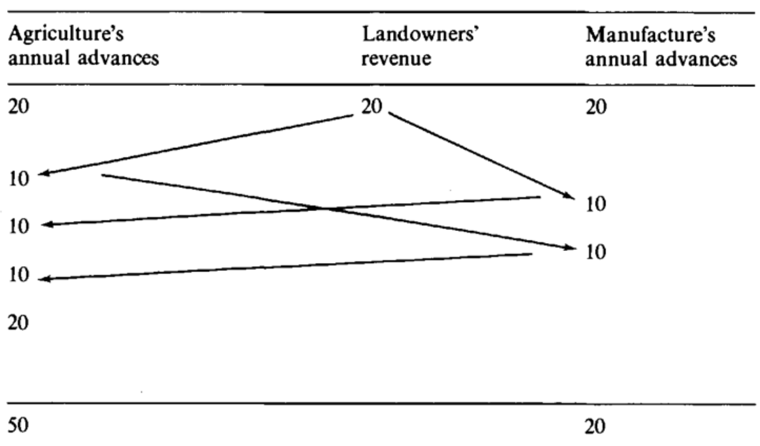
Lecture Note 3

Tableau Economique and other 18th century contributions

1. Quesnay's Tableau Economique in modern terms

As is often the case, contributions of the early thinkers may be difficult to grasp in their original formulation. The Tableau Economique inspired several generations of economists, from Marx to contemporary input-output analysts, and it has been reformulated in several ways. Here is one version, due to Negishi (1989), which is based on an example of the type given by Quesnay and shown in the figure.



What goes on is the following: The agricultural sector produces annually 50 units from input of 20 units of agricultural products (considered as working capital) and 100 units of manufactured products (fixed capital, not seen in the figure), of which 10 must be replaced annually. Moreover, it pays 20 as rent to landowners. Out of these 20 units, landowners spend 10 on agricultural products and 10 on manufactured products. The manufacturing sector produces 20 units of output from the 20 units of input, of which 10 is used for buying agricultural products and the remaining 10 is the replacement of fixed capital in the agricultural sector. Since the agricultural sector gains 20 units of money, they can replace the 20 units of working capital, and everything can be repeated year after year (in our contemporary language, this is a steady state).

In this model, every sector recovers exactly its outlays on producing the goods, except possibly the landowner who receives rent, but even this may be considered as a payment for the services of the land. In any case, no profit is earned anywhere. This has been explained in several ways, either by referring to the feudal agricultural society of France at that time (Quesnay himself pointed to the English way of organizing agriculture using large-scale capitalist farmers as a model to be followed by France). Alternatively contemporary economist might interpret Quesnay's model as a long-run competitive equilibrium where profits have been eliminated.

It would be tempting to rewrite the *tableau* as a linear production economy as used in input-output analysis, so that a_{ij} denotes the input of sector i to one unit of sector j's output, for $i, j \in \{1, 2, 3\}$, giving a table of the form

$$\begin{pmatrix} 0.4 & 0.4 & 0.2 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix}$$

(per unit of output, agriculture uses 0.4 of its own products, 0.4 units of value delivered from landowners, and 0.2 units of manufactured products). This is however not very useful, and the coefficients are not all technical or behavioral constants. Moreover, it doesn't really fit with the way in which Quesnay presented his circulation model. It was not considered as a realistic picture of the economy, rather it was suggested as an ideal situation to be achieved in the future.

To follow this, Negishi reformulates the model as an optimization problem, where we want to maximize the surplus under the given constraints. If this surplus is $\min\{c_1, c_3\}$ with c_i net output of sectors $i = 1, 3$, then the maximization problem can be written as

$$\begin{aligned} &\max \min\{c_1, c_3\} \text{ subject to} \\ &\quad X_1 \leq L, \\ &\quad 2X_1 \leq F, \\ &\quad 0.4X_1 \leq K_1, \\ &\quad c_1 + K_1 + K_3 \leq X_1, \\ &\quad X_3 \leq K_3, \\ &\quad c_3 + 0.1F \leq X_3, \end{aligned}$$

where X_i is production and K_i is initial working capital (to be reproduced) in sector i, L is available amount of land, and F is fixed capital in sector 1. Before analysing the problem, we notice that if $c = \min\{c_1, c_3\}$ then decreasing X_i if necessary we may assume that we maximize $c = c_1 = c_3$. This transforms the problem to linear programming problem with variables $c, X_1, X_3, K_1, K_3, L, F$. We write the matrix of coefficients below, with the objective function above, all inequalities are have variables to the left of the inequality sign. In the leftmost column, we have introduced the dual variables with suggestive notation, namely r (rent), p (price of fixed capital), w_i price of working capital in sector and p_i price of output in sector $i = 1, 3$.

	c	X_1	X_3	K_1	K_3	F	
	1	0	0	0	0	0	
r		1					L
p		2				-1	0
w_1		0.4		-1			0
p_1	1	-1		1	1		0
w_3			1		-1		0
p_3	1		-1			0.1	0

In the dual problem, the coefficients of the constraints are the columns of the matrix, and the right-hand side is given by the first row, with equality when the relevant variable can be assumed nonzero. We then have that

$$r + 2p + 0.4w_1 - p_1 = 0 \tag{1}$$

from the X_1-column, and from the X_3-columns we get that $w_3 - p_3 = 0$ or $w_3 = p_3$, from K_1 we have $w_1 = p_1$, from K_3: $p_1 = w_3$, and from F: $p = 0.1p_3$. Finally, the first column tells us that $1 - p_1 - p_3 = 0$, from which we find that $p_1 = \frac{1}{2}$.

