A simple model of Shadow Banking

In this note, we present a simple model of an economy where shadow banking plays a role, based on Plantin (2015). It shows that shadow banking arises as a way of bypassing capital requirements which represent the society's choice but may not reflect the preferences of the banks.

We consider an economy over 2 periods, $t = 1, 2$. The economy has (only) 4 agents, namely:

(1) A *consumer* who starts in $t = 1$ with a stock W of commodity 1 and a utility function which depends on consumption of commodity 1 and another commodity 2. Commodity 1 always has the price 1, and the consumer perceives the two commodities as equally good.

(2) a *producer* with a technology to produce commodity 2, output is delivered at $t = 2$ but input must be decided upon in $t = 1$, (hence notation N_1). If this decision is revised at $t = 2$, a cost of $\frac{k}{2}(N_2 - N_1)^2$ will be added, otherwise the production cost is c times output. The actual production takes place at $t = 2$, there is no need for a loan.

It is assumed that $c < 1$ (otherwise it was not interesting to produce). But the consumer cannot buy commodity 2 at $t = 2$ and pay at $t = 1$, because one can not trust that the manufacturer will deliver as agreed. Trade must therefore take place at $t = 2$, and the consumer must store commodity 1 for this purpose. This can be done by using one of the two financial agents:

(3) There is a *bank* which has two investment options:

(i) the first one requires a downpayment of I (per unit) at $t = 1$ and pays back at $t = 2$. If the investment is of type G it gives $L + l$, but if it is of type B it gives only L. The type cannot be observed and all agents assume that the investment is G with a certain probability p.

(ii) The second investment option requires a downpayment of x and provides repayment

$$\begin{array}{ll} x + f(x) & \text{with probability } q \\ x & \text{with probability } 1 - q. \end{array}$$

This last investment cannot be used as collateral, so it must be financed entirely by the bank's equity.

(4) In addition to the bank, there is also another financial agent, an trust fond, who can store the commodity so that one unit is returned if one unit has been handed in.

Summing up, what happens in the economy is that the consumer puts commodity 1 in either bank or trust fund (or both) and takes the stock out again at $t = 2$ as W_2. The manufacturer then makes an adjustment to the originally chosen production plan and offers the consumer a trade.

It is quite obvious that the producer will set the price of commodity 2 to 1 (same price as commodity 1) and adjust output from N_1 to W_2 so that profit is the result of production minus adjustment costs,

$$(1-c)W_2 - \frac{k}{2}(W_2 - N_1)^2.$$

Since the desired output W_2 depends on how the consumer placed the money at disposal at $t = 1$, it will appear to the manufacturer as a random variable, $\widetilde{W}_2$, and the producer will choose N_1 as its mean value (it is assumed that all agents are risk neutral). The expected profit then becomes

$$(1-c)\mathsf{E}\widetilde{W}_2 - \frac{k}{2}\mathsf{E}(\widetilde{W}_2 - \mathsf{E}\widetilde{W}_2)^2 = (1-c)\mathsf{E}\widetilde{W}_2 - \frac{k}{2}\mathrm{Var}(\widetilde{W}_2).$$

Until now, there has been no financial activity, let alone shadow banking. It comes here:

As the system works, the consumer will never get more at $t = 2$ than W_2. If more than W_1 is to be obtained, this must happen using the financial sector.

The bank offers a security which is based on an investment portfolio with two parameters μ and λ, so that the repayment is μL if the portfolio is B and $\mu L + \lambda l$ if it is G (so that the holder of the security receives a share λ of the fixed gain and a share μ of the additional gain on the investment). For the consumer who assumes that the investment is G with probability p, this paper is worth its average return, which is $\mu L + p\lambda I$, and consequently this is what the consumer pays. The bank gets the difference between this and I, which was the amount to be invested, and this difference is subsequently used to invest in the other technology (where only the bank's own money can be used).

The consumer puts the remainder into the trust fund, and this remainder $W - \mu L - \lambda pl$ is transferred to $t = 2$, where it together with the investment return amounts to $\widetilde{W}_2$ given by

$$\begin{array}{ll} W - \mu L - \lambda pl + \mu L + \lambda l = W + (1-p)\lambda l & \text{with probability } p, \\ W - \mu L - \lambda pl + \mu L = W - p\lambda l & \text{with probability } 1-p, \end{array}$$

and its mean value is $p(W + (1-p)\lambda l) + (1-p)[W - p\lambda l] = W$, while the variance is $p(1-p)\lambda^2 l^2$.

The bank receives $\mu L + \lambda pl$ from the consumer and receives $(1-\mu)L$ (if B) or $(1-\mu)L + (1-\lambda)l$ (if G) from the investment, total $L + pl$, but has laid out I. In addition, the return from the second investment, so in total the average profit

$$L + pl - I + qf(\mu L + \lambda pl - I).$$

It is seen that it is increasing in μ and λ, which is therefore set to 1 (f is an increasing function).

One can see μ and λ as an expression of how much of the investment is left by the bank for the consumers to finance, so that they give an expression of the leverage of the bank's investment. We also have that if there is no regulation, the leverage will be maximally high.

