Lecture 4:
Measuring risk; Interest rate risk

This general model of risk measurement givses us an expression for the probability distribution of losses. But for use in day-to-day risk management, one needs a simple, yet trustworthy measure of risk. There are the *notional* measures which focus on the size of particular assets or liabilities subject to risk. Further there are *sensitivity measures,* which show how much the aggregate value is changed after a small change in selected risk factors, using what economists know already as elasticities. As a third type we have measures derived from the loss distribution, among which in particular Value at Risk, which has been used very widely for several decades. In recent years, it is slowly being replaced by Expected Tail Loss which gives more better information about possible large losses. Finally, the scenario-bases measures, known from the occasional stress-tests of financial institutions, consider worst-case results of selected changes in risk factors. Read the part dealing with VaR and ETL thoroughly since we use it repeatedly, the remaining parts may be read more superficially.

We now consider a particular type of risk, namely *interest rate risk,* which may be considered as the simplest case. Interest risk arises from the changes in the market rates of interest, assuming that the underlying assets or liabilities are not subject to risk (such risks are taken care of as market and credit risk).

A very first step in measuring interest rate risk is the so-called gap analysis which is too straightforward to merit attention now. A more interesting yet simple measure of exposure is the *duration* of an asset or a liability, which can be used as a tool for *immunization* against interest rate fluctuation through the so-called *duration matching.*

The last section in Chapter 3 (which we do not read) deals with what is called *coherent risk measures.*

If time permits, we proceed to out next topic, which takes us back to the micro-economics of banking, namely a discussion of *loan contracts.* On the face of it, there is nothing to discuss – a contract just stipulates how much should be paid back and when. But things are as always more complicated – what if the borrower cannot pay back? One could argue that this possibility of defaulting on the repayment is taken care of straightforwardly by standard rules – if the borrower cannot pay the full sum we let him pay what he has. But at this point we have implicitly assumed that what the borrower has is observable, which may not be the case. Once again, asymmetric information complicates the situation, and we have to consider several different such cases.

Before doing so, we look closer at the ideal case where there are no complications in the form of asymmetric information, it may be considered as an ideal with which the less perfect reality should be compared. This is classical economics, actually economics of insurance, dealing with characterizing an efficient insurance contract. Interestingly, the *economics* of insurance, not the *mathematics* of insurance, is a rather new field, dating back only to the 50es of the last century. It tells us that the slope of the repayment function depends on the second derivatives of the utility functions of borrower and lender, respectively. This second derivative (which is negative for a risk averse individual) expresses the attitude towards risk – the more risk averse, the larger numerical value. The particular case where it is zero occurs when the individual is risk neutral, and this could happen if the lender is a bank with a large number of different borrowers, each subject to a particular, independent risk. In this case the slope is 1, meaning that if outcome for the borrower increases by some amount, the repayment increases by the same amount, in other words, the borrower delivers everything to the lender except possibly for a constant sum which is independent of the outcome. If the lender is risk averse as well, the contract is one of risk-sharing where any increase in outcome is divided between borrower and lender in a way which depends on the degree of risk aversion.

We read:

Chapter 3, Sections 2 and 3, and (if time permits) Chapter 5, sections 1 and 2.