Lecture Note 1:

Accounting

This note is a digression, since accounting and bookkeeping is a little out of our scope, not to speak of their mathematical generalizations. But there are several reasons for looking outside the frames given by standard texts. First of all, we should allow ourselves to be distracted by funny and thought-provoking aspects of our topic, even if they lead nowhere. And secondly, old ideas often come back in new clothes, in the present case the notion of a ledger, central in bookkeeping, has gained actuality in the form of the *blockchain.*

As with economics, and perhaps even more so, the history of accounting has its roots in very ancient times, tokens of bookkeeping were used in Mesopotamia, and also in China and India accounting became increasingly necessary for purposes of trade and taxation.

1. Double entry bookkeeping

The techniques of double entry bookkeeping is an old technique, usually connected with the name of Luca Pacioli (1445–1517), although it was known already at his time[1], but Pacioli organized and presented the matter in his book *Summa de arithmetica, geometria, proportioni et proportionalita* from 1494, which was subsequently used for centuries to come (a facsimile can be found in Google Books).

The basic idea of double entry bookkeeping is that one sets up a system of accounts, and the events of business are recorded in two accounts, once to the right and once to the left.

A simple example goes as follows: We start a business with DKK 1000 paid in cash (the example is somewhat oldfashioned). There are for the moment two accounts each having a left (debit) and a right (credit) side, one for cash and another for capital,

1000	0	0	1000

We now buy a certain purchase goods, to the amount of DKK 800, which we put into our inventory. Opening up a new account for purchase of goods (put as the third one in the row), the situation is

[1] A work by Benedetto Cotrugli from 1458 laying out the principles has recently been translated and published, see Sangster and Rossi (2018)

1000	800		0	1000		800	0

Next we sell out this inventory, which gives us DKK 700 (we are university people, not very successful as businessmen), and the accounts now show the following, where we have a further account for sale of goods,

1000	800		0	1000		800	0			700
700										

We now decide to close down the business. The cash account shows our net position DKK 900, and the capital position can be obtained if we post DKK 100 to the left on the capital account and to the right on the sales account (a fictive "should have been" sale). In practice this is done differently but it suffices for our purposes. Notice that the sum of all left-hand sides equals the sum of all right-hand sides all the way through.

Pacioli published another work in 1509, dealing with *the golden ratio* (which also was not his own invention, it had been around since Euclid): Two quantities are in the golden ratio ϕ if their ratio is equal to the ratio of their sum to the largest of them, that is if for $a > b > 0$

$$\frac{a}{b} = \frac{a+b}{a} = \phi.$$

This equation can be solved to $\phi = \frac{1+\sqrt{5}}{2} = 1.618....$ The golden ratio turns up in many different contexts – not however in economics, so we leave it there.

2. The mathematics behind double entry bookkeeping

Turning back to book-keeping, we notice that except for closing down the business, no subtraction is necessary. Subtraction was of course well-known also in medieval times, but negative numbers as such came into use only much later, around 1600. Actually, the use of debit-credit accounts is a way of having negative numbers without saying it explicitly: If we have only positive numbers and 0, then we can define new objects of the form

a	b

or simpler written, (a, b). We can add such pairs, and there is an obvious zero object $(0, 0)$.

The crucial next step is now to consider pairs (a, b) and (c, d) as equivalent if $a + d = b + c$ (Intuitively, if a bank account has DKK 100 on the debit side, DKK 70 on the credit side, then it would be no better or worse than an account with debit DKK 120 and credit DKK 90). Then all pairs (a, a) are equivalent to zero, and for any (a, b)

we can find another one, for example (b, a) such that their sum is (equivalent to) zero. A mathematician would say that the (equivalence classes) of all pairs (a, b) constitute a group (every nonzero element has an inverse), called the Pacioli group in Ellerman (1985).

For $a > 0$, the equivalence class of $(a, 0)$ has as inverse the class containing $(0, a)$, which then can be considered as the negative of a; we have thus invented negative numbers. This is of course no sensation, and the introduction of negative numbers through the group of differences was used by mathematicians from the first half of the 19th century and onwards, however without reference to double entry bookkeeping, and it is widely used in contemporary mathematics (but of course in very different contexts) nowadays known as the Grothendijk[2] group.

It may be noticed as a curious fact that the work of Pacioli had a profound influence on the development of mathematics, in particular on the introduction of symbols in mathematical reasoning, see e.g. Heeffer (2009).

3. Triple entry bookkeeping

Once we have introduced double entry bookkeeping, the logical next step would be to extend the system to *triple entry* or perhaps to systems with four, five or arbitrary number of entries. Fortunately for practicians, this did not happen, and serious proposals for a meaningful triple entry have appeared only in recent times.

The triple entry bookkeeping system proposed in Ijiri (1986) takes as point of departure that a posting in a standard double entry system involves an account dealing with *wealth* and another account which refers to *income,* that is a stock and a flow account. Before double entry, one would keep track only of wealth, but adding the income account meant that changes in wealth could be "accounted for". Pursuing this line of reasoning, it would seem useful to add another account which could explain changes in income, allowing the firm to keep track the rate at which its earnings are undergoing changes.

More specifically, Ijiri operates with concepts inspired by classical mechanics, where the momentaneous change in wealth (income in an infinitesimal time interval) is called *momentum,* and where the similar momentaneous change in momentum is *force.* Keeping track of the force over a period gives an assessment of how income will change, with subsequent effects on wealth.

