Solutions to Exercises in
Economics of Banking
Chapter 3

1. Buildings and other real estate properties may figure on the balance as assets, and their value may change, giving rise to losses. The relevant risk factors depend on the way in which real estate is assessed in the balance, but changes in market value must be one of them. The model used in setting up a loss function will then correspond to that used when assessing a portfolio of bonds.

2. The obvious notional measures are the holdings of foreign exchange of different types, as well as the liabilities in foreign currency. Here the underlying risk factors are the exchange rates, so the amount of foreign exchange does not capture the risk connected with holding foreign exchange. A sensitivity measure would give the percentage change of value of balances in a particular foreign currency per percentage change of the relevant exchange rate, and this gives a more useful picture of the risks connected with holding foreign exchange, even if it is not perfect.

3. We need to find the probability distribution of the total loss. The loss on stock A is uniform in $[-100, 100]$, the loss on B is uniform in $[-150, 150]$, and that on C is uniform in $[-50, 50]$ (all in €1000).

The density g of distribution of the sum of losses on B and C, which has support $[-200, 200]$ can be found from the standard convolution formula as

$$g(t) = \int_{x:t-x\in[-150,150]} \frac{1}{100}\,\frac{1}{300}\,\mathrm{d}x$$

and $g(t)$ is symmetric around 0 and goes linearly from 0 to $\frac{1}{300}$ for t going from -200 to -100 and is constant $= \frac{1}{300}$ in $[-100, 0]$. Taking convolution once more with the distribution of losses from A we get a distribution h, again symmetric around 0, and with linear segments, growing from 0 to $\frac{1}{4}\,\frac{1}{300}$ in $[-300, -200]$, from $\frac{1}{4}\,\frac{1}{300}$ to $\frac{3}{4}\,\frac{1}{300}$ in $[-200, -100]$, and from $\frac{3}{4}\,\frac{1}{300}$ to $\frac{1}{300}$ in $[-100, 0]$.

From the distribution h, $\mathrm{Var}_{0.99}$ is found as €250,000.

4. Since the sum of assets exceeds the sum of liabilities, the there must be some liabilities missing from the list, possibly in form of retained earnings to the amount of 30. The *liquidity gap* is the difference between liquid assets and liquid liabilities, which in this case are identified with interest-earning assets and liabilities, so

$$\text{Liquidity gap} = (75 + 35) - (30 + 40) = 40,$$

and the fixed interest gap is similarly found as

$$\text{Fixed interest gap} = 75 - 30 = 45.$$

If the market rate of exhange changes by 1%, then interest earnings on assets increase by $35 * 0.01 = 0.35$, and interest cost increases by $40 * 0.01 = 0.40$, so the total result is a loss of 0.05.

5. Duration matching means that assets and liabilities are composed in such a way that

$$V_A(1 - D_A) = V_L(1 - D_L),$$

where V_j is present value and D_j is duration, for $j = A, L$, where the subscript A denotes assets and L liabilities. Duration is the elasticity of the present value with respect to the repayment rate $(1 + r)$ where r is the market interest rate, and by its very nature, the elasticity measures the effects of a small (infinitesimal) change. Consequently, it cannot capture the effects of major shift in interest rates, where most approximations will be incorrect. Adding the second order effects by computing convexity as well would not be of much help.

An additional source of incorrectness comes from the general approach, where one assumes a flat interest rate structure, so that the elasticity is investigated assuming that the yield curve is shifted up and down rather than being subject to arbitrary changes.

To be safe against interest rate changes of larger magnitude, the bank will have to perform scenario analysis.

6. Writing the investments as (x_1, x_2), where x_i is outcome in state s_i, $i = 1, 2$, we have that an investment is satisfactory if

$$x_j < 0 \Rightarrow x_i \geq 5(-x_j), i \neq j, j = 1, 2,$$

so that the set of satisfactory investments can be written as

$$\{x \mid x_1 + 5x_2 \geq 0\, 5x_1 + x_2 \geq 0\}.$$

This set clearly contains $\mathbb{R}^2_+$ (Axiom 1), it contains no vectors negative in both coordinates (Axiom 2), it is convex as the intersection of two halfspaces (Axiom 3) and it is a cone (Axiom 4), since halfspaces are cones, and intersections of cones are also cones.

Defining the measure of risk as the smallest m such that m times $(10, -1)$ added to the risk becomes satisfactory as an investment, we get that for the point $(-4, 6)$ it is determined by

$$5(-4 + 10m) + (6 - m) = 0$$

or $m = 0.29$. For $(5, -5)$, we need to intersect the other halfspace, so that m is found from $(5 + 10m) + 5(-5 - m) = 0$, so that $m = 4$, and for $(2, -3)$, m is similarly found from $(2 + 10m) + 5(-3 - m) = 0$ to $m = 2.6$.

7. The standard deviation of a risk (a random variable considered as outpayments depending on the uncertain state) fails to satisfy Translation Invariance, since adding a certain (that is non-random) amount as a fixed payment will leave the standard deviation unchanged whereas it should be reduced by this amount to satisfy the axiom. Actually, it fails to satisfy the other axioms as well, only Positive Homogeneity is satisfied since $\sigma(\lambda X) = \lambda\sigma(X)$.