By the fundamental theorem of linear programming, the optimal value of the maximization problem equals the minimal value of the dual problem, so we obtain that

$$c = rL = 0.4p_1X_1 = 0.2X_1,$$

so all the surplus is paid as rent. To see that profit is eliminated in the optimum, we change F to some F' smaller than its optimal value, then also X_1' must be lower. The dual inequality for the F-column tells us that $-p + 0.1p_1 \geq 0$ or $p < 0.1p_1$, so buying more fixed capital and increasing output accordingly is profitable, and this will be the case until the profit is eliminated.

It is seen that in the Quesnay model, we are still far from a theory of income determination, and there is no hint of exploitation in the sense of Marx.

2. Some other contributions of interest

The Milanese School: Beccaria. We have dealt with English and a French schools, actually there was also a lively school of economic thinkers in Italy, which at that time was not yet unified, so these schools have been named according to the city-states.

As we have seen, Economics was not considered as a separate scientific discipline, so most authors were engaged in other matters. This goes also for Cesare Beccaria (1738 – 1794) who has become famous for his work on crime and punishment from 1764, arguing against torture and death penalty. Closer to our field is are some minor works treating the problem of smuggling. The approach here is very modern, not only for proposing the use of algebra in economic reasoning about quantities and for setting up a model the see how the tariffs will influence the volume of smuggling: Given that a proportion of the goods which a merchant tries to smuggle is seized while the remainder passes the border, how should this proportion be if the merchant should break even?

Let u be the value of the goods and x the amount smuggled successfully, and let t be total value of the duties to be paid if there was no smuggling. The amount saved and therefore earned by the smuggler is $\frac{x}{u}t$, and break-even occurs when

$$u - x = \frac{x}{u}t$$

which gives us that

$$x = \frac{u^2}{u+t}.$$

Beccaria then goes on to consider how the size of the tax influences the amount of smuggling. If the tax exceeds the value of the goods, say $t = u + d$ for some $d > 0$, then

$$x = \frac{u^2}{2u+d} < \frac{u}{2}$$

and smuggling becomes more attractive since less than half of the quantity of goods is needed to break even. Similarly, a tax smaller than u then a larger amount of must avoid capture in order to make smuggling worthwhile. In other words, the size of the tax has an impact on the amount of smuggling to be expected.

Beccaria's analysis was followed up by other writers who introduced also the determination of a punishment for smuggling. If the smuggler has a probability p of success, in which case the tax t is saved, then expected profit is pt. If the probability of being caught is P (which typically would equal $1 - p$) and a fine f must be paid, then expected loss is Pf. If $Pf = pt$ or

$$f = \frac{pt}{P},$$

then the gamble (against the authorities) may be considered as just, and consequently, the total payment of tax and fine,

$$S = t + \frac{pt}{P}$$

can be considered as a just punishment for smuggling.

Edmé Mariotte: Another scientist, now from the 1600s, who has gone into history known for very different contributions, is Edmé Mariotte (1620-1684), known for Boyle-Mariotte's law in physics, which actually was proposed by Boyle earlier than by Mariotte. A genuine contribution of Mariotte is the discovery of the blind spot in the eye. All this has of course nothing to do with economics, but in his writings on Moral Sciences he has several principles which can be seen as forerunners of later utility theorists, Indeed, one of the principles say that one 'good' is equal to an 'evil' if, when they are joined together, one is indifferent either to pursue them or to avoid them. This statement sounds surprisingly modern, and the same goes for other of his statements, cf. Theocharis (1983).

Daniel Bernoulli and expected utility. We shall return to Bernouilli much later when discussing Game Theory. His theory of expected utility arose as a solution to the so-called St.Petersburg paradox: What would you pay to participate in a gamble where a coin is tossed repeatedly until the first occurrence of heads, after which you will be paid the amount of 2^n, with n the number of trials. Expected payoff (over possible lengths of the game) is

$$\frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 4 + \cdots + \frac{1}{2^n} \cdot 2^n + \cdots$$

which is infinitely large. Bernoulli's approach to preventing gain from growing beyond limits was to assume that the increase in advantage $\mathrm{d}y$ caused by an increase in gain $\mathrm{d}x$ is inversely proportional to the wealth already existing, so that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}b,$$

with b the proportionality factor. This equation has the solution

$$y = b \ln x + C,$$

and if overall advantage at the initial level a wealth is normalized to 0, we get the formula

$$y(x) = b \ln\left(\frac{x}{a}\right)$$

for Bernoulli's "advantage" function (the version given here is due to Laplace, another famous mathematician of the century).

Another interesting contribution of the probability theorists is a first approach to two-person games, found in a letter from James Waldegrave to Nicolas Bernoulli (a cousin of Daniel) from 1713, where dominated strategies of both players are exhibited, and even mixed strategies and the concept of minimax were introduced, more than two hundred years before game theory emerged. More about this can be found in Dimand and Dimand (1996).

References

Dimand, M.A. and R.W.Dimand (1996), A history of game theory, Volume I, Routledge, London.

Theocharis, R.D. (1983), Early Developments in Mathematical Economics, Macmillan, London.

Negishi, T, History of Economic Theory, North-Holland, 1989.