Exercises to Chapter 13

(1) The economy can be identified with a two-goods, two consumer exchange economy as treated in Chapter 3. In a Pareto optimal allocation, the marginal rates of substitution must be equal, so that if the allocation is $(x_1(s_1), x_1(s_2), x_2(s_1), x_2(s_2))$, then

$$\frac{\pi_2 u_1'(x_1(s_2))}{\pi_1 u_1'(x_1(s_1))} = \frac{\pi_2 u_2'(x_2(s_2))}{\pi_1 u_2'(x_3(s_1))}$$

which is satisfied whenever

$$\frac{u_1'(x_1(s_2))}{u_1'(x_1(s_1))} = \frac{u_2'(x_2(s_2))}{u_2'(x_3(s_1))}. \tag{1}$$

Together with the equations

$$x_1(s_1) + x_2(s_1) = 2,\ x_1(s_2) + x_2(s_2) = 1$$

the expression (1) characterizes all the Pareto optimal allocations in the economy, in particular they do not depend on the exact values of π_1 and π_2.

(2) We may consider the economy as a usual one with 6 goods (4 of which are contingent commodities). In the Walras equilibrium allocation, marginal utilities of consumer 1 are

$$u_{11}' = \frac{1}{x_{11}}, u_{12}' = 1, u_{13}' = \frac{1}{2x_{13}}, u_{14}' = 1, u_{15}' = \frac{1}{2x_{15}}, u_{16}' = 1,$$

and if prices are normalized so that $p_1 = 1$ we get that the equilibrium price has the form

$$\left(1, p, \frac{1}{2}, \frac{p}{2}, \frac{1}{2}, \frac{p}{2}\right) \tag{2}$$

and we get that $x_{11} = x_{13} = x_{15} = p$, $x_{12} = x_{14} = x_{16}$. From the budget equation of consumer 1 we obtain that

$$p\left(x_{12} + \frac{1}{2}x_{14} + \frac{1}{2}x_{16}\right) + 2p = 4 + 6p$$

or

$$x_{12} + \frac{1}{2}x_{14} + \frac{1}{2}x_{16} = \frac{4}{p} + 4.$$

It is easily checked that consumer 2 cannot have an interior maximum, using only commodities $2, 4, 6$, so that

$$\frac{4}{p} + 4 = 9.5$$

from which we get that $= \frac{4}{5.5} = 0.727$. We obtain a Walras equilibrium with price vector (2) and consumer 1 consuming the amounts $x_{21} = 3 - p = 2,273, x_{23} = 3 - p = 2.273, x_{25} = 6 - p = 5.273, x_{22} = x_{24} = x_{26} = 0$, whereas $x_{12} = 4, x_{14} = 4, x_{16} = 7$.

(3) Health insurance gives the insured person a right to be treated, or a sum of money which is equivalent to the cost of treatment, for any given disease. In principle, this can be seen as the purchase of a contingent commodity, namely treatment delivered in the case of disease. However, it may not always be simple to establish that the event which triggers delivery has actually occurred, and on the other hand it may not be obvious when the service delivered can be considered as a treatment, in the sense that the purpose of treatment is fulfilled.

These two specific circumstances have as a consequence that healthcare only in very special cases can be organized as a market-based system of contingent trades. Indeed, the problems of moral hazard and adverse selections which are basic in healthcase are cases of market failure, so that competitive markets cannot allocate efficiently.

(4) We consider the economy as one with two consumers and two commodities (no uncertainty). The endowments are $(2, 1)$ and $(2, 2)$. Normalizing the price as $(1, p)$, we have that in the equilibrium

$$p = \frac{x_{11}}{x_{12}} = \frac{x_{21}}{x_{22}},$$

and from the budget equation of the two consumers,

$$x_{11} + px_{12} = 2 + p,\ x_{21} + px_{22} = 2 + 2p$$

we get that $x_{12} = \frac{2+p}{2p}$, $x_{22} = \frac{2+2p}{2p}$. Now, market balance for commodity 2 demands that

$$\frac{2+p}{2p} + \frac{2+2p}{2p} = 3,$$

which gives that $p = \frac{4}{3}$. Inserting we get that

$$x_{11} = \frac{10}{6},\ x_{12} = \frac{10}{8},\ x_{21} = \frac{14}{6},\ x_{22} = \frac{14}{8}.$$

Selling 2 units of security A gives an amount of 6, which may be used to buy 6/5 units of security B. The obligations are then $(2,2)$ (in the two states) from A against a yield of $\left(\frac{12}{5}, \frac{18}{5}\right)$ from security B, giving a net surplus of $\left(\frac{2}{5}, \frac{8}{5}\right)$.

We find first the amounts a and b of securities of type A and B needed to yield the Walras equilibrium net trades of consumer 2:

$$a\begin{pmatrix}1\\1\end{pmatrix} + b\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}\frac{2}{6}\\-\frac{2}{8}\end{pmatrix},$$

with solution $a = \frac{3}{2}$, $b = -\frac{7}{12}$, so that the Walras equilibrium allocation can be obtained if consumer 1 sells 3/2 units of security A and buys 7/12 units of security B. This security trade will be self-financing if security prices are $q_A = \frac{2}{3}$, $q_B = \frac{12}{7}$.

To show that these trades are equilibrium trades, we notice that if a trade (a, b') satisfies the self-financing condition $\frac{2}{3}a' + \frac{12}{7}b' = 0$ or $b' = -\frac{7}{18}a'$, then the value at equilibrium prices of the security yields is $(1,p)\cdot(1,1) = \frac{7}{3}$ and $(1,p)\cdot(2,3) = 6$, we get that the value of the resulting net trade is

$$\frac{7}{3}a' - 6\frac{7}{18}a' = 0,$$

so that security trade will yield only net trades obtainable at the equilibrium prices. Consequently, the given security trades are the best possible.

(5) Since the consumer has utility only of x_1 and x_2, we may assume that $x_0 = 0$. To find a Pareto optimal allocation, we maximize representative utility

$$\pi u(x_i) + (1-\pi)u(x_2)$$

over all (x_1, x_2) satisfying

$$\begin{aligned}\pi x_1 &= 1 - I\\(1-\pi)x_2 &= RI\end{aligned}$$

Inserting x_1 and x_2 in the utility function gives an expression which is maximized w.r.t. I. First order conditions at the optimum (x_1^*, x_2^*) are

$$u'(x_1^*) = Ru'(x_2^*).$$

The optimum may be realized as a contract where consumers deliver their endowment of one unit and receive the amount x_1^* if impatient, x_2^* if patient. The conditions on u show that $Ru'(R) < 1u'(R)$ so that $x_1^* > 1$, $x_2^* < R$, and since $u'(x_2^*) < \frac{u'(x_1^*)}{R} < u'(x_1^*)$, we have that $x_1^* < x_2^*$, so that patient consumers will lose if pretending to be impatient.

Assume that there are two states of nature at $t = 2$, namely s_1, which is the situation as above, investment gives R per unit invested, and s_2, where investment is lost and the yields outcome 0. The beliefs $(p, 1-p)$ over these states are formed only at $t = 1$ and only for patient consumers. For any belief such that

$$px_2^* > x_1^*$$

it will be advantageous for the patient consumer to wait until $t = 2$ for claiming the consumption, and if $px_2^* < x_1^*$ the consumer will run the bank (which consequently will fail, thereby supporting the beliefs).