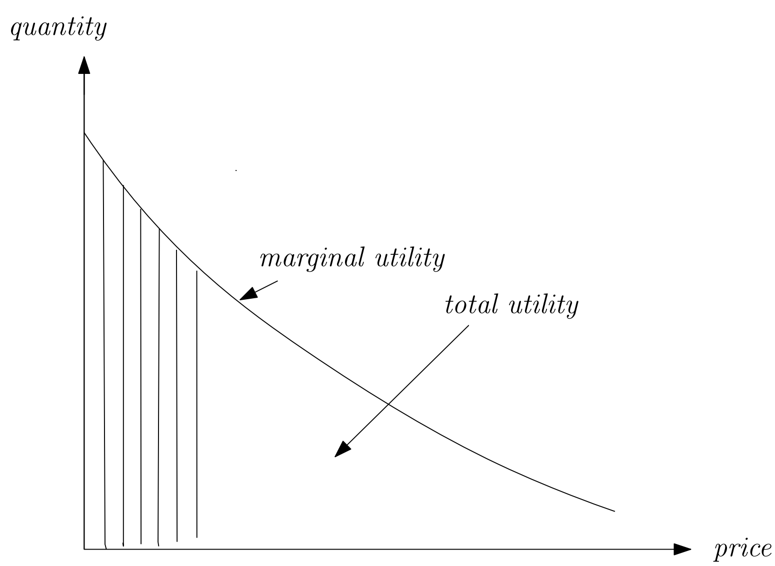
Lecture Note 9

Early marginalists

Jules Dupuit (Arsène Jules Étienne Juvenel Dupuit, 1804 – 66) was born in Piemonte when it was under French control, and the family emigrated to France. He was led to a closer study of demand through the work on public projects, and he is considered as the initiator of both cost-benefit analysis and marginal cost pricing.

Demand was already known, not only from Cournot but also from Rau (see below). The important contribution of Dupuit (in 1844), who didn't know Cournot or Rau, was the connection of demand to *utility*. He first introduced demand as function $y = f(x)$ with y the quantity and x the price, subject to two 'laws' – law 1 says that demand slopes downwards, and law 2 says that a given fall in price induces larger increases in quantity the lower the price, by Dupuit attributed to the distribution of income, since at low prices larger groups of individuals are involved som that the reactions become larger.



Given this demand curve, Dupuit could refute the proposal by Say that the market price of a good measures the utility of each unit consumed. Instead, it must measure the utility of *last* unit. Summing over the successive demand prices as one moves upward, one obtains the area under the demand curve as a measure of the *total utility*, what today is known as the (approximate) consumers' surplus. Dupuit subsequently extended this analysis to public works, noticing that the benefits of such projects could not be measured by their costs, one would need to consider the area under the demand curve.

The analysis of marginal utility could also be applied to taxes. The first result obtained was that a tax results in a loss in consumers' surplus which is larger that the tax revenue, giving rise to a deadweight loss. The second result estimated this loss as the square of the tax rate: If the tax rate is t, then this area can be approximated by the area of a triangle with height t and basis equal to the reduction Δy in demand, which has the size

$$\Delta y = \frac{t}{\alpha},$$

where α is the (numerical) slope of the demand function as we usually draw it, with quantity at the horizontal axis, so that $\frac{1}{\alpha} = f'(x)$. Writing $f'(x) = k$, we get Dupuit's formula

$$\Delta U = kt^2$$

for the deadweight loss.

The third result is the Laffer-curve type relationship between tax rate and tax revenue, which is a consequence of the quadratic deadweight loss which eventually swallows up the revenue. Indeed, if the tax revenue at the price x is $txf(x+t)$, then its derivative is

$$tx^2 f'(x+t) + xf(x+t),$$

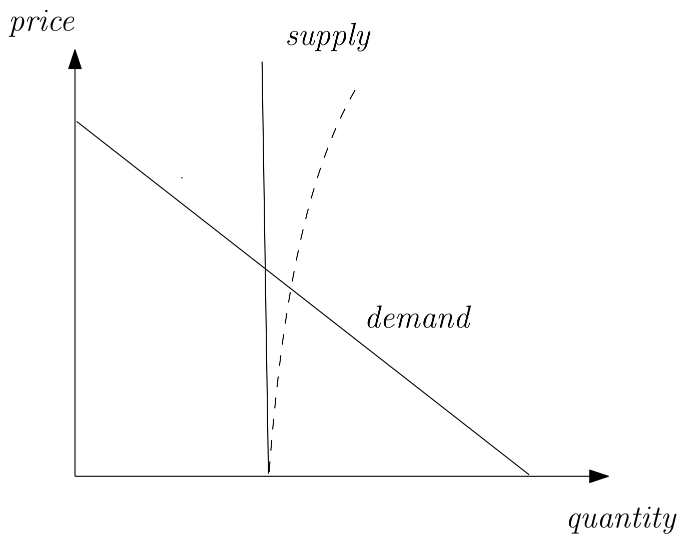
the first term of which is negative by law 1 and increasing (that is numerically decreasing) by law 2, whereas the second term is positive and decreasing (law 2 once more). It has therefore a unique maximum at