The system proposed by Ijiri seems to have been too complicated to obtain any practical adherence, but triple entry accounting has acquired a new actuality mainly from computer scientists (see e.g. Faccia and Mosteanu (2019), Maiti, Kotliarov and Lipatnikov (2021)). Here the central concept is that of a *distributed ledger:* Instead of

[2] Alexander Grothendijk (1928 – 2014), the founder of modern algebraic geometry, was a colourful personality (mathematics has more of them than economics).

the traditional way of accounting, where the business firm had one single ledger keeping track of all transactions and modified only by one single agent, modern internet based and decentralized systems need an account which is immediately accessible and which can be modified by all agents but which can still be trusted.

An example of a transaction in a distributed system of accounts would go as follows: Assume that *B* borrows DKK 1000 from *A*. Then the DK 1000 are posted on *A*'s book on the debit side as a loan to *B* but his bank account is credited with DKK 1000. At the same time, *B*'s book gets a post on the debit side with 1000 DK to his bank account, and the same amount is posted in the credit side as a debt to *A*. Thus, the transaction, in this case a loan, gives rise to two different types of postings, so it may be considered as a form of double entry accounting (where in each, the postings may or may not have been made using traditional double entry bookkeeping).

This is all very well, and each of the agents may keep their books correctly, but they may also make errors or even outright fraud. Consequently, the books cannot be trusted unless there is a way of transforming the individual books to a public book, and if we are dealing with a distributed system with no central authority, a specific approach is needed, and this where the *blockchain technology* comes into the picture.

Here is a *very* brief explanation of how a blockchain works in BitCoin, see Fig.1. We need the concept of a *hash function,* which is a procedure mapping very long sequences of data into rather short ones in such a way that even small changes in input will result in large changes in output (so that hashing a block of data can be used to track whether it has been manipulated). Finding the input corresponding a given output is very time-consuming. Computer scientists already used hash functions in other contexts.

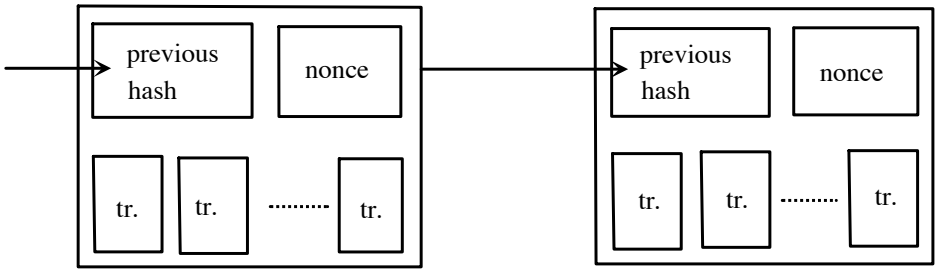

Fig.1. Two blocks in a cryptocurrency blockchain, each consisting of pointer, transactions, and the nonce

A new block is added to the blockchain if any participant in the network collects a number of new transactions together with a pointer to the last block in the chain into potential block. To finish the block, one has to add the *nonce,* which is a sequence of signs, so that when added to the data of the block and hashed gives a preassigned number of zeros. Finding the a nonce is a task which takes up considerable computer time, but the result is easy to check (using the hash function), so the network may verify that the data in the blockchain have not been tampered with.

We cannot do justice to blockchain here, and indeed we are discussing history rather than future of economic theory, so we leave it here. An introduction to blockchain can be found e.g. in Drescher (2017). Curiously, the emergence of cryptocurrencies such as BitCoin based on blockchain technology takes us back to accounting: The detailed registration of all previous transactions is what gives the cryptocurrency its status as *money*. Indeed, as it has been shown by Kocherlakota (1998), all feasible allocations which can be sustained using money could as well be achieved if sufficient memory about previous transactions is available to the agents.

4. The accounting view on money and macroeconomics

Before leaving accounting altogether, we should notice that in recent years there has been a certain revival in using concepts from accounting when treating problems of economics. According to Bezemer (2016), an 'accounting view' is a mode of macroeconomic analysis which explicitly uses accounting definitions and identities. It recognizes that credit is also debt, that flows of a variable gives rise to changes in the stock of this variable, or it uses accounting methods such as decomposing different kinds liabilities or linking flows of liquidity to transactions.

This may sound uncontroversial, but it does conflict with the traditional approaches in many cases, getting closer to what is called heterodoxical schools of thought. One example of this is the theory of money, where the standard (neoclassical) approach is to consider money as arising from transactions, a particular commodity is selected to simplify what otherwise would be a long sequence of barter trades. The accounting view comes closer to what is known as the *chartalist* theory of money (to which we shall return later), seeing money as arising from debt. It so happens that this view is confirmed by archeological findings which confirm that debt money existed long before any money commodity came around.

Bezemer subsumes the accounting view into three distinct directions: The first one is to distinguish between different versions of finance rather than treating it as a simple one, 'credit'. Business loans and mortgages have very different functions in the economy, in particular when one keeps considering the other side of the balance. Also, different types of debts can have very different degree of liquidity, so that there functions as money can differ. The second direction emphasizes the distinction between stocks and flows which is too often neglected, and the third direction deals with the different effects of assets and liabilities. Here an important topic is the growth of the financial sector and the balance (or lack of balance) between the real and financial sectors, as well as the rising income inequality, topics which are not well accounted for by the accepted economic theory.

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