Economics of Banking
Lecture 10

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Reduced form models

Simplest version: Default follows a Poisson process

Loan with maturity T repayment F,

default intensity λ (probability of default in $\triangle t$ is size $\lambda \triangle t$):

Expected present value of the loan is

$$e^{-\lambda T} F e^{-rT} = F e^{-(r+\lambda)T},$$

Riskiness can be assessed using the intensity as a *default spread.*

Structural models

Value of loan is derived from what happens to the borrower

The options approach: A loan can be seen as consisting of

(1) purchase of the assets of the firm,

(2) an option for the firm to buy back its assets at a price equal to the repayment.

Loan F to be paid back at T:

if value $V_T < F$, then borrower leaves V_T to lender

if $V_T \geq F$, then borrower pays back

Using Black-Scholes

Assume that value of assets V follows a geometric Brownian motion

$$dV_t = \mu V_t \, dt + \sigma V_t \, dZ_t$$

Value of option at at time t is

$$E_t(V, T, \sigma, r, F) = V_t N(d_1) - Fe^{-r(T-t)} N(d_2), \text{ where}$$

$$d_1 = \frac{\ln\left(\frac{V_t}{F}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = d_1 - \sigma\sqrt{T-t},$$

(with $N(\cdot)$ the standard normal pdf)

Valuation of debt

From this we get

$$D_t(V_t, T) = V_t - E_t(V, T, \sigma, r, F) = V_t - V_t N(d_1) + Fe^{-r(T-t)} N(d_2)$$
$$= V_t N(-d_1) + Fe^{-r(T-t)} N(d_2) = V_t \left[N(-d_1) + \rho_t N(d_2) \right].$$

where $\rho_t = \frac{Fe^{-r(T-t)}}{V_t}$ is known as the *quasi-debt ratio.*

Default spread

Define yield-to-maturity $y_t(T)$ by

$$D_t(V,T) = Fe^{-y_t(T)(T-t)}.$$

Take logarithms and rearrange:

$$y_t(T) = -\frac{1}{T-t}\ln\left(\frac{D_t}{F}\right),$$

then *default spread* $s_t(T) = y_t(T) - r$ is

$$s_t(T) = -\frac{1}{T-t}\ln\left(N(d_2) + \frac{V_t}{Fe^{-r(T-t)}}N(-d_1)\right).$$

Here $V_t/Fe^{-r(T-t)} = \rho_t^{-1}$ is known as the *expected relative distance to loss.*

Thus credit spread depends on volatility $\sigma\sqrt{T-t}$ and the quasi-debt ratio.

Capital regulation and credit risk

Two alternative methods of assessing credit risk:

(i) The standardized approach: Fixed weights for types of loans

(ii) Internal ratings method (IRB): Bank must assess

probability of default (PD),

exposure at default (EAD),

loss given default (LGD),

maturity (M).

There are two versions of IRB:

(a) Foundational: only PD is calculated by bank

(b) Advanced: all is calculated by bank

CreditMetrics

Basic tool: A *transition matrix* for credit ratings:

	Ratings one period later		
	1	...	k
Ratings now			
1	p_{11}	...	p_{1k}
⋮	⋮	...	⋮
k	p_{k1}	...	p_{kk}

Given the observed rating we may find a probability distribution of values after several rounds

The distribution can be used to find mean, variance, VaR.

The KMV methodology

Based on the Merton approach, but value of firm not observed.

Value of equity E is a call option on the assets of the firm, so that

$$E = f(V, \sigma, K_l, T, r)$$

with K_l nominal value of liabilities. Using market data for E, this may be solved for V and σ. This is used to find a critical value K of assets where the firm defaults. Then define *distance-to-default* as

$$DD = \frac{\mathrm{E}[V_1] - K}{\sigma V_0}$$

Final step: Link DDs to historical default rates to get EDF (expected default frequency).

CreditRisk

Reduced model, but with stochastic default rates μ.

For each engagement i:

- Probability of default P_i
- Loss given default L_i
- Expected loss $\lambda_i = L_i P_i$

Collect all engagements with the same L_i, λ_i, μ_i.

Gives an expression for the (Poisson) probability of losses.