$$t^* = -\frac{xf'(x+t)}{f(x+t)}.$$

Still another application of the connection between demand and utility was price discrimination (of the type known nowadays as third-degree, where the consumer groups can be separated), which he considered in connection with designing a ticket system for railways. Dupuit was aware that the possibility of charging different prices might actually increase total welfare when it led to increased output, results which were obtained only in 1920s by other authors.

Karl Heinrich Rau (1792 – 1870) also used demand curves, and he considered stability questions concerning intersection of supply and demand, publishing a note on this topic in 1841, seemingly without knowledge of Cournot's work.

Rau had as his point of departure the work of Canard, determining prices by the fraction of the price latitude going to the seller by want and number of buyers and sellers. Rau found the methods for determining the latitude unsatisfactory and analyzed the situation in diagrams, where the demand curve ("line of demand") determines the highest possible price. Supply is taken as unelastic, but Rau also contemplated the possibility of elastic supply.



The intersection of the demand curve with the vertical axis clearly gives the highest possible price in the market, that is the upper bound of the price latitude.

Apart from price theory, Rau was a pioneer in the economic analysis of location, considering the determination of market areas from product prices and market rates, a topic which was later continued and extended by **Wilhelm Launhardt** (1832 – 1918). Moreover, Rau introduced what he called the *equations of exchange,*

$$u \cdot g = w \cdot p,$$

where g is the amount of money, u its velocity, while w is the quantity of goods in the market and p the price. It can be seen as one of the earliest mathematical formulations of the quantity theory of money, it has later been referred to as the "Rau-Fisher" equation.

Carl Heinrich Hagen (1785 – 1856) considered prices as obtained from needs and availability, in the earlier version from 1822 expressed as

$$P = \frac{B}{V},$$

where P is the price and B (Bedarf) and V (Vorrat) express the need for and the available stock of the good, respectively. We are here still far from the demand and supply curves proposed by later contributors. The explanation of price formation was however not the principal aim of Hagen, as it can be seen from his later work The Necessity of Free Trade (Die Notwendigkeit der Handelsfreiheit) from 1844.

Hagen's point of departure is the notion of *profitability*, expressed as

$$\frac{P-C}{C},$$

where P is price at C is average cost. A change in market conditions may result in changes ΔP in price and ΔC in average cost, and profitability has improved if

$$\frac{P+\Delta P-(C+\Delta C)}{C+\Delta C} > \frac{P-C}{C},$$

which Hagen transforms to

$$\frac{P+\Delta P}{C+\Delta C} > \frac{P}{C}$$

where he uses the implicit assumption $\Delta P/P = \Delta C/C$. In the particular case of interest, namely the opening up of international trade, this criterion may be used to assess the effects.

Hagen is however aware that the overall welfare effects depends on more than profitability. Assuming that demand for the good was D but changes to $D - \Delta D + E$ (where E is the export) after the opening up of trade (price changes give rise to a change in demand), the exporters' gain can be written as

$$\begin{aligned}&((P+\Delta P)-(C+\Delta C))(D-\Delta D+E)-(P-C)D\\&\quad=(\Delta P-\Delta C)(D+E-\Delta D)+(P-C)(E-\Delta D).\end{aligned}$$

However, the non-exporters will have a loss since the exporters use more capital, namely in total $(C+\Delta C)(D-\Delta D+E)$, so that non-exporters lose the profits otherwise earned on this capital, which is

$$\frac{P-C}{C}\left[(C+\Delta C)(E-\Delta D)+\Delta C\right].$$

To this one must add to loss of the consumers who have to pay a higher price, thereby losing

$$\Delta P(D-\Delta D).$$

(from which we see that Hagen understates the welfare loss). The total loss can then be found as

