Lecture 4:
Measuring risk; Interest rate risk

We almost finished the discussion of the general loss model for risk assessment, leaving only a few comments, and we start there, using a few minutes recapitulating the model and finishing it. Then we move on:

We begin with the general model of measuring losses subject to risk, which was commented upon in the previous handout. This model gives us an expression for the probability distribution of losses. But for use in day-to-day risk management, one needs a simple, yet trustworthy measure of risk. Therefore, we consider in section 2 several different ways of measuring risk, under the condition that the result must take the form of a single or a few numbers which can be communicated easily to the decision makers.

Traditionally one distinguish between several forms of such risk measures: There are the *notional* measures which focus on the size of particular assets or liabilities subject to risk. Further there are *sensitivity measures,* which show how much the aggregate value is changed after a small change in selected risk factors, using what economists know already as elasticities. As a third type we have measures derived from the loss distribution, among which in particular Value at Risk, which has been used very widely for several decades. In recent years, it is slowly being replaced by Expected Tail Loss which gives more better information about possible large losses. Finally, the scenario-bases measures, known from the occasional stress-tests of financial institutions, consider worst-case results of selected changes in risk factors. Read the part dealing with VaR and ETL thoroughly since we use it repeatedly, the remaining parts may be read more superficially.

We now consider a particular type of risk, namely *interest rate risk,* which may be considered as the simplest case. Interest risk arises from the changes in the market rates of interest, assuming that the underlying assets or liabilities are not subject to risk (such risks are taken care of as market and credit risk). Financial institutions are subject to risk if their assets or liabilities have variable rate of interest (changing with the market rate) and also if the assets or liabilities with fixed interest reach maturity and have to be renewed at the market interest rate prevailing at that time. Changes in the interest rate play a crucial role for the profitableness of asset management, and it is therefore important to have tools for monitoring and controlling this risk. Banks have a different exposure depending on whether they are net borrowers or net lenders.

A very first step in measuring interest rate risk is the so-called gap analysis which is too straightforward to merit attention now. A more interesting yet simple measure of exposure is the *duration* of an asset or a liability. On the face of it, duration is just another formula, slightly different from that of present value, but it has an interesting application. First of all, it measures the sensitivity of the portfolio to interest rate changes which already is something. But secondly, it can be used to provide a simple tool for *immunization* against interest rate fluctuation, the so-called *duration matching.* If the durations of the asset and the liability side balance, when computed in the proper way, then the bank cannot be hurt by movements of the interest rate. Unfortunately, the criterion, though useful as a first test, is insufficient, since it works only when all interest rate structure moves up and down by a given amount ("parallel shift in interest rates") and only for small changes. More sophisticated risk management must involve other and less simple methods (simulations etc.).

The last section in Chapter 3 (which we do not read) deals with what is called *coherent risk measures*, and it is there to show that although risk management is a field with many practical applications, there is also theoretical research. You may (or may not) have a quick glance at it, it looks very abstract (and yes, it is very abstract) but it is a field where interesting research is going on, so it may be useful to know what is behind.

If time permits, we proceed to out next topic, which takes us back to the microeconomics of banking, namely a discussion of *loan contracts.* On the face of it, there is nothing to discuss – a contract just stipulates how much should be paid back and when. But things are as always more complicated – what if the borrower cannot pay back? One could argue that this possibility of defaulting on the repayment is taken care of straightforwardly by standard rules – if the borrower cannot pay the full sum we let him pay what he has. But at this point we have implicitly assumed that what the borrower has is observable, which may not be the case. Once again, asymmetric information complicates the situation, and we have to consider several different such cases.

Before doing so, we look closer at the ideal case where there are no complications in the form of asymmetric information, it may be considered as an ideal with which the less perfect reality should be compared. This is classical economics, actually economics of insurance, dealing with characterizing an efficient insurance contract. Interestingly, the *economics* of insurance, not the *mathematics* of insurance, is a rather new field, dating back only to the 50es of the last century. The results of the section are due to Arrow and emerged in connection with considerations of health insurance contracts. You may skip the details of the proof which is anyway not very complicated, and go directly to the result.

What matters here is how to interpret this result: It tells us that the slope of the repayment function depends on the second derivatives of the utility functions of borrower and lender, respectively. This second derivative (which is negative for a risk averse individual) expresses the attitude towards risk – the more risk averse, the larger numerical value. The particular case where it is zero occurs when the individual is risk neutral, and this could happen if the lender is a bank with a large number of different borrowers, each subject to a particular, independent risk. In this case the slope is 1, meaning that if outcome for the borrower increases by some amount, the repayment increases by the same amount, in other words, the borrower delivers everything to the lender except possibly for a constant sum which is independent of the outcome. If the lender is risk averse as well, the contract is one of risk-sharing where any increase in outcome is divided between borrower and lender in a way which depends on the degree of risk aversion.

We read:

Chapter 3, Sections 2 and 3, possibly also Chapter 5, sections 1 and 2.