Solutions to Exercises in

Game Theory
Chapter 3

1. We find the intersection points of the lines given by the inequalities (with = instead of ≤ or ≥ together with the lines $x = 0$ and $y = 0$ given by the nonnegativity constraints, and discard those of the points which violate some of the inequalities, leaving us with the points

$$(0,0), (0,11), (5,16), (15,12), (27,0). \tag{1}$$

The set if feasible solutions is convex as the intersection of all the halfspaces defined by the inequalities and the nonnegativity constraints, and it contains the above points. If the convex hull of the points was strictly contained in the feasible set, then some there would be some point (x_0, y_0) in the interior of the feasible set but not in the convex hull og the points, meaning that there would be a further inequality with coefficients p, q such that

$$px + qx < px_0 + qx_0$$

for all (x, y) in the convex hull of the above points. Since new inequality differs from those defining the feasible set, there must be at least one of the intersection points which does not satisfy the inequality, a contradiction, showing that the feasible points is the convex hull of the points in (1).

Writing the first inequality as $-x + y \leq 11$ we can immediately write down the dual as

$$\begin{aligned}
\min 11z_1 + 27z_2 + 90z_3 \ s.t.& \\
-z_1 + z_2 + 2z_3 &\geq 2 \\
z_1 + z_2 + 5z_3 &\geq 3 \\
z_1, z_2, z_3 &\geq 0.
\end{aligned}$$

2. The LP problem in canonical form

$$\begin{aligned}
\max c_1x_1 + \cdots + c_nx_n& \\
a_{11}x_1 + \cdots + a_{1n}x_n &\leq b_1 \\
&\vdots \\
a_{m1}x_1 + \cdots + a_{mn}x_n &\leq b_m \\
x_1, \ldots, x_n &\geq 0
\end{aligned}$$

can be transformed to a problem in standard form by introduction of a slack variable z_j, $j = 1, \ldots, m$, in each of the inequalities, so as to become

$$\begin{aligned} &\max c_1 x_1 + \cdots + c_n x_n \\ &\quad a_{11}x_1 + \cdots + a_{1n}x_n + z_1 = b_1 \\ &\qquad \vdots \\ &\quad a_{m1}x_1 + \cdots + a_{mn}x_n + z_m \leq b_m \\ &\quad x_1, \ldots, x_n, z_1, \ldots, z_m \geq 0. \end{aligned}$$

If the primal problem is in the standard form (10), then it can be transformed to a problem in canonical form by replacing each equality $a_{i1}x_1 + \cdots + a_{in}x_n = b_n$ by the two inequalities

$$\begin{aligned} a_{i1}x_1 + \cdots + a_{in}x_n &\leq b_n \\ -a_{i1}x_1 - \cdots - a_{in}x_n &\leq -b_n, \end{aligned}$$

$i = 1, \ldots, m$, to get the LP problem

$$\begin{aligned} \max c \cdot x& \\ Ax &\leq b \\ -Ax &\leq -b \\ x &\geq 0 \end{aligned}$$

and its dual problem has the form

$$\begin{aligned} \min\ b \cdot y - b \cdot w& \\ yA &\geq c \\ -wA &\geq c \\ y, w &\geq 0. \end{aligned}$$

This is seen not to be an LP problem in standard form. Introducing new variables $u = y - w$, one gets an LP in canonical form but *without* nonnegativity constraints.

3. Assuming independence of drug use and test accuracy, we get the system of simultaneous probabilities

	User	Non-user
Positive	0.093	0.036
Negative	0.007	0.864

The probability of a positive test is therefore $0.093 + 0.036 = 0.139$. Given a positive test result, the probability that the tested individual is a drug user is $0.093/0.139 = 0.482$, and the probability of being a user given negative test result is $0.007/(0.007 + 0.864) = 0.008$.

The assessment of the testing procedure (considered as an information method) depends on the utility function. However, for the standard assessment of the value of information methods it is assumed that the results are obtained before a decision (about participation of the athlete) whereas the test results as considered here are usually obtained only afterwards.

4. Consider the vector game with the two payoff matrices

$$\begin{pmatrix} -3 & -3 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ -3 & -3 \end{pmatrix}$$

(the outcome does not depend on player 2). The two halfspaces $\{(x_1, x_2) \mid x_1 \geq 0\}$ and $\{(x_1, x_2) \mid x_2 \geq 0\}$ are approachable, since for any mixed strategy q of player 2, there are (pure) strategies of player 1 giving the vector payoff $(2, -3)$ and $(-3, 2)$, but the intersection $\mathbb{R}^2_+$ is not approachable, since there is no mixed strategy of player 1 giving a nonnegative payoff.