**Solutions to Exercises in**

# Economics of Banking
# Chapter 1

**1.** Intuitively, an additional cost of issuing debt will make it attractive to borrow in banks at a loan rate which is smaller than $r + d$. Assume for simplicity that the cost is paid at $t = 1$.

Consumers have unchanged budget constraints

$$x_0 + b^c + s \leq \omega$$
$$px_1 \leq (1 + r)b^c + (1 + r_D)s + \pi^p + \pi^b,$$

but the producer's profit ( is

$$\pi^p = p(g(b^p + l) - (1 + r + d)b^p - (1 + r_L)l.$$

Similarly, the bank's profit measured at $t = 1$ is

$$\pi^b = (1 + r_L)l - (1 + r + d)b^b - (1 + r_L)s,$$

where again it is assumed that the cost is paid at $t = 0$. In an equilibrium with $r + d > r_L > r_D > r$ no bounds will be issued, all investments are financed through the bank.

**2.** If an individual belongs to a group of $k$ investors choosing $G$, then the expected payoff of an investment of type $G$ is

$$\pi_G \left[ \sum_{j=0}^{k-1} \binom{k-1}{j} \pi_G^{k-1-j} (1 - \pi_G)^j \left( G - \frac{1}{j} \right) \right].$$

provided that all the other individuals in the group choose $G$. Here we sum over the probabilities that $j$ of the group members are unsuccessful, for $j = 0, \dots, k - 1$, and the other individuals will have to pay the debts of the others so far as this is possible. A similar expression can be set up for investments of type $B$,

$$\pi_B \left[ \sum_{j=0}^{k-1} \binom{k-1}{j} \pi_B^{k-1-j} (1 - \pi_B)^j \left( R - \frac{1}{j} \right) \right].$$

Comparing with the situation in 1.3.2, it is seen that the payoff in case of success is reduced considerably, roughly by the mean value of $1/j$ where $j$ has a binomial distribution with parameter $1 - \pi_G$ or $1 - \pi_B$. Since this reduction is larger for $B$, this investment becomes less attractive relative to $G$, so that $G$ may be chosen at repayment rates lower than $R^*$ of section 1.3.2.

**3.** We have here a very simple overlapping generations model: Assuming that all individuals in a generation are equal, we the use notation $(-1, c_{t+1}, c_{t+2})$ to denote investment and consumption (in the case of impatience or patience) for the representative individual in the generation born at date $t$.

The allocation found in the static model gives rise to a steady state allocation $(-1, c_1^*, c_2^*)_{t=1}^{\infty}$ (each generation gets the same) which maximizes expected utility of the representative individual $\pi u(c_{t+1}) + (1-\pi)u(c_{t+2})$ under the constraints

$$\begin{aligned} \pi c_{t+1} &= 1 - I \\ (1-\pi)c_{t+2} &= IR \end{aligned}$$

for $t = 1, \ldots$.

Suppose that there is a bank run at date 2, so that the bank must pay $(1-\pi)c_1^*$ to patient individuals unexpectedly wanting their money at 2 instead of 3. Suppose first that $(1-\pi)c_1^* \leq 1 - I$. Then the bank can pay out $(1-\pi)c_1^*$ from the investment of the newborn and still invest $I$ at $t = 2$. At $t = 3$ and all subsequent periods the bank can pay the impatient using the new investment and invest as usual. It is therefore possible to survive the bank run.

If $(1-\pi)c_1^* > 1 - I$, the investment at $t = 2$ is $< I$, giving rise to problems at $t = 4$, unless the surplus $(1-\pi)(c_2^* - c_1^*)$ is large enough to cover the deficit. If not, the bank cannot cover its obligations in the long run.

**4.** The new investment project may pose a problem to the bank if the payoff is sufficiently large: If

$$\frac{1}{2}u(c_1^*\overline{\rho}) > u(c_2^*),$$

then the original banking contract will be upset by the patient individuals who will withdraw their investment from the bank and use the new investment. The bank will then default, and to prevent this, the banking contract $(c_1^*, c_2^*)$ must be revised to another one $(c_1', c_2')$ with $\pi c_1' = 1 - I$, $(1-\pi)c'_{=}IR$ such that

$$\frac{1}{2}(c_1'\overline{\rho}) \leq u(c_2'),$$

and $c_1' \geq 1$ (to ensure that individuals prefer to use the bank rather than the market solution).

If $\overline{\rho}$ is sufficiently large, then the bank can propose the individuals an investment where only the new investment is used, so that the deposits are used only at $t = 1$ and for the new risky technology. The contract $(c_1'', c_2'')$ should then maximize

$$\pi u(c_1) + (1-\pi)u(c_2)$$

subject to the constraints

$$\begin{aligned} \pi c_1 &= 1 - I, \\ (1-\pi)c_2 &= I\frac{\overline{\rho}}{2}, \end{aligned}$$

so that the bank insures the individuals not only against impatience but also against the investment risk.

**5.** The situation outlined is one which may reasonable by modeled by the moral hazard model in 1.3.2. The general interest rate level is such that investors in the industry considered are induced to choose the risky technology, which on average will invoke losses to the lenders, which therefore do not extend credit to firms in this industry,

**6.** When choosing between $G$ and $B$, the investor must compare expected payoff, choosing $G$ if

$$pi_G(1-q)G \geq \pi_B(1-q)B$$

or equivalently if $\pi_G G \geq \pi_B B$, meaning that the investor will always choose the investment which is the best for society. With our standard assumptions, this means that $G$ will be chosen.

The investments can be financed if expected repayment of funding, $\pi_G qG$, is at least as large as the general funding rate $R$. Assuming that $\pi_G G > R$, this gives an interval for the feasible values of $q$.

**7.** This situation is parallel to what is described in the Diamond-Dybvig model of liquidity insurance. The bank can enter a contract under which the depositor gets a certain (small) interest payment if withdrawal is early and a larger one if it is late, at this will work just as in the contracts described in 1.2. The instability problem arises if depositors become doubtful as to whether they will eventually get their money back and consequently all demand early withdrawal, which will strain the liquidity of the bank.