A *regulation* of the bank is now introduced, whereby the regulator can set μ and λ. The regulation is assumed to be made so as to maximize agents' total expected payoff. Since the system works in such a way that the consumer always ends up with a net gain of 0 (the producer sets prices so that the entire endowment of $t = 2$ is used to pay for the consumption), and the trust fund also has no gains, this sum is just the expected income of the bank and the producer, that is

$$L + pl - I + qf(\mu L + \lambda pl - I) + W(1 - c) - \frac{k}{2}p(1 - p)\lambda^2 l^2.$$

This expression is increasing in μ, so that the optimal value of μ must be 1. The optimal λ^* is found by the first order condition

$$qf'(\lambda pl + L - I)pl - kp(1 - p)\lambda l^2 = 0$$

In other words, the regulation consists only in fixing λ, and this can be done indirectly, as an upper limit on the consumer's share of the investment at $\mu = 1$, namely

$$\frac{L + \lambda^* pl}{L + pl},$$

ensures that the portfolio's λ does not exceed the optimal λ^*, and as the bank's earnings are increasing in λ, it will be exactly equal to that.

The result is therefore that by introducing capital regulation one can achieve a welfare improvement in this economy.

Until now, the trust fund has been completely passive, but now we allow it to take a more active role, as it can buy a share in the portfolio (as the consumer cannot buy more than corresponding to the capital constraint). If the price is < 1 a type G bank without positive other investment will not be interested, so the probability that the bank will sell at all is $1 - p + pq$ (the bank is B or it is G with positive other investment), and for the expected value of the investment to be ok for the trust fund, the price must correspond to the probability of getting the money back given that the bank will sell,

$$r = \frac{pq}{1 - p + pq}.$$

The bank then chooses how much it wants to sell, namely λ', by maximizing the expected income, which as before consists of income from the sale of securities less expense for the investment plus income from the other investment based on the profit from the first one, plus income from own holdings of securities in the first investment, in total

$$(p\lambda + r\lambda')l + L - I + f((p\lambda + r\lambda')l + L - I) + (1 - \lambda - \lambda')l,$$

The first order condition with respect to λ' becomes

$$rl + f'((p\lambda + r\lambda')l + L - I)rl - l = 0,$$

or

$$f'((p\lambda + r\lambda')l + L - I = \frac{1 - r}{r}.$$

Here one may insert the formula for r so that the right-hand side becomes $\dfrac{1-p}{pq}$, and introducing the notation φ for the inverse of f', one gets the expression

$$(p\lambda + r\lambda')l + L - I = \varphi\left(\frac{1 - p}{pq}\right). \tag{1}$$

With this form of shadow banking, an extra element of uncertainty has come in: the investment of the trust fund is based on the event that a certain type of bank does not show up at all, so that all investment is based in the remaining types, and in this way the second investment and its uncertainty also becomes a part of the basis for the security.

With the expression (1) in hand, we return to the regulator. For the trust fund's share of the portfolio - which can be interpreted as the extent of shadow banking - to be between 0 and $1 - \lambda$ (the remaining portfolio after consumers have received their share), it must be the case that λ, that share that regulator allows the bank to sell to consumers, must lie between the limits

$$\overline{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right) + I - L}{pl} \quad \text{and} \quad \widehat{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right) + I - L - rl}{(p - r)l},$$

which are obtained by inserting $\lambda' = 0$ and $\lambda' = 1 - \lambda$ in (1). It is seen that $\overline{\lambda}$ is the larger of the two (in $\widehat{\lambda}$ the same amount has been subtracted from the denominator and numerator in the fraction, and if the fraction is ≤ 1 it becomes smaller).

We thus have that if $\lambda \geq \overline{\lambda}$, then no shadow banking occurs at all. Therefore, we assume in the following that $\overline{\lambda} < 1$.

We now return to the welfare considerations. As before, we have that the consumer and the trust fund have no welfare gain, so it is still just the sum of the bank's and the producer's expected profit that counts. But the producer's profit is affected by more uncertainty (the variance on the consumer's period-2 inventory increases), and it gives greater cost, how much this weighs depends on k. It can however be seen immediately that the optimal regulation of λ will not be one where shadow banking has a scope greater than 0 and less than the maximum: If λ is in the range $\widetilde{\lambda} > \lambda > \overline{\lambda}$, and we let λ grow, then the bank will be better off because the consumer pays a better price and the producer will be better because there is a little less uncertainty, so the variance decreases. Thus, the optimal value for the regulator of λ must either be $< \widetilde{\lambda}$, and shadow banking takes over the rest, or $\lambda \geq \overline{\lambda}$ and shadow banking is not used at all.

If in this situation it holds that λ^* (optimum before shadow banking came on the scene) is $\geq \overline{\lambda}$, then it is still optimum. But if $\lambda^* < \overline{\lambda}$, the old optimum will have to be revised.

Plantin, G. (2015), Shadow banking and bank capital regulation, Review of Financial Studies 28, 146 – 175.