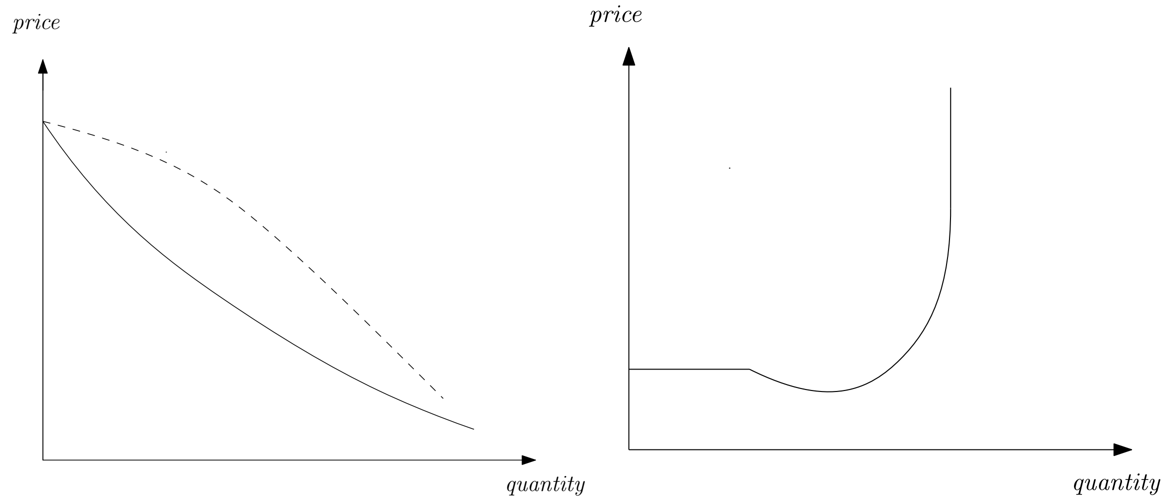
$$\Delta PE - \left(P\frac{\Delta C}{C}\right)(D-\Delta D+E),$$

which may be positive or negative, so that export is not necessarily beneficial for the country. The break-even condition that the loss is 0 can be expressed as

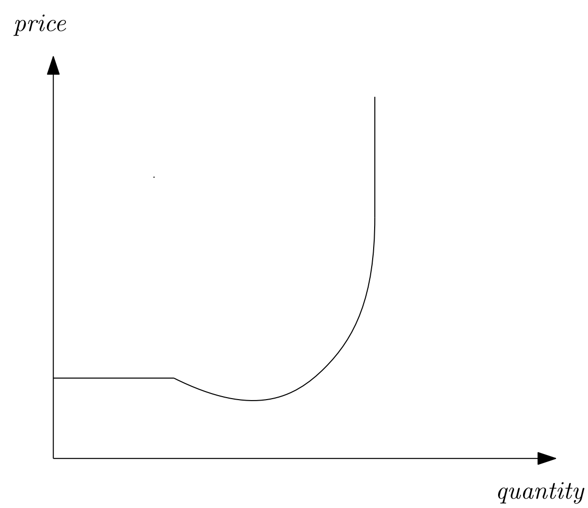
$$\frac{D-\Delta D+E}{E} = \frac{\Delta P/\Delta C}{P/C}.$$

The right-hand side of the expression is formally an elasticity (of the price as dependent on cost), and gain or loss from opening up international trade can be found by comparing relative change of output to this elasticity. There seems to be no connection to later use of elasticities in assessing changes in international trade.

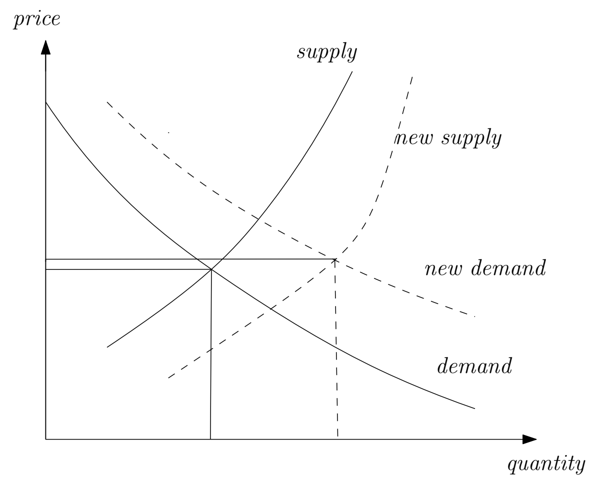
Hans von Mangoldt (1824 – 68) followed up on Rau with an explicit treatment of demand and supply curves in what nowadays is called "the Marshallian cross". In his main work from 1855, revised in 1863, Mangoldt made extensive use of demand curves, acknowledging that they might have many different shapes, among which he also considered what was later known as the Giffen case where demand is backward bending (explained by conspicuous consumption or fear of price changes).



