t^{IT} + (1 + v) \widetilde{\mathcal{W}}_t^{GT}, \\
 \text{Flexible inflation targeting + durables smoothing} & : \widetilde{\mathcal{W}}_t^{ITDS} = \widetilde{\mathcal{W}}_t^{IT} + S \left( \Delta \widetilde{d}_t \right)^2,
 \end{aligned}$$

where

$$\begin{aligned}
 \pi_t^{IT} & = \{ \pi_t^{core}, \pi_t^{agg} \} \\
 \pi_t^{core} & = \varpi \pi_t^n + (1 - \varpi) \pi_t^d, \\
 \pi_t^{agg} & = \phi \pi_t^n + (1 - \phi) \pi_t^d,
 \end{aligned}$$

and

$$\begin{aligned}
 \widetilde{x}_t^{GT} & = \{ \widetilde{x}_t^c, \widetilde{x}_t^p \} \\
 \widetilde{x}_t^c & = \omega \widetilde{c}_t^n + (1 - \omega) \widetilde{c}_t^d, \\
 \widetilde{x}_t^p & = \phi \widetilde{y}_t^n + (1 - \phi) \widetilde{y}_t^d.
 \end{aligned}$$