CreditPortfolioView

Default probabilities for a debtor in industry or country j at time t are given by

$$P_{j,t} = \frac{1}{1 + e^{Y_{j,t}}},$$

$Y_{j,t}$ country specific index:

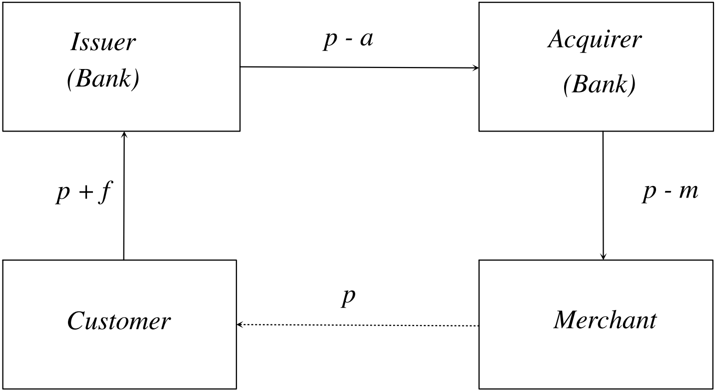
$$Y_{j,t} = \beta_j^0 + \begin{pmatrix} \beta_j^1 & \beta_j^2 & \cdots & \beta_j^m \end{pmatrix} \begin{pmatrix} X_{j,t}^1 \\ X_{j,t}^2 \\ \vdots \\ X_{j,t}^m \end{pmatrix} + v_{j,t},$$

$X_{j,t}$ macrovariables, $v_{j,t}$ error term.

Payments

- (Payments and money)
- (Payments between banks, RTGS system)
- Payment cards
- Cryptocurrencies

The payment card model



Details

(a) Customers. Benefit b_B of card use, $\underline{b}_B \leq b_B \leq \overline{b}_B$ with density $h(b_B)$.
Fee f, $D(f) = \int_f^{\overline{b}_B} h(b_B)\, db_B$ use card, average benefit

$$\beta(f) = \frac{\int_f^{\overline{b}_B} b_B h(b_B)\, db_B}{D(f)},$$

(b) Issuers. Some kind of oligopolistic market, fee is set to

$$f = f^*(c_I - a).$$

f^* is increasing, greater a results in lower f.

(c) Acquirers. Perfectly competition, $m = a + c_A$,
c_A cost of transferring money.

(d) Merchants. Two merchants with cost d,
Buyers in the interval $[0, 1]$ (all buying 1 unit), firms in 0 and 1, transportation cost t.

Equilibrium

Let

$$m^n(a) = m - b_S = c_A + a - b_S, \text{ (net cost to merchant)}$$

and the specific value $\overline{a}$ of a by

$$\beta(f^*(c_I - \overline{a}) = m^n(\overline{a}).$$

If $a < \overline{a}$, then average buyer satisfaction exceeds the net cost to the merchant.

THEOREM *There is an equilibrium where both merchants accept cards if and only if* $a \leq \overline{a}$.

Outline of proof

If both take cards, then the symmetric equilibrium price is

$$p^* = [d + D(f^*(c_I - a))m^n(a)] + t,$$

total profit t, each has profit $\frac{t}{2}$.
If merchant 1 changes to *not* taking cards, prices become

$$\begin{aligned} p_1 &= t + d - \frac{1}{3}D(f)[\beta(f) - m^n(a)], \\ p_2 &= t + d + \frac{1}{3}D(f)[\beta(f) + 2m^n(a)]. \end{aligned}$$

Profit of merchant 1 satisfies

$$\frac{1}{2} - \frac{D(f)[\beta(f) - m^n(a)]}{6t} \leq \frac{t}{2}$$

exactly when $\beta(f) \geq m^n(a)$!

Welfare considerations

Maximize social surplus

$$W(f) = [\beta(f) + b_S - c_I - c_A]D(f) = \int_f^{\overline{b}_B} [b_B + b_S - c_I - c_A]h(b_B)\, db_B$$

First order condition is

$$W'(f) = -[f + b_S - c_I - c_A]h(f) = 0.$$

How to achieve optimum?

Two cases to consider:

(i) $f(c_I - \overline{a}) \leq c_I + c_A - b_S$: Reduce a from $\overline{a}$, f will increase.

(ii) $f(c_I - \overline{a}) > c_I + c_A - b_S$: a should be $> \overline{a}$, but then merchants cease to accept cards.