Moreover, he investigated the supply curve in detail, specifying its form as a consequence of assumptions on cost conditions:



The intersection of demand and supply and the resulting equlibrium price was analyzed by Mangoldt in the same way as it is done today, inclusive comparative statics with shifts in the demand and supply curves:



Mangoldt also considered cases of joint production and joint demand, the treatment here was however less straightforward.

In his treatment of the problems of international trade, Mangoldt considered the determination of the equilibrium price resulting from trade between two countries, each specialized in the trade of the good in which it has a comparative advantage. In his (somewhat specific) case, the situation is given by labour input coefficients (thought of as a case where country 2 experiences a technological imporvement in production of good B): The price in international trade must be such that one unit A

	Country 1	Country 2
Good A	p	q
Good B	p	$q/2$

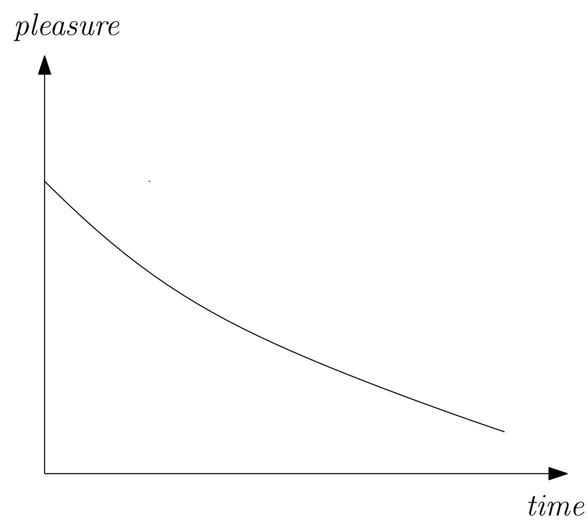
exchanges for between one and two units of B, but in order to find the exact price one must also involve the demand side. Mangoldt assumes unit elasticity (so that the expenditure on each good is independent of price), and if before trade the country 1 used m units of B and country 2 n units of A, then after opening of trade the ratio is such that m units of A is exchanged for $2n$ units of B.

Mangoldt can be considered as decidedly innovative in the use of diagrams, having introduced many of the tools which were later generally adapted by economic theorists. Unfortunately, his contributions were largely ignored at least until rediscovered by Edgeworth around the turn of the century.

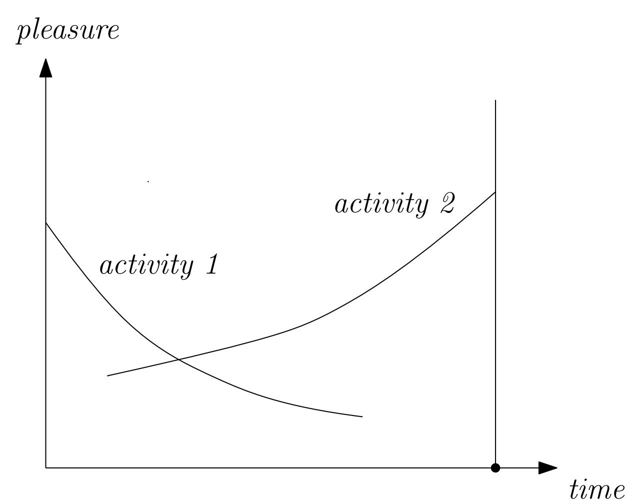
Fleeming Jenkin (1833 – 85) used supply and demand curves extensively in detailed considerations of market equilibria, applied to trade unions and to welfare effects of taxes. His work, published in 1870, contained most of what was later published by Marshall and was later considered as his contributions. Marshall himself admitted that he had seen Jenkin's work in 1870, long before his own work was published, but insisted that he had lectured on the topic as early as 1863. Due to the authority of Marshall, Jenkin was almost totally forgotten.

Hermann Heinrich Gossen (1810 – 58) was similarly neglected by his contemporaries. His main work which appeared in 1854 was rediscovered only much later.

Gossen's main achievement was to introduce the individual into the analysis of demand, introducing a function which gives the subjective enjoyment or pleasure from using time on a particular actitivity, possibly the consumption of a good or a bundle of goods. Gossen notices that the pleasure derived from using one hour on the actitivity falls as the total number of hours increases. This is an instance of what is known as Gossen's first law on diminishing marginal utility.



If the individual can choose between several activities, then the choice will be such that the pleasure of the last unit is the same in each activity:



The analysis can be extended so that the choice is not time but units of a good, and then the principle for choosing the right quantities translates to Gossen's second law, saying that the marginal utility divided by price is the same for all goods, were rediscovered only later but are now fundamental parts of microeconomics.

Even though Gossen did not enter into a explicit analysis of consumer behavior, the analysis goes a long way towards what is in contemporary textbooks on microeconomics.